Gaps in Protoplanetary Disks as Signatures of Planets

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ABSTRACT

We examine the observational consequences of partial gaps being opened by planets in protoplanetary disks. We model the disk using a static $\alpha$-disk model with detailed radiative transfer, parametrizing the shape and size of the partially cleared gaps based on the results of hydrodynamic simulations. Shadowing and illumination by stellar irradiation at the surface of the gap leads to increased contrast as the gap trough is deepened by shadowing and cooling and the far gap wall is puffed up by illumination and heating. We present a formula to derive the scale height and inclination angle of an imaged disk using simple geometric arguments. A gap produced by a 200 $M_\oplus$ planet at 10 AU can lower/raise the midplane temperature of the disk by up to $\sim 30\%$ by shadowing in the gap trough and illumination on the far shoulder of the gap. At the distance of Taurus, this gap would be resolvable with $\sim 0.01''$ angular resolution. The gap contrast is most significant in scattered light and at thermal continuum wavelengths characteristic of the surface temperature, reducing or raising the surface brightness by up to order of magnitude. Since gaps sizes are correlated to planet mass, this is a promising way of finding and determining the masses of planets embedded in protoplanetary disks. Gap are currently being observed in resolved images of disks, such as the transitional disk in LkCa 15. Applying our models, we put a lower limit 0.5 Jupiter masses on a planet in its inner cavity.

Subject headings: planetary systems: formation — planetary systems: protoplanetary disks — radiative transfer — stars: individual (LkCa 15)

1. Introduction

With the advent of adaptive optics systems on large ground-based optical and near-IR telescopes, we are beginning to be able to image protoplanetary disks around young stars
(e.g. Thalmann et al. 2010, LkCa 15). These gas-rich disks are where we expect giant planets to form, since gas giants need that large reservoir of gas from which to accrete their massive envelopes. However, the interpretation of structure in these imaged disks can be problematic, because scattered light traces only the optically thin and diffuse surface layers of the disks rather than the overall structure of the disk (Jang-Condell & Boss 2007). On the other hand, if growing planets do indeed significantly perturb the disks in which they are born, observation of these signatures can mean the detection of nascent planets in protoplanetary disks.

Jang-Condell (2008, henceforth Paper I) calculated shape of a dimple created by the gravitational potential of a planet embedded in a disk and the accompanying thermal perturbations in shadowed and illuminated regions of the dimple. (Jang-Condell 2009, henceforth Paper II) predicted the observability of these dimples. Neither of the papers considered the clearing of an annular gap in the disk by tidal forces. Planets above about 30 M⊕ are able to open partial gaps in disks, as demonstrated in numerical hydrodynamic simulations (e.g. Bate et al. 2003). These gaps are much larger in scale than the local dimples modeled in Papers I and II, and are therefore a promising way of detecting and characterizing planets in protoplanetary disks. If the gap is well resolved enough to determine its depth and width, then we can determine the mass of the planet to within a factors of a few.

Numerous people have carried out hydrodynamic simulations of gap opening in disks by planets (e.g. Paardekooper & Papaloizou 2008; Ayliffe & Bate 2009; de Val-Borro et al. 2006; Bate et al. 2003; Edgar & Quillen 2008) However, the effects of illumination of these gaps by the central star has not been well-studied. Wolf et al. (2002) examine this problem in a flat disk with constant $H/r = 0.05$ and find that the far edge of the gap is not illuminated. On the other hand, Varnière et al. (2006) find that the far edge in a flared disk, which is more typical of T Tauri disks, is heated and puffed up, creating a positive feedback loop that enhances the appearance of the gap. In both these cases, the gap was created by a Jupiter-mass planet, which clears nearly all the material from its orbital, creating a deep and wide gap. In this work, we focus on partially cleared gaps created by sub-Jupiter mass planets.

A full three-dimensional hydrodynamic simulation including radiative transfer is very computationally intensive, so we simplify the problem by assuming a fixed disk structure in terms of its radial surface density profile, $\Sigma(r)$. To further simplify the computation, we assume an axisymmetric disk structure. This means that we do not include the behavior of spiral density waves, but only the large scale gap opened by the planet.
2. Method

The disk model incorporates radiative transfer based on the methods of Jang-Condell & Sasselov (2003) and Jang-Condell & Sasselov (2004) and adopted to self-consistently determine the disk’s thermal and pressure structure as described in Paper I. Stellar irradiation is an important heating source in protoplanetary disks, and the amount of heating is sensitive to the angle of incidence at the disk surface. If the shape of the surface of the disk changes on scales smaller than the disk thickness, the plane-parallel approximation fails. To account for this kind of disk structure, radiative heating is integrated piecewise over the surface of the disk rather than assuming a thin plane-parallel disk model. This method of calculating radiative transfer is fully-three dimensional and not directionally dependent on a fixed coordinate system. For a detailed explanation of the method, see Jang-Condell & Sasselov (2003) and Jang-Condell & Sasselov (2004). Calculation of the observables is done as detailed in Paper II.

2.1. Initial Conditions

We follow the same method as used in Papers I and II to calculate the initial disk structure. The stellar parameters are mass $M_\ast = 1 M_\odot$, radius $R_\ast = 2.6 R_\odot$, and effective temperature $T_{\text{eff}} = 4280$ K, consistent with a protostar with an age of 1 Myr (Siess et al. 2000). We assume a constant-$\alpha$ disk model where the viscosity is parameterized as $\nu = \alpha_{\text{ss}} c_s H$ (Shakura & Sunyaev 1973). The disk parameters are accretion rate $\dot{M} = 10^{-8} M_\odot \, \text{yr}^{-1}$ and viscosity parameter $\alpha_{\text{ss}} = 0.01$, parameters typical for T Tauri stars.

The initial conditions are calculated in a two-step process. First, we generate a locally plane-parallel model for the entire disk. Then, we select a radially and azimuthally limited wedge of this disk to calculate in more detail, including the three-dimensional curvature of the wedge, but assuming axisymmetry. In both steps, we iteratively calculate the density and temperature structure of the disk, including radiative transfer of the stellar irradiation at the disk surface. Viscous heating is included, but is only important inwards of a few AU. Beyond this distance, the primary heating source is stellar irradiation, which depends sensitively on the angle of incidence at the surface of the disk. The surface is defined to be where the Planck-averaged optical depth to stellar irradiation is $2/3$.

For the initial locally plane-parallel model we calculate the disk at logarithmically spaced intervals of $\sqrt{2}$ from 0.25 AU to 256 AU. From this initial overall disk structure, we select a slice with radial range ($r$) from 3 to 20 AU and vertical distance from the midplane ($z$) from 0 to 4.6 AU, in order to model the gap created by a planet at 10 AU. We sample
the initial density and temperature at 100 grid points in both $r$ and $z$ directions, using cylindrical coordinates. In order to ensure that this initial disk is in thermal and hydrostatic equilibrium, we recalculate the heating from stellar irradiation, now removing the assumption that the disk is locally plane parallel and including the full three-dimensional curvature of the disk. The resulting density profile is $\Sigma_0(r)$.

2.2. Gap Modeling

The depth of a gap that can be opened by a planet depends not only on the mass ratio between planet and star, but also on the disk properties. In a low viscosity disk, the critical threshold for gap-opening is generally considered to be the mass at which the planet’s Hill radius equals the thermal scale height of the disk. The Hill radius is $r_{\text{Hill}} = q^{1/3} a$ where $q \equiv M_p/M_*$ is the mass ratio of the planet to the star. The thermal scale height is $H = c_s/\Omega_K$ where $c_s \equiv \sqrt{kT/\mu}$ is the thermal sound speed and $\Omega_K \equiv \sqrt{GM_*/a^3}$ is the Keplerian orbital angular speed. Here, $k$ is Boltzmann’s constant; $T$ is the disk’s midplane temperature; $\mu$ is the mean molecular weight of the gas, assumed to be primarily molecular hydrogen; and $G$ is Newton’s gravitational constant.

The gap opening criterion for a viscous disk has been found empirically to be

$$\frac{3}{4} \frac{H}{r_{\text{Hill}}} + \frac{50}{qR} \lesssim 1$$

where Reynolds number is defined as $R \equiv r^2 \Omega_P/\nu$. (Crida et al. 2006). Their results are also consistent with those of Varnièere et al. (2004).

For the purposes of this paper, we adopt gap profiles similar to those calculated by Bate et al. (2003), who model gap opening by planets of varying masses in a disk using three-dimensional hydrodynamic simulations, focusing specifically on planets that only partially open gaps in disks. A gap is modeled as an ad hoc perturbation imposed on the initial conditions. The surface density of a disk modified by a gap of width $w$, depth $d$, and position $a$ is

$$\Sigma(r) = \Sigma_0(r) \left\{ 1 - d \exp\left[ -(r - a)^2/(2w^2) \right] \right\}.$$

This is consistent with the results of (Crida & Morbidelli 2007), who find that planets whose masses are less than that of the disk open local gaps in disks rather than clearing inner cavities. Since the study presented here addresses planets under a Jupiter mass ($M_J$), they are well under the disk mass.

The depth and width of the gap compared to planet mass are determined from the results of Bate et al. (2003). In Figure 1, we reproduce the surface density profiles of gap
opening by planets from Figure 2 of Bate et al. (2003), having used DataThief\(^1\) to acquire the values. The unperturbed density profile is plotted as a solid line, and goes as \(\Sigma_0(r) \propto r^{-1/2}\). The planet is located at \(r/a = 1\). The gaps opened by planets of 0.03, 0.1, 0.3 and 1 \(M_J\) are plotted as dashed, dotted, dot-dashed, and long-dashed black lines, respectively. We fit all but the 1 \(M_J\) planet to Equation (2), and plot these fits as magenta lines in Figure 1. The best fit parameters are tabulated in Table 1. The maximum deviation between the empirical gap profile and the fit, expressed as a percentage of the unperturbed disk density, is 0.04\% for the 0.03 \(M_J\) planet, 3\% for the 0.1 \(M_J\) planet, and 9\% for the 0.3 \(M_J\) planet. The deviation for the 0.3 \(M_J\) planet is greatest at \(r/a < 1\), indicating asymmetry of the gap that is not accounted for in the model presented here. Nevertheless, fitting the gap to a Gaussian is a useful model for parametrizing the disk response to a planet without the need for running a full hydrodynamic simulation.

In (Bate et al. 2003), \(R = 10^5\) and \(H/a = 0.05\). Using Eq. (1) as the gap opening criterion, this gives a gap-opening threshold of \(q = 1.06 \times 10^{-3}\), or slightly more than 1 \(M_J\). For comparison, \(r_{\text{Hill}} = H\) when \(q = 3.75 \times 10^{-4}\), so the viscous gap opening criterion gives a mass more than twice as large as for an inviscid disk. In the disk model adopted in this paper, at 10 AU the midplane temperature is 49K, \(H/r = 0.046\), and \(\alpha_{\text{ss}} = 0.01\), so \(R = 4.4 \times 10^4\) and the gap-opening threshold is \(q = 1.97 \times 10^{-3}\), or almost 2 \(M_J\).

Given the difference in disk properties, we cannot assume that the same mass planets open the same size gaps in each disk. We estimate the masses of planet according to the following procedure. Defining

\[
G \equiv \frac{3}{4} \frac{H}{r_{\text{Hill}}} + \frac{50}{qR},
\]

we assume that \(G\) is the relevant scale for determining the depth of the gap opened by a planet, so that planets with the same value of \(G\) open similarly sized gaps. Then, the 0.03, 0.1, and 0.3 \(M_J\) planets in the disk modeled by Bate et al. (2003) have mass ratios of \(q = 3 \times 10^{-5}, 1 \times 10^{-4}\) and \(3 \times 10^{-4}\), respectively, and have \(G\) values as tabulated in Table 1. For the disk parameters adopted in this paper, the equivalent planet masses are then approximately 20, 70 and 200 \(M_\oplus\), assuming that \(M_\oplus/M_\odot = 3 \times 10^{-6}\).

Holding \(\Sigma(r)\) constant and assuming axisymmetry, we recalculate the temperature structure of the disk, using the same iterative radiative transfer method as described in §2.1. The effect of the gap is to create a shadow within the trough in density created by the gap. The far side of the gap, which is now exposed to more direct stellar illumination, is brightened. A schematic of this is shown in Figure 2. This results in cooling within the trough and heating

\(^1\)B. Tummers, http://datathief.org
on the far wall. The gas that composes the bulk of the disk material responds to this cooling and heating by contracting and expanding. This changes the vertical density profile of the gap region, and the illuminated surface now must be recalculated. For this reason, the heating and cooling of the disk must be calculated self-consistently with the density structure of the disk.

2.3. Observables

Calculation of synthetic images of disks with gaps is done as in Paper II. We assume that the scattered light emission comes from an optically thin surface layer of the disk. Then the intensity of the scattered light of frequency $\nu$ from this layer is

$$I_s^\nu = \frac{\omega^\nu \mu R^2 B^\nu(T^\star)}{4r^2(\mu + \cos \eta)}, \quad (4)$$

where $\omega^\nu$ is the albedo, $B^\nu(T^\star)$ is the intensity emitted at the stellar surface (here assumed to be a blackbody), $r$ is the distance from the star, $\mu$ is the cosine of the angle of incidence of stellar light, and $\eta$ is the angle between the line of sight to the observer and normal to the surface. The angle $\eta$ is distinct from the inclination angle of the overall disk, which we represent as $i$. The angles are illustrated in Figure 3, which shows that for larger values of $\eta$ ($\eta_2 > \eta_1$), the observer sees more scatterers along the line of sight, resulting in an apparent brightening of the surface layer.

Thermal emission is calculated by integrating the equation of radiative transfer along the line of sight,

$$\frac{dI^\nu_t}{dl} = \kappa^\nu \rho B^\nu(T_d) \exp(-\tau_{obs}) \quad (5)$$

where $\kappa^\nu$ is the absorption coefficient at frequency $\nu$, $\rho$ is the local density, $T_d$ is the local disk temperature and $B^\nu(T_d)$ is the thermal emission at that temperature, and $\tau_{obs} = \int \chi^\nu \rho \, dl$ is the line of sight optical depth, $\chi^\nu$ being the extinction coefficient.

The total intensity is then $I^\nu = I^\nu_s + I^\nu_t$, although as a general rule scattered light dominates in optical to near-IR, and thermal emission dominates at longer wavelengths.

2.4. Validation

The radiative transfer technique used for calculating the disk structure for this work is fully three-dimensional, in the sense that radiation impinging on the surface of the disk is allowed to propagate in all directions throughout the disk. However, it relies on a locally
one-dimensional analytic solution to the radiative transfer equation and adopts a number of simplifying assumptions (see also Jang-Condell & Sasselov 2003, 2004). The novel approach presented here is to iteratively calculate the self-consistently the density and temperature structure. As shown by the significant temperature perturbations created by shadowing and illumination on gaps, the self-consistency is an important aspect to consider in the analysis of radiative transfer in disks.

***MODIFY THIS PARAGRAPH*** A direct comparison to a Monte Carlo radiative transfer simulation (e.g. Walker et al. 2004; Dullemond & Dominik 2004; Tannirkulam et al. 2008; Pinte et al. 2008; Mulders et al. 2010) would be the next step to conclusively verify the results presented here.

The Monte Carlo approach is similar to that of Pinte et al. (2006). We find the radiative equilibrium temperatures by following a large number of photon packets from the star through scattering, absorption and re-emission till escape to infinity. In this way the energy is conserved exactly. We sum the radiation energy absorbed all along the packet paths (Lucy 1999) and relax to equilibrium by choosing each re-emitted packet’s frequency to adjust the local radiation field for the updated temperature (Bjorkman & Wood 2001). The scattering is assumed to be isotropic. Each of the calculations shown involves $10^7$ photon packets.

3. Results

3.1. Gap Structure

The full radial and vertical temperature structure of our models are shown in Figure 4. The gap opened by the 20 M$_\oplus$ planet is less than 2% in depth, and produces no more than 2% excursions in temperature from the unperturbed disk, so small as to be negligible. We thus set 20 M$_\oplus$ as the lower bound on a detectable planet in this disk at 10 AU, or more generally, planets with $r_{\text{Hill}}/H \leq 0.43$ are undetectable, whereas $r_{\text{Hill}}/H \geq 0.64$ do significantly perturb the disk.

Figure 4a shows the unperturbed temperature structure of the disk, while Figure 4b and c show the temperatures as contours and temperature deviations ($\Delta T$) as colors for the 70 and 200 M$_\oplus$ planets, respectively. The shadowed cooled region is radially inward of the planet position, while the brightened heated region is radially outward of the planet position. The minimum $\Delta T$ (i.e. the maximally cooled point) occurs on the inner shoulder of the gap, that is, at inner edge of the gap and near the surface of the disk. For the 70 M$_\oplus$ planet, the maximum amount of cooling is $-14$ K (-24%) at $(r, z) = (9.1 \, \text{AU}, 1.25 \, \text{AU})$, and for the 200 M$_\oplus$ planet, it is $-24$ K (-41%) at $(8.6 \, \text{AU}, 1.11 \, \text{AU})$. The maximum $\Delta T$ occurs on the outer
shoulder of the gap, but further away from the gap position than the most cooled point. For the 70 $M_\oplus$ planet, the maximum amount of heating is $+19$ K ($+38\%$) at (12.2 AU, 1.94 AU), and for the 200 $M_\oplus$ planet, it is $+28$ K ($+54\%$) at (12.4 AU, 1.98 AU). For reference, the surface of the disk at 10 AU is at $z = 1.90$ AU, about 4 times the disk thermal scale height.

The heating on the outer shoulder of the gap is particularly significant. In both gap models, the maximum $|\Delta T|$ is on the heated far shoulder of the gap, rather than in the cooled trough. Additionally, the volume of disk material that is heated is greater than the volume of disk material that is cooled. This is because the amount of heating caused by stellar irradiation is sensitive to the cosine of the angle of incidence at the surface of the disk. This is generally $\mu \ll 1$ in protoplanetary disks, but cannot drop below 0, thus limiting the amount of cooling caused by shadowing. However, the upper limit in $\mu$ is 1, allowing for much greater heating than cooling to occur.

In contrast to the results of Paper I, the midplane temperature is significantly affected by gap opening. Paper I considered planets up to 50 $M_\oplus$ in the absence of a gap, but here we find that a 70 $M_\oplus$ planet should open a significant gap. In a disk with lower viscosity, a 50 $M_\oplus$ might open a similarly sized gap. This shows the importance of including large scale non-linear dynamical interactions in planet-disk models.

The midplane temperature is summarized in the lower plot of Figure 5, illustrating the S-shaped perturbation to the midplane temperature profile. In the case of both gaps, the temperature at the position of the planet is lowered, although the temperature minimum is inward of the planet position. The minimum and maximum temperature deviations at the midplane are $-7$ K ($-13\%$) at 9.1 AU and $+9$ K ($+20\%$) at 13.3 AU, respectively, for the 70 $M_\oplus$ planet, and $-15$ K ($-27\%$) at 8.9 AU and $+13$ K ($+31\%$) at 14.0 AU, respectively.

Figure 4 also indicates that the puffed-up outer disk edge shadows the disk behind it, resulting in additional cooling radially outward of the gap. This shadowing and cooling effect is real, but since the shadow extends beyond the scope of the simulation boundaries, quantifying this effect is beyond the scope of this paper. Nevertheless, this ringing effect is worth noting.

### 3.2. Simulated Images

#### 3.2.1. 0° Inclination Disks

In Figures 6 and 7, we show synthetic images of face-on disks without and with gaps at wavelengths from 1 $\mu$m to 1 mm, as indicated. The star is omitted in each image, but
its brightness is that of a 4280 K blackbody with radius $2.6 R_\odot$. Although the apparent brightness varies as the inverse square of the distance, the surface brightness in Jy/asec$^2$ is distance-independent.

The top row in Figure 6 shows the scattered light images of the disks at 1 $\mu$m. The 3 $\mu$m image is very similar to this one, just scaled to the stellar brightness at 3 $\mu$m. The stellar brightness at 1 $\mu$m at a distance of 140 pc is 0.79 Jy. The shadow in the gap and the brightening outward of the gap are quite apparent. The brightening in the outer edge of the gap also creates a further shadowing of the disk beyond the gap. Because the disk is not well-modeled outside 20 AU, it is unclear whether or not the full extent of the disk beyond the gap rim is fully shadowed or not.

The middle and bottom rows in Figure 6 show the thermal emission from the surface of the disk at 10 and 30 $\mu$m, respectively. The stellar brightness at these wavelengths at a distance of 140 pc is 0.055 Jy at 10 $\mu$m and 0.0068 Jy at 30 $\mu$m. Compared with the near-infrared, the overall radial falloff in surface brightness is steeper because the outer disk is too cold to emit efficiently. Also, scattered radiation is fainter due to the star’s lower flux and the grains’ lower albedo at mid-infrared wavelengths. Still, the cooling and heating in the gap structure is readily apparent.

The gap contrast is highest at 30 microns. This is because the surface temperatures of the disk are $\sim$ 100 K, where the blackbody peak is 30 microns. Thus, observations at 30 microns will be the most sensitive to temperature perturbations at the surface of the disk. At 1 and 10 microns, scattering is the dominant mode of emission from the disk, so the gap contrast is driven by shadowing and illumination at the surface, rather than actual temperature changes.

In Figure 7, the top, middle and bottom rows show thermal continuum emission at 0.1, 0.3, and 1 mm, respectively. At these wavelengths, the stellar emission is negligible. The disk becomes more optically thin toward longer wavelengths, so the different wavelengths probe different layers in the disk as well as different temperature regimes. In contrast to the dimple images calculated in Paper II, the gaps are still apparent at 1 mm wavelengths because the shadowing effect in the gap is on a much larger scale than the localized dimple. Thus, searching for gaps in disks with ALMA is a promising way of finding planets in during the planet formation epoch.

The radial surface brightness profiles for each of the disks imaged in Figures 6 and 7 are also plotted in Figure 8 so as to better directly compare the effect of inserting planets of various masses in the disks. The perturbation from the gap shadow is evident across all wavelengths, reflecting the cooling and heating that take place in the shadowed trough and
brightened rim of the gap. The puffing up of the far rim of the gap is significant enough to shadow the outer disk, as evident in the steeping of the brightness profile at 1 micron. The brightening of the far rim of the gap is consistent with the results of Varni`ere et al. (2006), with feedback from cooling and shadowing resulting in a puffing up of the gap edge.

One might expect that the shadow produced by the gap is offset by the brightened outer edge of the gap, creating a net zero effect on the spectral energy distribution (SED). Because the simulated region is limited in radius, a full SED cannot be produced for the entire disk, as any emission from interior to 4 AU or exterior to 20 AU will be omitted. Nevertheless, we can assess how a gap in a disk changes the SED by integrating over the face on disk images. We approximate the contribution from the inner 4 AU by assuming that the brightness profile goes as $r^{-1}$, is normalized to the brightness at 4 AU for the gapless disk, and is unaffected by the presence of the gap. The star’s emission is approximated as a blackbody of 4280 K and radius $2.6 R_\odot$. In Figure 9, we show the resulting SEDs assuming that the system is at a distance of 140 pc. The upper panel shows that the gapped SEDs (dotted/dashed line for $70/200 M_\oplus$) are generally brighter than the gapless SED (solid line), particularly at around 30 $\mu$m. The lower panel of Figure 9 shows the difference between gapped and gapless SEDs in magnitudes, with negative values being brighter. These magnitude differences should be interpreted as upper limits on the amount of brightening that can occur, because emission from the outer disk beyond 20 AU has been omitted, and this disk contribution may swamp out the relative differences. Shortward of 10 $\mu$m, stellar flux (dot-dashed line) dominates the SED so the disk contribution is negligible. Beyond the submillimeter regime, emission from the outer disk becomes increasingly important, so the effect of the gap is relatively less than that shown.

The conclusion is that the brightened outer rim of the gap more than compensates for the lack of emission in the gap trough. The reason for this is the greater amount of heating that occurs on the outer shoulder of the gap, as described in §3.1. Moreover, the contribution to the brightening is amplified because the surface area of the outer rim is greater than the surface area of the gap shadow. However, the contribution of the gap to SED is subtle, and not easily distinguished from a more massive disk. The SED alone cannot be used to identify a partially cleared gap.

3.2.2. Inclined Disks: No Gap

In Figure 10, we demonstrate the effect of inclination on disks in the absence of a gap. In each image, the disk is inclined at 45° with the top edge tilted away from the observer. The images at shorter wavelengths are vertically asymmetric because the emission is from the
disk surface, which is essentially bowl-shaped. A schematic for the geometry is illustrated in Figure 11, which shows a cartoon image of a disk around a star. The lines of sight marked by the angles $i_1$ and $i_2$ intersect the disk at the same distance from the star, but because the surface is above the midplane, the projected distance on the sky is not equal, as can be seen by comparing the distance between the solid lines to the dashed line intersected in the star. Moreover, since $i_2 > i_1$, the near side appears brighter than the far side. This is can be seen in the 1 $\mu$m images in the top left panel of Figure 10. The foreshortening of the near side of the disk is also evident in the 10 and 30 $\mu$m images. At 0.1 mm and longer, the optical depth decreases with increasing wavelength, so the vertical asymmetry diminishes. The apparent thick rim on the near side of the disk in these images is a numerical artifact resulting from the simulation boundary and should be ignored.

The surface brightness profile at 1 $\mu$m along the major axis of the tilted disk is systematically brighter by a factor of about 1.4 as compared to the face on disk, as seen in the upper left plot of Figure 12. However, this is not reproduced at wavelengths longward of 30 $\mu$m. This can be explained as follows. At 1 $\mu$m, the disk image is purely scattered light. Thus, Equation (4) governs the brightness profile. Assuming $\mu \ll 1$, which is generally the case, then the brightness depends on the angle of scattering with respect to the surface $\eta$ as $\sim 1/\cos \eta$. While Eq. (4) is derived in detail in Paper II, another representation for the scattered light brightness is illustrated in Figure 3. In essence, the star illuminates an optically thin layer of scatterers on the surface of the disk. For $\mu \ll 1$, the brightness is roughly proportional to the number of scatterers along the line of sight, which is in turn proportional to $1/\cos \eta$. When the disk is inclined at angle $i$, $\cos \eta = \cos \alpha \cos i$, along the major axis (at maximum elongation), as shown in the Appendix. Since $\alpha$ is angle of the disk surface with respect to the disk midplane and is intrinsic to the disk itself, the brightness profile along the major axis of the inclined disk scales as $1/\cos i$ as compared to the face on disk. When $i = 45^\circ$, this factor is $\sqrt{2} \approx 1.4$.

A similar effect occurs with the thermal emission to a limited extent. As seen in Figure 4, the surface of the disk is generally hotter than the optically thick interior. At wavelengths close to the blackbody peak of the surface temperature, an inclination effect similar to that of the scattered light will occur. Since temperature decreases with distance in the disk at both the surface and the midplane, the wavelength of observation may be tuned to probe the surface at a particular distance in the disk. At 10 AU in this disk model, the surface temperatures are above 100K while the interior is below 100K, so 30 $\mu$m imaging is a good probe of the surface of the disk. Wavelengths longer than 30 microns are sensitive to the interior of the disk both because the disk becomes optically thin and because the interior is cooler than the surface. This is illustrated in the disk profiles in Figure 12, where the cyan lines indicating the major axis of the tilted disks are all systematically brighter than
the face-on disk for ≲ 30 microns, whereas at longer wavelengths the major axis profiles are nearly equal to the face-on profiles.

In Table 2 we list the heights and slopes of the disk surface at 10 AU at the listed wavelength. For wavelengths dominated by scattered light, we define the surface to be where the disk becomes optically thick to stellar light at the given wavelength. For thermal wavelengths, the surface is defined to be where the optical depth to observer in a face-on orientation becomes optically thick. At all wavelengths, we set \( \tau_\lambda = \frac{2}{3} \) as the optically thick limit. At scattered light wavelengths, \( \mu \approx \partial z_s/\partial r - z_s/r \), and typically \( \mu \ll 1 \).

In addition to the brightness asymmetry, the projected distance a given radial distance on the disk’s surface is foreshortened on the near side of the disk compared to the far side. If \( r_n \) and \( r_f \) are the projected distances along the near and far sides of the disk, respectively, \( r_d \) is the actual distance, and \( \tan \beta = z_s/r \), then

\[
\begin{align*}
  r_n &= r_d \cos(i + \beta)/\cos \beta \\
  r_f &= r_d \cos(i - \beta)/\cos \beta.
\end{align*}
\] (6)

(7)

In Figure 12, we show the deprojected surface brightness profiles along the far and near sides of the disk as dashed red and blue lines, respectively. In the 1 \( \mu \)m plot, the near side brightness profile becomes brighter than the major axis, while the far side becomes dimmer, as expected.

The brightness asymmetry between the near and far sides of the disk, represented by red and blue lines in Figure 12, can be quantified in terms of the disk geometry as well. Suppose the slope of the surface \( \partial z_s/\partial r = \tan \alpha \) at \( r = r_d \) and the disk is inclined at an angle \( i \). Let \( \eta_1 \) and \( \eta_2 \) be the cosines of the angles between the surface normal and the observer on the far side and near side, respectively. This is illustrated in Figure 11. By geometry, \( \eta_1 = i - \alpha \) and \( \eta_2 = i + \alpha \). A more general formulation for \( \eta \) is discussed in the Appendix.

Let \( F_0(r_d) \) be the surface brightness of the face-on disk at the projected distance \( r_d \). For a tilted disk, the same three-dimensional radial distance is seen in projection at \( r_d, r_n, \) and \( r_f \) along the major axis, near side minor axis, and far side minor axis, respectively. The corresponding surface brightnesses are \( F_m(r_d), F_n(r_n), \) and \( F_f(r_f) \). While these points are at the same physical distance from the star on the disk, they appear to be at different distances when seen in projection and appear to be different brightnesses because the values of \( \eta \) are different in Eq. (4). In particular, \( F \propto 1/(\mu + \cos \eta) \). For the face-on disk, \( \eta = \alpha \). Thus, if we find that Major axis, near side, and far side are, respectively,

\[
\begin{align*}
  F_m(r_d)/F_0(r_d) &= (\mu + \cos \alpha)/(\mu + \cos \alpha \cos i) \\
  F_n(r_n)/F_0(r_d) &= (\mu + \cos \alpha)/[\mu + \cos(i + \alpha)]
\end{align*}
\] (8)

(9)
\[
F_f(r_f)/F_0(r_d) = (\mu + \cos \alpha)/[\mu + \cos(i - \alpha)].
\]

Eq. (8) is the harmonic mean of Eqs. (9) and (10), so find the relation

\[
\frac{1}{2} \left[ \frac{1}{F_n(r_n)} + \frac{1}{F_f(r_f)} \right] = \frac{1}{F_m(r_d)}.
\]

This means that given the correct values of \(i\) and \(\beta\) to deproject \(r_n\) and \(r_f\) according to Eqs. (6)-(7), then the harmonic mean of the deprojected surface brightness profiles on the near and far sides of the disk should equal the brightness profile along the major axis. This harmonic mean is plotted as black dashed lines in Figure 12, which does indeed prove to be a good fit to the cyan line at all wavelengths.

We can make use of this relation to construct a simple geometric model for a tilted disk image, and then apply this model to derive the inclination angle and thickness of a disk image. In order to reduce the number of free parameters, we make the assumption that \(\partial z_s/\partial r \approx z_s/r\) or \(\alpha \approx \beta\). We determine the best fits for \(i\) and \(\beta\) over 4 AU < \(r_d\) < 20 AU by minimizing \(\chi^2\) between the brightness profile along the major axis and the harmonic mean of the brightness profiles along the near and far minor axes, using the MPFIT package (Markwardt 2009). The results of this fitting procedure are tabulated in the last two columns of Table 2. At all wavelengths, the derived inclination angles are within 1° of the actual inclination. The fitting was done with no presumption about which side of the disk was near or far, but the correct orientation was still found.

The values in the third and fourth columns of Table 2 were determined by first calculating \(z_s\). For wavelengths at which scattering dominates the disk emission (1 and 10 microns), this was defined as the surface at which the disk becomes optically thick along the line of sight to the star. At thermal wavelengths, (≥ 30 microns), the was defined as the height at which the disk becomes optically thick to an observer viewing the disk face-on. This last assumption leads to less accuracy in the fitted values for \(z_s/r\) at thermal wavelengths, since the height at which the disk becomes optically thick to the observer is in general dependent on the disk’s inclination.

This fitting only works well if \(z_s/r\) varies slowly with \(r\). Otherwise fitting to a a single value of \(\alpha\) is inaccurate. Another effect that was not considered for these models was forward-scattering of light by small dust grains. A prediction of Mie theory is that small dust grains can be strongly forward-scattering, but scattering is assumed to be completely isotropic in the models presented here. On the other hand, if the anisotropy of the scattering can be

\[\text{http://purl.com/net/mpfit}\]
well-described by a single parameter, such as in the Henyey-Greenstein model (Henyey & Greenstein 1941), the anisotropy may be treated as an additional fitting parameter and solved for accordingly. However, this is outside the scope of this paper.

### 3.2.3. Inclined Disks: With Gaps

Simulated images of inclined disks with gaps imposed on them by 70 and 200 $M_\odot$ planets are shown in Figures 13 and 14, respectively. The images are shown at the same wavelengths of observation as in Figure 10, for reference. In the images of tilted gapped disks, the asymmetry in brightness between the near and far sides of the disk is more apparent than in the gapless disk. From 1 to 30 microns, the brightened shoulder of gap is brighter on the lower half of the disk, the side that is tipped toward the observer. In addition, the width of the shadow within the gap is narrower on the same side because of the geometric foreshortening of the disk. At 100 microns and beyond, these asymmetries become less apparent as the disk becomes optically thin.

We can again make use of Eqs. (8-10) to estimate the brightness of the shadow in the gap and the brightened far shoulder as a function of PA. The gap in the disk makes the determination of disk thickness and inclination angle easier, because the shadow and brightening in the gap are points of reference that we can use to compare the profiles along different PA to each other.

Along the major axis, near minor axis, and far minor axis, we measure the surface brightness profile at each wavelength of observation as a function of projected radius. For each of these surface brightness profiles, we find the local minimum in brightness caused by the gap, and the local maximum on the far gap shoulder, labeling these points $r_{\text{min}}$ and $r_{\text{max}}$, respectively. The values of $r_{\text{min}}$ and $r_{\text{max}}$ are plotted for all axes for each gap model at all wavelengths in the second panel of Figure 15. For reference, the values of $r_{\text{min}}$ and $r_{\text{max}}$ for the face-on disk are also plotted in black. Along the major axis, $r_{\text{min}}$ and $r_{\text{max}}$ are nearly identical to the face-on case. Both the near and far sides of the disk are foreshortened, but because of the thickness of the disk, $r_{\text{min}}$ and $r_{\text{max}}$ are smaller on the near side than on the far side.

We define the flux ratio to be the brightnesses at these minima and maxima compared to the disk brightness at 10 AU on the gap-less disk seen face-on. The flux ratios at $r_{\text{min}}$ and $r_{\text{max}}$ for each axis in each disk model at all wavelengths are shown in the top panel of Figure 15. For comparison, the relative brightness at $r_{\text{min}}$ and $r_{\text{max}}$ for a face-on gapped disk is also plotted on the same graph in black. The brightnesses along the major axis are
slightly higher than for the face-on disk, because \( \cos \eta = \cos \alpha \cos \iota \) (see Appendix). We also find that the near/far side of the disk is systematically brighter/dimmer at \( r_{\min} \) and \( r_{\max} \) than along the major axis up to 30 microns, because \( \eta \) is larger on the near side than on the far side. At longer wavelengths, the disk becomes optically thin, and geometric brightening is no longer important.

As in §3.2.2, we can use the geometric relations between \( r_d \), \( r_n \), and \( r_f \) to back out the inclination and aspect angles of the disk. From Eqs. (6-7),

\[
\frac{1}{2} (r_f + r_n) = r_d \cos \iota \quad \text{and} \\
\frac{1}{2} (r_f - r_n) = r_d \sin \iota \tan \beta = r_d \sin \iota \left( \frac{z_s}{r} \right).
\]

Using the local minimum in the radial brightness profile as a reference point, we can then solve for the inclination and aspect ratio of the disk from Eqs. (12-13).

The calculated values of \( \iota \) and \( \beta \) based on measurements of the simulated disk images are plotted in Figure 15 and tabulated in Table 3. Comparing these values to those in Table 2 we find that although it may be easier to calculate \( \iota \) and \( z_s/r \) by using the brightness maxima and minima as reference points, the values returned are not necessarily more accurate than those obtained by comparing the overall surface brightness profiles as a whole. Moreover, if the gap is eccentric, as might be produced by an eccentric planet, additional errors in the fitting can be introduced.

### 3.3. LkCa 15

To demonstrate the application of the simulated gapped disk images to real observations, we examine the case of LkCa 15. LkCa 15 is a T Tauri star that has been identified as a transitional disk because it appears to have an inner cavity of radius 46 AU as inferred from its SED (Espaillat et al. 2007) and directly imaged by radio interferometry (Andrews et al. 2011; Piétu et al. 2006; Isella et al. 2009), and more recently in scattered light (Thalmann et al. 2010). It is also sometimes referred to as a “pre-transitional” disk because the inner cavity is not completely cleared (Espaillat et al. 2008). In Figure 16, we reproduce the scattered light image from Thalmann et al. (2010) and the SMA image from Andrews et al. (2011), both of which reveal a large inner clearing of the disk. In both images, northwest side of the gap appears brighter than the southeast side.

One enticing possibility for the clearing of the gap in LkCa 15 is planet formation. Pott et al. (2010) observed no stellar companion down to 3.5 AU separations from the star.
Bonavita et al. (2010) put an upper limit of $5 \, M_{\text{Jup}}$ on a possible companion using NACO observations. From a theoretical standpoint, planets more massive than $6 \, M_{\text{Jup}}$ should cut off any accretion onto the star (Lubow et al. 1999), but Espaillat et al. (2007) find that the disk is still accreting onto the star at a rate of $\dot{M} = 2.4 \times 10^{-9} \, M_\odot \, \text{yr}^{-1}$. Gas has been detected inside the cavity in the form of CO lines (Piétu et al. 2007), indicating an incompletely cleared inner disk. LkCa 15 also boasts a warm dust component at 0.12-0.15 AU as inferred from its near-infrared excess (Espaillat et al. 2008), further evidence that the inner cavity is not completely cleared. All these lines of evidence point to the suggestion that one or more planetary-mass companions are responsible for the inner cavity in LkCa 15, with an upper limit on a single planet of $5 \, M_{\text{Jup}}$.

Supposing that a single embedded planet is responsible for the inner clearing in LkCa 15, then we can model its appearance in scattered light following the methods outlined above. Comparing simulated images of gaps caused by varying planet mass to the observed images will constrain the mass of a possible planet in LkCa 15.

LkCa 15 has a stellar mass of $M_*=0.97 \pm 0.03 \, M_\odot$, effective temperature $T_{\text{eff}}=4350$ K, and luminosity $L_*=0.74 \, L_\odot$ (Simon et al. 2000). Its disk is inclined at approximately 52° (Piétu et al. 2007) and its properties are well-fit with an accretion rate of $\dot{M} = 2.4 \times 10^{-9} \, M_\odot \, \text{yr}^{-1}$ and $\alpha_{\text{ss}} = 0.0007$ (Espaillat et al. 2007; Thalmann et al. 2010). For the purposes of this study, we round the stellar mass to $1 \, M_\odot$ and derive a stellar radius of $R_*=1.5 \, R_\odot$ from the effective temperature and luminosity. The initial disk model is calculated using these parameters as in §2.1. To simplify the calculation, the initial surface density profile is simply interpolated from the locally-plane parallel disk model rather than recalculating the full structure in detail. The radial range of this slice is from 9.5 to 99.5 AU, and the vertical range is 0 to 28.9 AU.

Since the width of a gap created by a planet varies with planet mass, a larger planet would be located further from the gap wall than a smaller planet. Thus, for a given gap size, we position it so that its half depth on the far side is at 46 AU. That is, we position the gap so that

$$\Sigma(r = 46 \, \text{AU})/\Sigma_0(r = 46 \, \text{AU}) = 1/2$$

with the gap trough interior to this distance. A planet more massive than $M_{\text{crit}}$ opens a gap that is not well-modeled with Equation (14). Bate et al. (2003) calculate the surface density profile of a $2.7 \, M_{\text{crit}}$ planet in their hydrodynamic simulations, and we adopt this as an axisymmetric gap profile without fitting to a Gaussian. The calculated gap parameters are summarized in Table 4 and their surface density profiles are plotted in Figure 17.

A planet much less than $M_{\text{crit}}$ is unlikely to be a good fit for the gap in LkCa 15 because such a planet does not clear sufficient material to account for the deep clearing observed in
the SED and in scattered light. We therefore restrict our comparison to the 0.94 $M_{\text{crit}}$ and 2.7 $M_{\text{crit}}$ models, which which we will refer to as the 50 $M_\odot$ and 0.5 $M_J$ models for simplicity.

In Figure 18, we show simulated H-band images of our LkCa15 disk model with a 50 $M_\oplus$ and 0.5 $M_J$ planet. The disks are oriented so that the top part of the disk is the near edge, while the far edge is below. The brightness in anisotropy is solely due to geometric effects. This model successfully reproduces the morphology of the H-band image obtained by Thalmann et al. (2010) (see left panel of Figure 16), with the disk brighter on one side, with some emission coming from the far side of the disk as well. The orientation of our disk model is consistent with that of Mulders et al. (2010), but with geometric effects causing the brightening rather than forward-scattering grains.

In our simulated images in Figure 18, the blacked-out inner circle represents the 0′′.055 FWHM PSF as reported in Thalmann et al. (2010). In our simulated images, the inner disk is quite bright and extends past the PSF circle, but this is not seen in the Thalmann, et al. observation. Moreover, detailed analysis of LkCa 15’s spectrum indicates that the dust in the inner disk extends no further than 5 AU (Espaillat et al. 2008). It is difficult to determine which model image (50 $M_\oplus$ or 0.5 $M_J$) is a better match to the observation, because the method used by Thalmann et al. to gain high contrast in order to image the disk does not preserve total flux. However, both models over-predict emission from the inner disk compared to the observational evidence. This suggests that either the gap in LkCa 15 is caused by a single planet more massive than 0.5 $M_J$, or there are multiple planets in the gap.

If the orange and red ellipses drawn by Thalmann et al. (2010) to indicate the LkCa 15 disk represent the true shape of the gap, then the brighter side of the disk is about equidistant from the opposite side of the disk, which does not fit the model presented in this paper. This inconsistency could be solved by invoking an eccentric cavity, but still another solution is possible. The east sides of the orange and red ellipses are guided by a bright blob which is similar in size and brightness to blobs both interior and exterior to the ellipses. If that blob is ignored, then the ellipses might alternatively be guided by a different blob southwest of the star. Then the shape of the gap might be traced by the cyan ellipse instead. Using the cyan ellipse, we find that $r_f$, $r_n$ and $r_d$ are 44, 20, and 55 AU, respectively. Using Eq. (12), we find an inclination of 54°, consistent with values found in radio continuum and CO observations (Piétu et al. 2006, 2007). From Eq. (13), $z_s/r = 0.28$. In the 0.5 $M_J$ disk model, $z_s/r \approx 0.2$ interior to the gap, and rises quickly at the far edge of the gap to $z_s/r \approx 0.28$, so this large disk scale height is completely consistent with the model.

In Figure 19, we show predicted radio images of the LkCa 15 disk model with a 0.5 $M_J$ planet, convolved with Gaussian PSFs to represent a range of spatial resolution from infinite
to 0″.2. The images are shown in reverse video, so the brightened and heated rim of the gap appears as the dark lanes in each image. Comparing the highest angular resolution images (left) to the lowest (right), we see that the far edge of the gap (lower side) appears brighter at lower angular resolution not because that edge is intrinsically brighter, but because the frontally illuminated gap wall is appears larger in size than the near side.

Because of the limited radial range of the simulation and large beam sizes of current radio observations of LkCa 15, it is difficult to make a direct comparison between observations and models. Since we exclude the very innermost radii of the disk from the simulation, convolving the simulated image with a larger beam size would force us to exclude an equivalently large inner region of the disk as well. We do reproduce the flattening of the brightness profile in the inner region of the disk, as reported by Isella et al. (2009) and Piétu et al. (2006). Andrews et al. (2011) image the inner hole with the SMA at 880 μm with a beam size of 0″.41 × 0″.32, revealing bright ansae along the major axis of the disk at the location of the gap wall (see right panel of Figure 16), qualitatively consistent with the model. In their image, the gap appears brighter along the northwest minor axis, opposite to the model prediction. This discrepancy remains to be solved. With higher resolution, such as with ALMA, we might be able to resolve the shadow in the trough of the inner disk as suggested by the 0″.05 and 0″.1 convolved images. Resolving the sizes of the inner disks will provide additional constraints on the number of masses of possible planets lurking in the inner cavity.

A definitive measurement of the position of a planet in LkCa 15 could be obtained by determining the orbital velocity of non-axisymmetric structure of the disk. A planet would be expected to raise spiral arms in the disk, whose pattern speed would be that of the planet, rather than the local Keplerian orbital speed. Thus, a planet with semi-major axis a would complete an orbit in \((a/1 \text{ AU})^{3/2}\) years, so a feature on the inner wall of the disk at 46 AU should orbit at a rate of 2″ yr\(^{-1}\) × \((a/1 \text{ AU})^{-3/2}\), or 11, 9, or 8 mas/yr for a planet at 33, 38, or 41 AU, respectively, with some variation due to the inclination of the disk.

4. Discussion

Our example of LkCa 15 demonstrates that gaps in disks are currently detectable in scattered light. However, the gap in LkCa15 is at a much larger radius (≈ 50 AU) compared to the gap at 10 AU as modeled here. Supposing that the 10 AU gap model was at 140 pc, the distance of Taurus, would the gap be detectable? The gaps modeled were 1.1 and 1.7 AU in width, respectively, or 8 and 12 mas, respectively. Observing at 1 micron at the diffraction limit, this would require a 17 – 27 m telescope to resolve. At 0.3 mm, the baseline required would be 5 – 8 km. Large optical telescopes such as the LBT, GMT, and TMT,
and the radio array ALMA would achieve such resolving power, but prospects for imaging gaps at mid- to far-infrared wavelengths are small. In the optical, high contrast imaging would also be necessary. An inner working angle of 0′′.05 would block out the inner 7 AU of the disk. At sub-mm to mm wavelengths, stellar contrast is not an issue, but the contrast within the gap itself is much less than at shorter wavelengths. At 0.3 and 1 mm, the required sensitivity is on order of 1 and 0.01 Jy/asec², or 1.4 and 0.15 K respectively. In band 9 of ALMA (0.45 mm), the required integration time for 10 mas resolution with 1.4 K sensitivity using 50 antennas is 22 hours. At 1 mm (band 7), 0.15 K sensitivity requires 13 hours.

In Paper I, we carried out similar calculations on local perturbations to the disk caused by less massive planets than the gap producing ones modeled in the present work. Since gaps are much larger in physical scale, they produce both a more significant temperature perturbation on the disk and a larger footprint on the disk itself. Thus, planets that produce the gaps modeled in this paper are very likely to be observable, whereas the much more subtle perturbations modeled in Paper I are much harder to observe. Imaging is necessary for detecting these gaps, since the effects are too subtle to identify in the SED alone, as described in §3.2.1.

We have only modeled gaps at 10 AU in this present work, but we can make some extrapolations to how gaps at other distances in the disk would behave. The same planet mass opens a smaller gap at larger radii because the disk scale height increases faster than the Hill radius of the planet. Thus, the temperature variations would be relatively smaller as the planet is moved further out. On the other hand, gaps created by planets at larger radii should be more easily detectable because gap widths scale with distance, relaxing the angular resolution requirement, and the inner working angle is larger. For interferometers, sensitivity also improves with smaller baselines. We found that the greatest gap contrast was seen at the blackbody peak associated with the temperature of the surface of the disk. At 10 AU, the surface temperature is around 100 K, so the wavelength of greatest contrast is 30 microns. The surface temperature is driven by stellar irradiation, so it should vary roughly as $r^{-1/2}$. This means that to some extent, the wavelength of observation may be tuned to the planet’s orbital radius.

5. Conclusions

We have calculated the thermal effects of gap-opening by planets in protoplanetary disks. The thermal feedback leads to depression of gap troughs and puffing of gap walls, enhancing the observability of gaps carved by planets forming in disks. Planets less than one tenth of the critical gap-opening mass, or 10 $M_{\oplus}$ at 10 AU, do not create significant
gaps in disks. However, a modest gap of only 50%, created by a planet 27% of the critical mass or 70 M\(_{\oplus}\) at 10 AU, can induce a significant perturbation to the temperature profile of a protoplanetary disk. These gaps are observable in scattered light and thermal emission.

By comparing resolved images of disks to the models presented here, we can estimate the masses of planets that might be causing those gaps. In particular, we put a lower mass limit of 0.5 Jupiter masses on a planetary companion in LkCa 15 that would create the observed gap. If planets are responsible for the gap, then our results suggest that it is either caused by a more massive planet or by multiple planets with overlapping gaps.

The ability to determine the masses of planets in disks, together with the age of the disks and the locations of gaps, puts vital observational constraints on the time scales of planet formation. If we find that massive planets form early, this might indicate that giant planets form via gravitational instability, which is much faster than the competing paradigm of core accretion. On the other hand, if we only see gap-forming planets at late ages, this might indicate that core accretion is the dominant process. Upper limits on the luminosity of a planet embedded in the disk will determine whether giant planet form more like brown dwarfs with a hot start (Baraffe et al. 2003) or more quiescently with a cold start (Marley et al. 2007).

Temperature variations in the disk produced by shadowing and illumination can have profound effects on the forming planets. We address the consequences for Type I migration in a separate paper (Jang-Condell 2011). These temperature perturbations can also affect the condensation and sublimation of volatiles. Planetesimals just interior to the gap may become enriched in frozen volatiles, while those just outside the gap might become depleted in volatiles via the cold finger effect (Stevenson & Lunine 1988). This can affect the overall distribution of volatiles in the protoplanetary disk, shift the snow line, and lead to enhanced planet formation in the volatile enriched regions.

Topics for future work include carrying out radiative transfer modeling in a similar way on three-dimensional hydrodynamic simulations of gap-clearing by planets rather than relying on a simple analytic model. This would allow us to capture non-axisymmetric gap characteristics, particularly the region immediately around the planet itself. Our methods can also be applied to the inner walls of fully-cleared inner cavities in transitional disks. If the inner walls are sufficiently puffed up, they can shadow the outer disk and create flat radial surface brightness profiles seen in some disks (Grady et al. 2005). Polarized intensity images are a promising way to resolve protoplanetary disks (e.g. Oppenheimer et al. 2008), but geometrical effects make these images difficult to interpret (e.g. Jang-Condell & Kuchner 2010; Perrin et al. 2009). In a future paper, we will address the effects of both polarization and anisotropic scattering in scattered light images of protoplanetary disks.
The author thanks C. A. Grady for numerous helpful discussions in the preparation of this paper. The author also thanks an anonymous referee for constructive comments that greatly improved this paper. This work was performed under contract with the Jet Propulsion Laboratory (JPL) funded by NASA through the Michelson Fellowship Program. JPL is managed for NASA by the California Institute of Technology.

A. Scattering Angles

In this Appendix, we show how to calculate $\eta$, the angle between the surface normal and the observer, assuming an axisymmetric disk. For a general surface, $z = f(x, y)$, the unit surface normal can be expressed as

$$\hat{n} = \frac{-(\partial z/\partial x)\hat{x} - (\partial z/\partial y)\hat{y} + \hat{z}}{\sqrt{(\partial z/\partial x)^2 + (\partial z/\partial y)^2 + 1}}$$

Without loss of generality, we place the disk midplane in the $xy$ plane and the observer in the 1st quadrant of the $xz$ plane. Then, if the inclination angle is $i$, the vector toward the observer is $\hat{m} = \sin i \hat{x} + \cos i \hat{z}$. Then the cosine of the angle between the surface normal and observer is

$$\cos \eta = \hat{n} \cdot \hat{m} = \frac{-(\partial z/\partial x) \sin i + \cos i}{\sqrt{(\partial z/\partial x)^2 + (\partial z/\partial y)^2 + 1}}$$

We convert this to cylindrical coordinates, with

$$x = r \cos \theta$$
$$y = r \sin \theta$$
$$z = z$$

and using the chain rule to get

$$\frac{\partial z}{\partial x} = \cos \theta \frac{\partial z}{\partial r} - \frac{\sin \theta}{r} \frac{\partial z}{\partial \theta}$$
$$\frac{\partial z}{\partial y} = \sin \theta \frac{\partial z}{\partial r} + \frac{\cos \theta}{r} \frac{\partial z}{\partial \theta}$$

and find

$$\cos \eta = \frac{\cos i - [\cos \theta (\partial z/\partial r) + (\sin \theta/r)(\partial z/\partial \theta)] \sin i}{\sqrt{(\partial z/\partial r)^2 + (1/r^2)(\partial z/\partial \theta)^2 + 1}}.$$  \hspace{1cm} (A2)

In an axisymmetric disk, $\partial z/\partial \theta = 0$. Defining $\tan \alpha = \partial z/\partial r$,

$$\cos \eta = \cos \alpha \cos i - \cos \theta \sin \alpha \sin i.$$  \hspace{1cm} (A3)
Since the observer is located toward positive $x$, the far side of the disk is $\theta = \pi$ and the near side of the disk is $\theta = 0$. Hence, on the far side of the disk, $\eta_1 = i - \alpha$ and on the near side $\eta_2 = i + \alpha$. At maximum elongation, $\theta = \pm \pi/2$ and $\cos \eta = \cos \alpha \cos i$.

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Table 1: Best-fit parameters for gaps opened by planets.

<table>
<thead>
<tr>
<th>$q^1$</th>
<th>$d$</th>
<th>$w/a$</th>
<th>max. error$^2$</th>
<th>$G$</th>
<th>derived $q$</th>
<th>derived planet mass$^3$</th>
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<tr>
<td>$3 \times 10^{-5}$</td>
<td>0.014</td>
<td>0.078</td>
<td>0.04%</td>
<td>18</td>
<td>$6.7 \times 10^{-5}$</td>
<td>22 $M_\oplus$</td>
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<tr>
<td>$1 \times 10^{-4}$</td>
<td>0.56</td>
<td>0.11</td>
<td>3%</td>
<td>6.2</td>
<td>$2.2 \times 10^{-4}$</td>
<td>72 $M_\oplus$</td>
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<tr>
<td>$3 \times 10^{-4}$</td>
<td>0.84</td>
<td>0.17</td>
<td>9%</td>
<td>2.5</td>
<td>$6.2 \times 10^{-4}$</td>
<td>206 $M_\oplus$</td>
</tr>
</tbody>
</table>

$^1$As simulated in Bate et al. (2003)

$^2$Error = $(\Sigma - \Sigma_{\text{fit}})/\Sigma_0$

$^3$Actual masses used for this work. In the text, the masses have been rounded to 20, 70, and 200 $M_\oplus$ for convenience.

Table 2: Disk inclinations calculated from simulated images. Actual inclination is 45°.

<table>
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<tr>
<th>$\lambda$ (µm)</th>
<th>emission type</th>
<th>$z_s/r$ (at 10 AU)</th>
<th>$\partial z_s/\partial r$ (at 10 AU)</th>
<th>inclination (degrees)</th>
<th>$z_s/r$ (derived)</th>
</tr>
</thead>
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<tr>
<td>1</td>
<td>scattered</td>
<td>0.189</td>
<td>0.225</td>
<td>45.1</td>
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<tr>
<td>10</td>
<td>scattered</td>
<td>0.172</td>
<td>0.202</td>
<td>44.5</td>
<td>0.172</td>
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<td>30</td>
<td>thermal</td>
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<td>0.116</td>
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<td>0.181</td>
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<tr>
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<tr>
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<tr>
<td>1000</td>
<td>thermal</td>
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<td>45.5</td>
<td>0.038</td>
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This preprint was prepared with the AAS LaTeX macros v5.2.
Table 3: Gap and disk parameters from inclined disk images.

<table>
<thead>
<tr>
<th>$\lambda$ (microns)</th>
<th>Planet mass $(M_\oplus)$</th>
<th>max/min</th>
<th>derived incl. $(^\circ)$</th>
<th>derived $z_s/r$</th>
<th>actual $z_s/r$</th>
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<tr>
<td>1</td>
<td>70</td>
<td>min</td>
<td>44.0</td>
<td>0.186</td>
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<td>min</td>
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Table 4: Gap parameters used to model LkCa 15.

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<td>derived planet mass</td>
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<td>$36 \ M_{\oplus}$</td>
<td>$152 \ M_{\oplus}$</td>
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<sup>a</sup>Determined by wall creation at 46 AU

<sup>b</sup>Calculated from disk properties at planet position
Fig. 1.— Profiles of gaps carved by planets in disks and the Gaussian fits to them. The unperturbed disk profile is plotted as a solid line, while the dashed, dotted, dot-dashed, and long-dashed lines are gaps opened by planets of 8%, 27%, 80%, and 270% $M_{\text{crit}}$, respectively. The black lines are from Bate et al. (2003), and magenta lines are Gaussian fits, following Eq. (2). The gap opened by the planet more massive than $M_{\text{crit}}$ is not well fit by a a Gaussian.
Fig. 2.— Cartoon diagram of radiative feedback on disk structure. The star is represented as a yellow disk, and the planet by a blue dot. The disk surface represents a contour of constant density. The left image shows the initial gap opened in the disk, with the dotted line showing the original, unperturbed disk surface. Stellar illumination on the surface of the gap creates shadowed and brightened regions. Shadowing and cooling occurs in the disk trough, and the far side of the gap is illuminated and heated. The right image shows the response of the gaseous disk material to the cooling and heating: the shadowed region contracts and deepens the gap, while the illuminated far side expands and is elevated.
Fig. 3.— Illustration of how inclination affects brightness of scattered light. The path length through the illuminated layer represents the relative brightness seen by each observer. The cosine of the angle of incidence of stellar light at the surface is $\mu$. The angle between the surface normal and the observer is $\eta$. For $\eta_2 > \eta_1$, there are more scatterers along the path through the illuminated layer for the observer at $\eta_2$, so the disk appears brighter.
Fig. 4.— Temperature cross-sections of disks with and without gaps, where $r$ is the radial distance and $z$ is vertical height above the midplane. Contours indicate temperature in Kelvins and are spaced at 10K intervals for (a) the unperturbed disk, and in gaps created by (b) a $70 \, M_⊕$ and (c) $200 \, M_⊕$ planet. In both (b) and (c), the planet is at radius 10 AU and in the midplane of the disk. The color scale in shows the absolute disk temperature in (a), and the deviation from the unperturbed temperatures in (b) and (c).
Fig. 5.— Surface density (top) and midplane temperature (bottom) profiles for a disk with and without a gap created by a planet at 10 AU. Solid, dotted, and dashed lines indicate planet masses of 0, 70, and 200 $M_\oplus$, respectively. The thermal perturbation is caused by shadowing and illumination by stellar irradiation at the disk surface.
Fig. 6.— Simulated images of disks with and without gaps at 1 (top), 10 (middle), and 30 (bottom) microns. The left, center, and right images in each row show disks with 0, 70, and 200 M\(_\oplus\) planets, respectively. The contours are spaced at intervals of factors of two in brightness, and the colors trace brightness according to the displayed color bars.
Fig. 7.— Same as Figure 6, but at 0.1 (top), 0.3 (middle), and 1 (bottom) mm.
Fig. 8.— Radial surface brightness profiles of disks with and without gaps at the wavelengths imaged in Figures 6 & 7, as indicated. The gapless disk surface brightness are plotted with dotted lines. Surface brightness profiles of gaps created by 70 and 200 $M_\odot$ planet are plotted as dashed and solid lines, respectively.
Fig. 9.— SEDs of face-on disks with and without gaps. The SEDs are calculated by summing the total thermal emission over the simulated disks. Since the radiative transfer calculation includes only the annulus lying 3 to 20 AU from the star, the flux approaches that of the whole disk only at wavelengths from 10 to 100 microns, where the annulus emits most strongly. Nevertheless, the difference in fluxes should be representative. Top: the disk integrated spectrum of a disk without a gap is plotted as a solid line, while the disks with gaps carved by 70 and 200 $M_{\oplus}$ planets are plotted as dotted and dashed lines, respectively. The dot-dashed line shows the stellar photosphere. Bottom: the difference between the gapped disk models and the gap-less disk are plotted in magnitudes. The 70 and 200 $M_{\oplus}$ models are shown as dotted and dashed lines, respectively. The maximum deviation from the gapless model occurs at around 30 – 40 $\mu$m, corresponding to the peak of emission for temperature of the disk surface at the gap radius.
Fig. 10.— Images of inclined disks without gaps at 1, 10, 30, 100, 300, and 1000 μm. The disks are inclined at 45°, with the upper edge tilted away from the observer. The vertical asymmetry of the images is a geometric effect, as discussed in the text.
Fig. 11.— Schematic of inclination and aspect angles in the disk. The yellow circle represents the central star and the gray flared wedges represent the optically thick interior of the disk. Points that are equidistant from the star in the deprojected disk with a distance of $r_d$ appear to be at different distances on the near side versus the far side of the disk when the disk is inclined at an inclination angle of $i$. The observer sees only the surface of the disk above the optically thick region. The aspect angle $\alpha$ is the angle that this surface makes with the disk midplane. The points on the near and far sides of the disk appears to be at a distances $r_n$ and $r_f$, respectively. Moreover, the angles with respect to the surface normals along these lines of sight are different, with $\eta_1 = i - \alpha$ and $\eta_2 = i + \alpha$. Since $\eta_2 > \eta_1$, the nearer point appears brighter than the far point.
Fig. 12.— Radial surface brightness profiles of a disk without any gap at 1, 10, 30, 100, 300, and 1000 microns, as indicated. The solid black line is the profile for a face on disk. The cyan, red, and blue lines show the profiles for a disk tilted at 45°, along the major axis, far minor axis, and near minor axis, respectively. When the profiles along the minor axes are deprojected, accounting for both the inclination angle and the wavelength-dependent aspect angle, they shift to the red and blue dashed lines, respectively. The geometrical average of these deprojected curves is plotted as the black dashed line. At all wavelengths, this average deprojected profile is a good fit to the major axis. This relation may be an effective way to determine inclination angles of image protoplanetary disks.
Fig. 13.— Images of a disk tilted at 45° with a gap created by a 70 $M_⊕$ planet, at 1, 10, 30, 100, 300, and 1000 microns. Note that the near side of the disk (bottom edge) is both more foreshortened and brighter at shorter wavelengths.
Fig. 14.— Images of a disk tilted at 45° with a gap created by a 200 $M_{\oplus}$ planet.
Fig. 15.— Gap parameters for the tilted disk. The positions of the local minimum and maximum of the brightness profile along the major axis (cyan), near side minor axis (blue), and far side minor axis (red) are at $r_{\text{min}}$ ($\times$) and $r_{\text{max}}$ (+), respectively. Points are offset horizontally from the wavelength of observation so that the left and right points represent gaps produced by 70 and 200 $M_{\oplus}$, respectively. Top panel: The brightness at $r_{\text{min}}$ and $r_{\text{max}}$, scaled to the brightness of a face-on gap-less disk at 10 AU. Points in black are the values for face-on gapped disk. Second panel: Values of $r_{\text{min}}$ and $r_{\text{max}}$. Third panel: Inclination derived from values of $r_{\text{max}}$ and $r_{\text{min}}$. Actual inclination is 45°. Bottom panel: Aspect angle derived from values of $r_{\text{max}}$ and $r_{\text{min}}$. Actual values plotted as dotted lines.
Fig. 16.— Observations of the disk with inner hole in LkCa 15. Left: H band image by Thalmann et al. (2010), with the scale bar representing 140 AU. Right: 880 μm image by Andrews et al. (2011), with the scale bar representing 50 AU. Both images show a deficit of disk material in the inner ∼ 50 AU of the disk.
Fig. 17.— Surface density profiles of the disks used to model LkCa 15. The dotted line shows the unperturbed profile with no planet. The dot-dashed, dashed, and solid lines show gaps created by planets of 18, 51, and 155 $M_\oplus$ at 40.7, 38.3, and 32.5 AU, respectively.
Fig. 18.— H-band images of models for LkCa 15 with a 50 M⊕ (left) and 0.5 (right) M₉ planet. The disk is inclined at 52° and oriented so that the top edge of the disk is tipped toward the observer. The contours are spaced by factors of 2 in brightness. The images have been convolved with a Gaussian PSF of FWHM of 0′.055, as represented by the size of the black circle centered on the position of the star, which is indicated by the white cross. Both images over-predict emission from the inner disk, indicating that the gap is wider than one created by a single planet of 0.5 M₉.
Fig. 19.— Simulated radio images of LkCa 15 at 1.3 and 0.45 mm for the 0.5 $M_J$ planet model. The disk is oriented the same as in Figure 18, with the upper edge of the disk tilted toward the observer. The leftmost images are shown at infinite resolution, while the remaining panels show the images convolved with Gaussian PSFs with FWHM of $0''.05$, $0''.1$, and $0''.2$, as indicated by the circles in the lower left corner of each plot. The brightness is shown in reverse video, so the darker regions greater in intensity. The blacked out center ovals outlined in magenta indicate the regions outside the simulation boundaries and convolved with the beam size. The contours are spaced at intervals of 5 mJy-asec$^{-2}$ beginning at 10 mJy-asec$^{-2}$ in the 1.3 mm images and 0.1 Jy-asec$^{-2}$ beginning at 0.1 Jy-asec$^{-2}$ in the 0.45 mm images. In continuum, the far edge appears brighter at lower angular resolution not because of an inherent difference in brightness, but because the angular size of the frontally illuminated far side of the gap is larger than the near side.