Introduction to Atmospheres

ASTR 5420 – Stellar Evolution & Interiors

Friday, February 5, 2016

1 Einstein Coefficients

(references: LeBlanc §4.3.5, Rybicki & Lightman 1.6)

When we are discussing line absorption/emission, we are talking about photons emitted as an atom or molecule transitions between one quantum state and another: bound-bound transitions.

Draw energy levels $E_1 \to E_2$, energy difference $h\nu_{12}$, statistical weights g_1 and g_2.

Spontaneous emission:

$$A_{21} = \text{transition probability per unit time for spon. em. } [\text{s}^{-1}]$$

Absorption:

Depends on the radiation field J_ν, and the line profile. Draw an example line profile, $\phi(\nu)$.

$$\int_0^\infty \phi(\nu)\, d\nu = 1$$

Define

$$\bar{J} = \int_0^\infty J_\nu \phi(\nu)\, d\nu$$

then the transition probability for absorption is

$$B_{12}\bar{J}$$

Stimulated emission:

$$B_{21}\bar{J}$$

We have A_{21}, B_{12}, and B_{21} for our Einstein coefficients. They are related by

$$n_1 B_{12}\bar{J} = n_2(A_{21} + B_{21}\bar{J})$$

$$\bar{J} = \frac{A_{21}/B_{21}}{(n_1/n_2)(B_{12}/B_{21}) - 1}$$

Recall for the Boltzmann distribution, for two excitation states of the same species,

$$\frac{n_1}{n_2} = \frac{g_1}{g_2} e^{-(\mathcal{E}_1 - \mathcal{E}_2)/kT} = \frac{g_1}{g_2} e^{h\nu_0/kT}$$

So

$$\bar{J} = \frac{A_{21}/B_{21}}{(g_1/g_2)(B_{12}/B_{21})e^{h\nu_0/kT} - 1}$$

In thermodynamic equilibrium, $J_\nu = B_\nu$. We can treat $\phi(\nu)$ as a delta function. Then $\bar{J} = B_{\nu_0}$. So,

$$\frac{A_{21}/B_{21}}{(g_1/g_2)(B_{12}/B_{21})e^{h\nu_0/kT} - 1} = \frac{2h\nu_0^3/c^2}{e^{h\nu_0/kT} - 1}$$

And

$$A_{21}/B_{21} = 2h\nu_0^3/c^2$$

$$(g_1/g_2)(B_{12}/B_{21}) = 1$$

The absorption coefficient can be written

$$\alpha_\nu (= \rho\kappa_\nu) = \frac{h\nu}{4\pi}\phi(\nu)(n_1 B_{12} - n_2 B_{21})$$

Interpret: $\phi(\nu)$ is a sharply peaked function about ν_0. If $\tau = \int \alpha\, ds$, then this means that the opacity increases sharply around $\nu = \nu_0$.

Analogy to limb darkening:

```
------ tau=0         ------- -------- F(tau=2/3)
                           | |
                           |_|_______ F(tau_line=2/3)


------ tau_line=2/3


------ tau=2/3       ---------------- F(tau=0)
```

If the temperature at τ_{line} is less than temperature at $\tau = 2/3$, then then the flux at ν_0 is less than the continuum, and you see an absorption line. Similarly, if the temperature is higher at τline, then you see emission.

1.1 Inverted populations: masers, lasers

If the system is in thermal equilibrium, then

$$\frac{n_2 g_1}{n_1 g_2} = \exp(-h\nu/kT) < 1$$

$$\frac{n_1}{g_1} > \frac{n_2}{g_2}$$

Suppose the relation is inverted

$$\begin{aligned} \frac{n_1}{g_1} &< \frac{n_2}{g_2} \\ n_1 B_{12} &< \frac{g_1 n_2}{g_2} B_{12} = n_2 B_{21} \end{aligned}$$

Then the absorption coefficient is *negative*, so intensity increases along the ray. This is what creates a maser or laser (microwave/light amplification of stimulated emission of radiation). Seen in astrophysical sources like star formation regions, ISM of quasars. Inverted populations may be caused by shocks, radiative pumping. Not observed in stars themselves.

1.2 Oscillator Strengths

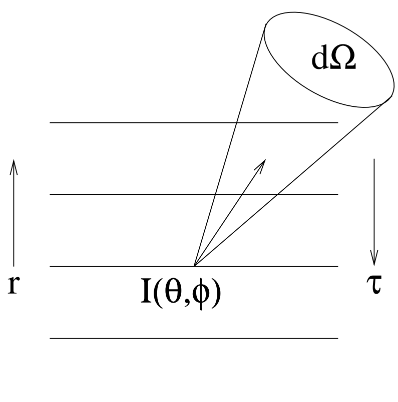
$$B_{ij} = \frac{4\pi^2 e^2}{h\nu_{ij} m_e c} f_{ij}$$

where f_{12} is the oscillator strength. It is a quantum mechanical correction to the classical value.

2 Plane-parallel Atmosphere

Imagine a plane-parallel atmosphere. The **plane-parallel approximation** relies on two assumptions:

1. Quantities vary only radially
2. The radius of the star is large compared to the distance over which quantities change so that successive layers are roughly planar.



- θ is the angle with respect to r
- ϕ is the azimuthal angle.
- Ω refers to solid angle, and $d\Omega = \sin\theta \, d\theta \, d\phi$.

Let $\mu = \cos\theta$, with θ= the angle with respect to $\hat{\mathbf{r}}$. Then

$$ds = \frac{dz}{\cos\theta} = \frac{dz}{\mu}$$

$$\mu \frac{dI_\nu(z,\mu)}{dz} = -\chi_\nu \rho (I_\nu - S_\nu)$$

with

$$S_\nu = \omega_\nu J_\nu + (1 - \omega_\nu) B_\nu$$

Also, $d\Omega = \mu \, d\mu \, d\phi$. Let's define τ to be the optical depth in the vertical direction,

$$\tau_\nu = -\int_{r_0}^{r} \chi_\nu \rho \, dr,$$

integrated downward vertically. Note, τ is unit-less, and $\chi_\nu = \kappa_\nu^a + \kappa_\nu^s$. Then, we have

$$\mu \frac{dI_\nu(z,\mu)}{d\tau_\nu} = I_\nu - S_\nu$$

$$I_\nu = S_\nu + \mu \frac{dI_\nu(z,\mu)}{d\tau_\nu}$$

In the deep interior, I_ν will change slowly with τ, so lhs vanishes and $I_\nu \approx S_\nu$. This also means that I_ν is roughly independent of angle, so $I_\nu \approx J_\nu$. In **local thermodynamic equilibrium** (LTE) we expect $J_\nu \approx B_\nu(T)$. Then we can sub in the above equation,

$$I_\nu(\tau) \approx B(\tau) + \mu \left(\frac{\partial B}{\partial \tau} \right)$$

Recall

$$\begin{aligned} F_\nu &= \int I_\nu \cos\theta \, d\Omega \\ &= \frac{4\pi}{3} \left(\frac{\partial B}{\partial \tau} \right) \\ &= -\frac{4\pi}{3} \frac{1}{\chi_\nu \rho} \frac{\partial B}{\partial r} = -\frac{4\pi}{3} \frac{1}{\chi_\nu \rho} \frac{\partial T}{\partial r} \frac{\partial B}{\partial T} \end{aligned}$$

We define the **Rosseland mean opacity** as

$$\boxed{\frac{1}{\chi_R} = \frac{\int_0^\infty \frac{1}{\chi_\nu} \frac{\partial B_\nu}{\partial T} \, d\nu}{\int_0^\infty \frac{\partial B_\nu}{\partial T} \, d\nu}} \tag{1}$$

Then

$$\begin{aligned} F = \int_0^\infty F_\nu \, d\nu &= -\frac{4\pi}{3\rho} \frac{\partial T}{\partial r} \int_0^\infty \frac{1}{\chi_\nu} \frac{\partial B}{\partial T} \, d\nu \\ &= -\frac{4\pi}{3\rho\chi_R} \frac{\partial T}{\partial r} \frac{\partial}{\partial T} \left(\frac{\sigma_B}{\pi} T^4 \right) = -\frac{4\pi}{3\rho\chi_R} \frac{\partial T}{\partial r} \frac{\partial}{\partial T} \left(\frac{ac}{4\pi} T^4 \right) \\ F_{\text{rad}} &= -\frac{c}{3\rho\chi_R} \frac{\partial a T^4}{\partial r} = -D \frac{\partial U}{\partial r} \end{aligned}$$

This is the **radiative diffusion equation**, with $D = c/(3\rho\chi_R)$ and $U = aT^4$ is the radiative energy density. Also, we used the relation $a = 4\sigma_B/c$. Note that $D \propto 1/\chi$.

2.1 When is the Diffusion Approximation Valid?

Holds true when material is optically thick and in **local thermodynamic equilibrium** (LTE).

First criterion is that we are in an optically thick regime, $\tau > 1$. Recall the optical depth:

$$\tau = -\int_R^r \chi\rho\, dr, = -\int_R^r \frac{1}{\ell_{\rm ph}}\, dr$$

Another way of stating this is that the mean free path is smaller than the distance to the stellar surface.

LTE: This means that the energy density of radiation changes slowly over a mean free path: $U = aT^4$.

$$\frac{\Delta U}{U} \approx \frac{\ell_{\rm ph}}{U}\frac{\partial U}{\partial r} = \frac{\ell_{\rm ph}}{aT^4} 4aT^3 \frac{\partial T}{\partial r} = \frac{4\ell_{\rm ph}}{T}\frac{\partial T}{\partial r}$$

Estimate for sun.