PHYSICS 1210-02 Final Exam

University of Wyoming

11 December 2013

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If you have a question about the test, please raise your hand. Please do not open this test booklet until everyone has received a booklet and the test administrator has indicated for you to begin. While you are waiting, make sure that your name is written clearly at the top of the first page.

Please write directly on the exam and attach other sheets of work if necessary. Multiple-choice problems have only one correct answer. You may choose to circle two answers on a multiple-choice problem and, if one of them is correct, receive half credit. Circle three and if one is correct, 1/3 credit, etc.

For worked problems, be complete and show all work, beginning with diagrams and fundamental, general equations used. Circle your final answer for each part of each worked problem.

Kinematics

$v_{avg} = \frac{x_2 - x_1}{t_2 - t_1} = \frac{\Delta x}{\Delta t}$ $\quad \vec{v} = \frac{d\vec{r}}{dt}$ $\quad a_{avg} = \frac{v_2 - v_1}{t_2 - t_1} = \frac{\Delta v}{\Delta t}$ $\quad \vec{a} = \frac{d\vec{v}}{dt}$

$x_1 = x_0 + v_0 t + \frac{1}{2} a t^2$ $\quad v_1 = v_0 + a t$ $\quad v_1^2 = v_0^2 + 2a(x_1 - x_0)$ $\quad a_{rad} = \frac{v^2}{R} = \frac{4\pi^2 R}{T^2}$

$\sum \vec{F} = m\vec{a}$ $\quad \vec{w} = m\vec{g}$ $\quad f_s \le \mu_s N$ $\quad f_k = \mu_k N$ $\quad f = kv$ $\quad f = Dv^2$ $\quad f_{spring} = -kx$

Momentum/Impulse

$\vec{p} = m\vec{v}$ $\quad J = \Delta(mv) = F\Delta t$ $\quad \vec{F} = \frac{d\vec{p}}{dt}$ $\quad x_{cm} = \frac{\sum m_i x_i}{\sum m_i}$

Work/Energy

$W = \vec{F} \cdot \vec{s} = Fs\cos\theta$ $\quad K_1 + U_1 + W_{other} = K_2 + U_2$ $\quad P = \frac{\Delta W}{\Delta t} = \vec{F} \cdot \vec{v}$

$W = \Delta K$ $\quad K = \frac{1}{2} m v^2$ $\quad U_{spring} = \frac{1}{2} k x^2$ $\quad U_{grav} = mgy$ $\quad F = -\frac{dU}{dx}$

Angular Motion

$\omega = \frac{d\theta}{dt}$ $\quad \alpha = \frac{d\omega}{dt}$ $\quad \theta_1 = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$ $\quad \omega_1 = \omega_0 + \alpha t$ $\quad \omega_1^2 = \omega_0^2 + 2\alpha(\theta_1 - \theta_0)$

$s = r\theta$ $\quad v = r\omega$ $\quad a_{tan} = r\alpha$ $\quad a_{rad} = \omega^2 r$ $\quad 2\pi = 360^\circ$

$I = \sum_i m_i r_i^2$ $\quad I = I_{cm} + Md^2$ $\quad \vec{\tau} = \vec{r} \times \vec{F} = rF\sin\phi$ $\quad \vec{\tau} = \frac{d\vec{L}}{dt}$ $\quad \sum \vec{\tau} = I\vec{\alpha}$ $\quad W = \tau \Delta\theta$

$K_{rot} = \frac{1}{2} I\omega^2$ $\quad K_{tot} = \frac{1}{2} m v_{cm}^2 + \frac{1}{2} I_{cm}\omega^2$ $\quad \vec{L} = \vec{r} \times \vec{p} = rmv = I\omega$ $\quad \Delta L = \tau \Delta t$ $\quad \text{power} = \tau\omega$

$g = 9.8\ \text{m/s}^2$
1 radian = 57.3°
1 mile = 1.609 km
1 m = 39.37 in = 3.28 ft
$\rho_{water} = 1000\ \text{kg/m}^3$
$\rho_{ice} = 920\ \text{kg/m}^3$
$\rho_{air} = 1.20\ \text{kg/m}^3$
$G = 6.6743 \times 10^{-11}\ \text{N-m}^2/\text{kg}^2$
$M_{Sun} = 1.99 \times 10^{30}$ kg
$M_{Earth} = 5.97 \times 10^{24}$ kg
$M_{Moon} = 7.35 \times 10^{22}$ kg
1 AU = 1.50×10^{11} m
$R_{Sun} = 6.96 \times 10^8$ m
$R_{Earth} = 6.37 \times 10^6$ m
$R_{Moon} = 1.74 \times 10^6$ m
Moon distance = 3.84×10^8 m

Fluids

$B = \rho V g$ $\quad p = \frac{dF_\perp}{dA}$

$\frac{dV}{dt} = A_1 v_1 = A_2 v_2$

$\frac{dm}{dt} = \rho_1 A_1 v_1 = \rho_2 A_2 v_2$

$p_2 - p_1 = -\rho g(y_2 - y_1)$

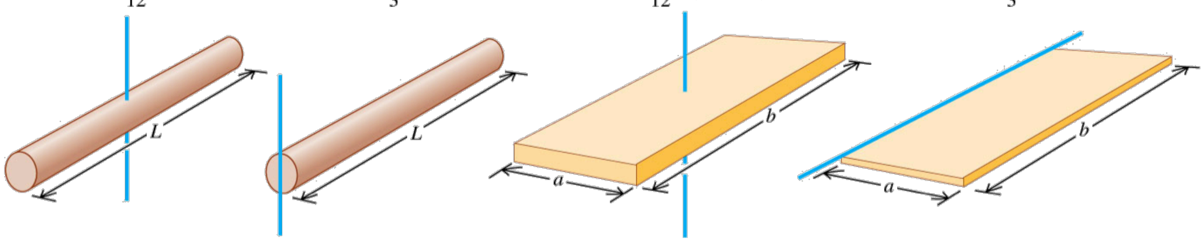
$p_1 + \rho g y_1 + \frac{1}{2}\rho v_1^2 = p_2 + \rho g y_2 + \frac{1}{2}\rho v_2^2$

(a) Slender rod, axis through center $\quad I = \frac{1}{12} ML^2$

(b) Slender rod, axis through one end $\quad I = \frac{1}{3} ML^2$

(c) Rectangular plate, axis through center $\quad I = \frac{1}{12} M(a^2 + b^2)$

(d) Thin rectangular plate, axis along edge $\quad I = \frac{1}{3} Ma^2$



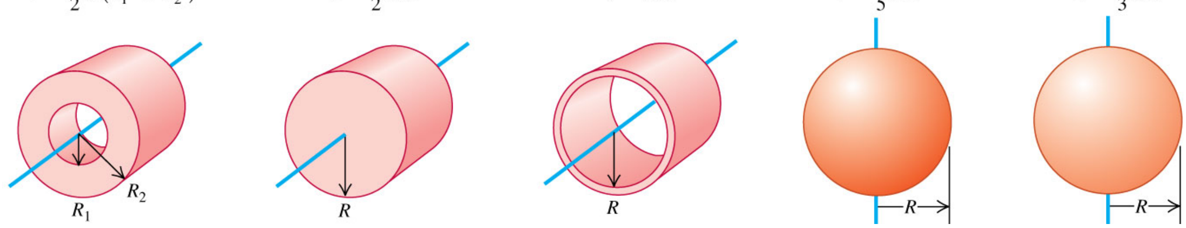
(e) Hollow cylinder $\quad I = \frac{1}{2} M(R_1^2 + R_2^2)$

(f) Solid cylinder $\quad I = \frac{1}{2} MR^2$

(g) Thin-walled hollow cylinder $\quad I = MR^2$

(h) Solid sphere $\quad I = \frac{2}{5} MR^2$

(i) Thin-walled hollow sphere $\quad I = \frac{2}{3} MR^2$



Gravity

$G = 6.67 \times 10^{-11}\ \text{N m}^2/\text{kg}^2$

$F_g = \frac{GM_1M_2}{r^2}$ $\quad U_g = \frac{-GM_1M_2}{r}$

$v_{escape} = \sqrt{\frac{2GM_1}{r}}$

$v_{orbit} = \sqrt{\frac{GM_1}{r}}$

$T_{orbit} = \frac{2\pi r}{v_{orbit}} = 2\pi\sqrt{\frac{r^3}{GM_1}}$

Periodic Motion

$f = 1/T$ $\quad \omega = 2\pi f$ $\quad x = A\cos(\omega t + \phi)$ $\quad E = \frac{1}{2} m v^2 + \frac{1}{2} k x^2 = \frac{1}{2} k A^2$

$\omega = \sqrt{\frac{k}{m}}$ $\quad$ Simple pendulum $\omega = \sqrt{\frac{g}{L}}$ $\quad$ Physical pendulum $\omega = \sqrt{\frac{mgd}{I}}$ $\quad$ Damped oscillations $x = Ae^{-(b/2m)t}\cos(\omega' t + \phi);\ \omega' = \sqrt{\frac{k}{m} - \frac{b^2}{4m^2}}$

Mechanical Waves

$v = \lambda f$ $\quad y(x,t) = A\cos(kx - \omega t)$ $\quad k = \frac{2\pi}{\lambda}$ $\quad v = \frac{\omega}{k}$ $\quad I = \frac{P}{4\pi r^2}$ $\quad \frac{I_1}{I_2} = \frac{r_2^2}{r_1^2}$

Waves on a string $\quad v = \sqrt{\frac{F}{\mu}}$ $\quad f_1 = \frac{v}{2L}$ $\quad f_n = nf_1, (n = 1,2,3,\ldots)$ $\quad P_{av} = \frac{1}{2}\sqrt{\mu F}\omega^2 A^2$

Sound

$p_{max} = BkA$ $\quad v = \sqrt{B/\rho}$ $\quad I = \frac{p_{max}^2}{2\rho v} = \frac{p_{max}^2}{2\sqrt{\rho B}}$ $\quad \beta = (10\ \text{dB})\log\frac{I}{I_0}$

Open pipe $f_n = \frac{nv}{2L}, (n = 1,2,3,\ldots)$ $\quad$ Closed pipe $f_n = \frac{nv}{4L}, (n = 1,3,5,\ldots)$ $\quad f_{beat} = f_a - f_b$ $\quad f_L = \frac{v + v_L}{v + v_S} f_S$

Name: ______________________________

MULTIPLE CHOICE: Questions 1-11 are 5 points each.

1. To double the total energy of a mass-spring system oscillating in simple harmonic motion, the amplitude must increase by a factor of

a. 16
b. 4
c. $2\sqrt{2}$
d. 2
e. $\sqrt{2}$
f. $\sqrt[4]{2}$

$$E = \frac{1}{2}kA^2$$

2. A person swings on a swing. When the person sits still, the swing oscillates back and forth at its natural frequency. If, instead, the person stands on the swing, the natural frequency of the swing is

a. greater.
b. smaller.
c. the same.

Mass moves closer to the pivot, increasing the frequency.

3. Two identical symmetric pulses of opposite amplitude travel along a stretched string and interfere destructively. Which of the following is/are true?

a. When the two pulses interfere, the energy of the pulses is momentarily zero.
b. There is an instant at which the string is completely straight.
c. There is a point on the string that does not move up or down.
d. There are several points on the string that do not move up or down.

When two equal but opposite pulse overlap, they cancel each other out exactly.

Name: ______________________________

4. Prof. Krabappel is delivering a physics lecture. Lisa is sitting 3m away in the front row, and Bart is sitting 30m away in the back row. How does the intensity of Prof. Krabappel's voice sound to Lisa as compared to Bart?

a. 1/100 the intensity.
b. 1/10 the intensity.
c. The same intensity.
d. 3 times the intensity.
e. 10 times the intensity.
f. 30 times the intensity.
g. 100 times the intensity. Intensity $\propto 1/r^2$
h. 1000 times the intensity.
i. 10000 times the intensity.

5. A hollow ball and a solid ball of the same mass and radius initially have the same total kinetic energy when they start rolling up the same ramp of constant angle above the horizontal. Which of the following is true?

a. They both go the same height up the ramp before coming to a stop.
b. They both have the same moment of inertia.
c. They both started with the same translational speed.
d. They both reach the top of their maximum height at the same time.
e. They both started with the same angular speed.

When they stop, all the kinetic energy (both translation and rotational) are converted into gravitational potential energy.

6. On a day when there is no wind, you are moving toward a stationary source of sound waves. Compared to what you would hear if you were not moving, the sound that you hear has

a. the same frequency and the same wavelength.
b. the same frequency and a shorter wavelength.
c. the same frequency and a longer wavelength.
d. a lower frequency and the same wavelength.
e. a lower frequency and a shorter wavelength.
f. a lower frequency and a longer wavelength.
g. a higher frequency and the same wavelength.
h. a higher frequency and a shorter wavelength.
i. a higher frequency and a longer wavelength.

Wavelength doesn't change if the source doesn't move. However, the time between arrivals of wave crests is compressed as you move toward the source.

7. A uniform solid sphere rolls without slipping along a level surface. What fraction of its total kinetic energy is rotational, and what fraction is translational?

a. 1/3 rotational and 2/3 translational.
b. 2/3 rotational and 1/3 translational.
c. 1/5 rotational and 4/5 translational.
d. 4/5 rotational and 1/5 translational.
e. 2/5 rotational and 3/5 translational.
f. 3/5 rotational and 2/5 translational.
g. 2/7 rotational and 5/7 translational.
h. 5/7 rotational and 2/7 translational.
i. 1/2 rotational and 1/2 translational.

$E_{trans} = ½mv^2$

$E_{rot} = ½I\omega^2 = ½(2/5)mr^2\omega^2 = (1/5)\ mv^2$

$E_{tot} = (1/2+1/5)\ mv^2 = (7/10)\ mv^2$

$E_{trans}/E_{tot} = (1/2)/(7/10) = 5/7$

$E_{rot}/E_{tot} = (1/5)/(7/10) = 2/7$

8. A fish bites at a baited hook and swims downward, pulling the fishing line and float down with it. As the fish pulls the float deeper below the surface, how does the inward pressure on the float change?

a. The pressure increases.
b. The pressure does not change.
c. The pressure decreases.

Pressure increases with depth: $p = p_0 + \rho g d$

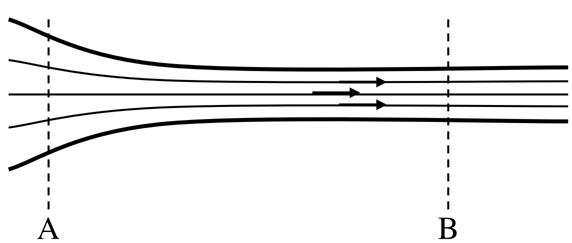
9. In the scenario of the previous question, how does the upward force of buoyancy acting on the submerged float change as the fish pulls the float deeper under water?

a. The buoyancy force increases.
b. The buoyancy force does not change.
c. The buoyancy force decreases.

Buoyancy depends only on mass of water displaced. Water is incompressible.

10. Water flows through a pipe with a narrowing radius, as shown in the diagram. Where is the pressure of the water the greatest?

a. At point A.
b. At point B.
c. It is the same at A and B.



Speed is higher at B, so according to Bernoulli's law, the pressure is lower at B.

11. The Kuiper Belt Object known as Pluto is about 1/5 the mass of Earth's Moon and averages about 40 times farther from the Sun than Earth's Moon. Compared to the average gravitational pull Earth's Moon exerts on the sun, how strong is the gravitational pull that Pluto exerts on the sun?

a. 5/40 the gravitational pull of Earth's Moon.
b. $1/(5 \cdot 40)$ the gravitational pull of Earth's Moon.
c. $5/40^2$ the gravitational pull of Earth's Moon.
d. $1/(5^2 \cdot 40^2)$ the gravitational pull of Earth's Moon.
e. $1/(5 \cdot 40^2)$ the gravitational pull of Earth's Moon.
f. 40/5 the gravitational pull of Earth's Moon.
g. $1/(5^2 \cdot 40)$ the gravitational pull of Earth's Moon.
h. $5^2/40$ the gravitational pull of Earth's Moon.
i. $5^2/40^2$ the gravitational pull of Earth's Moon.
k. $40^2/5$ the gravitational pull of Earth's Moon.

$F = Gm_1m_2/r^2$

m_1 is the mass of the sun, m_2 is the mass of the Moon or Pluto, r is the distance.

Pluto's mass is 1/5 that of the moon, so that decreases the force by a factor of 5. Pluto is 40 times further away, so that decreases the force by a factor of 40^2.

FREE RESPONSE: Questions 12-14 are 15 points each. Show all your work and clearly indicate your answer for each section.

12. You discover a new exoplanet orbiting a star that is 0.750 times the mass of the Sun, or 1.50×10^{30} kg. It completes one orbital period in 10 days, or 8.64×10^{5} s. Assuming the orbit is perfectly circular, what is the radius of its orbit in AU? (1 AU = 1.50×10^{11} m)

$v_{\mathrm{circ}} = (GM/r)^{1/2}$

$T = 2\pi r/v_{\mathrm{circ}} = 2\pi r/(GM/r)^{1/2} = 2\pi r^{3/2}/(GM)^{1/2}$

$r^3/GM = (T/2\pi)^2 \Rightarrow r = [GM(T/2\pi)^2]^{1/3}$

$r = [(6.67\mathrm{e}{-11}\ \mathrm{N\ m^2/kg^2})(1.50\mathrm{e}30\ \mathrm{kg})(8.64\mathrm{e}5\ \mathrm{s}/2\pi)^2]^{1/3}$

$= [(1.00\mathrm{e}20\ \mathrm{N\ m^2/kg})(1.89\mathrm{e}10\ \mathrm{s^2})]^{1/3} = (1.89\mathrm{e}30\ \mathrm{m^3})^{1/3}$

= 1.24e10 m * (1 AU/1.50e11 m)

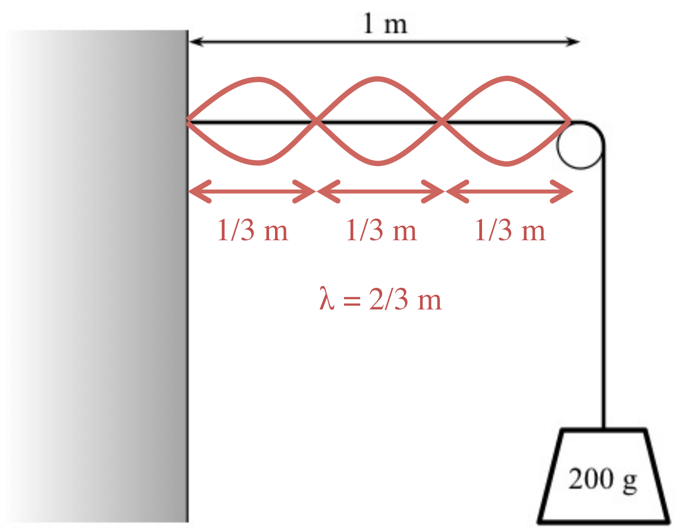
= **0.0824 AU**.

13. A stretchy cord of mass 10.0 g and unstretched length 2.00 m is tied to an immovable wall at one end. The other end is draped over a pulley 1.00 m from the wall, and then a 0.200 kg mass is attached. The cord has an elastic coefficient of k=5.00 kg/s^2.

a. [5 pts] What is μ (mass per length) of the stretched cord?

b. [5 pts] If the length of rope between the wall and the pulley is plucked like a guitar string, how long will it take for the wave pulse to travel from the wall to the pulley?

c. [5 pts] On the diagram, draw the appearance of the string if it is vibrating in the third harmonic (2nd overtone).



a) The string stretches like an ideal spring: $F = -kx$.
F = weight of the 0.200 kg mass = mg = (0.200 kg)(9.8 m/s^2) = 1.96 N
k = 5.00 kg/s^2
$x = F/k$ = (1.96 N)/(5.00 kg/s^2) = 0.392 m
New length = original length + x = 2.00 m + 0.392 m = 2.39 m.
Mass per length = mass / total length = (0.010 kg)/(2.39 m) = **0.00418 kg/m**

b) Need to know the speed of the wave: $v = (F/\mu)^{1/2}$ = (1.96 N/0.00418 kg/m)$^{1/2}$
v = 21.7 m/s.
Distance between the wall and the pulley = 1.00 m.
$t = d/v$ = (1 m)/(21.7 m/s) = **0.0462 s.**

c) See diagram above.

14. Dori, whose mass is 65 kg, climbs into an empty oak barrel of mass 60 kg. The barrel is sealed watertight and sent floating down a river. The barrel has an interior volume of 0.120 m^3, oak has a density of 770 kg/m^3, and river water has a density of 1000 kg/m^3.

Will the barrel sink or float?

If it floats, what percentage of the volume of the entire barrel will be above the water?

If it sinks, what minimum force must be applied to a rope attached to the barrel in order to raise it to the surface?

Whether the barrel will sink or float depends on how its total density compares to the density of water.

The total mass is $m_{Dori}+m_{barrel}$ = 65 kg + 60 kg = 125 kg.

The total volume is $V_{interior}+V$barrel.

$V_{interior}$ = 0.120 m^3

$V_{barrel} = m_{barrel}/\rho_{barrel}$ = 60 kg / 770 kg/m^3 = 0.0779 m^3

V_{total} = 0.120 m^3 + 0.0779 m^3 = 0.198 m^3

Therefore, the total density is $\rho = m/V$ = 125 kg/0.198 m^3 = **631 kg/m³**. Since this is less than 1000 kg/m^3, **the barrel will float**.

The floating barrel has two forces acting on it: the buoyant force pushing up, and the weight force pulling it down. These forces are equal and opposite.

The buoyant force is $B = \rho_{water}V_{disp}g$ where V_{disp} is the volume displaced by the barrel+Dori.

The weight force is $w = mg = \rho_{total}V_{total}g$.

Setting these equal, $\rho_{water}V_{disp}g = \rho_{total}V_{total}g \Rightarrow \rho_{water}V_{disp} = \rho_{total}V_{total}$.

The fraction of the barrel underwater is $V_{disp}/V_{total} = \rho_{total}/\rho_{water} = 0.631$.

Therefore, the fraction of the barrel above the water is $1 - V_{disp}/V_{total}$ = **0.369**, or **36.9%**.