A mass attached to a spring oscillates back and forth as indicated in the position vs. time plot below. At point P, the mass has



- I. positive velocity and positive acceleration.
- 2. positive velocity and negative acceleration.
- 3. positive velocity and zero acceleration.
- 4. negative velocity and positive acceleration.
- 5. negative velocity and negative acceleration.
- 6. negative velocity and zero acceleration.
- 7. zero velocity but is accelerating (positively or negatively).

Text 'PHYSJC' and your answer to 22333

Schwarzchild Radius

- $R_{\rm S} = 2GM/c^2$
- For the Sun (M = 2e30 kg):
 - $R_{\rm S} = 3 \text{ km}$
- For a 1,000,000 solar mass black hole:
 - $R_{\rm S} =$ 3,000,000 km

Lab This Week

- We are doing Lab A on fluids. This is **not** in your lab manual.
- Download a packet from the course website

Ch 14:1-3 -- Simple Harmonic Motion

PHYS 1210 -- Prof. Jang-Condell

Chapter 14

Periodic Motion

PowerPoint[®] Lectures for University Physics, Thirteenth Edition – Hugh D. Young and Roger A. Freedman

Lectures by Wayne Anderson

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Monday, April 18, 16

Goals for Chapter 14

- To describe oscillations in terms of amplitude, period, frequency and angular frequency
- To do calculations with simple harmonic motion
- To analyze simple harmonic motion using energy
- To apply the ideas of simple harmonic motion to different physical situations
- To analyze the motion of a simple pendulum
- To examine the characteristics of a physical pendulum
- To explore how oscillations die out
- To learn how a driving force can cause resonance

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What causes periodic motion?

• If a body attached to a spring is displaced from its equilibrium position, the spring exerts a *restoring force* on it, which tends to restore the object to the equilibrium position. This force causes *oscillation* of the system, or *periodic motion*.



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What causes periodic motion?

- If a body attached to a spring is displaced from its equilibrium position, the spring exerts a *restoring force* on it, which tends to restore the object to the equilibrium position. This force causes *oscillation* of the system, or *periodic motion*.
- Figure 14.2 at the right illustrates the restoring force F_x .

(a)



(b)



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- The **amplitude**, *A*, is the maximum magnitude of displacement from equilibrium.
- The **period**, *T*, is the time for one cycle.
- The **frequency**, *f*, is the number of cycles per unit time.



• The **frequency** and period are reciprocals of each other

$$f = 1/T$$
$$T = 1/f$$

Simple harmonic motion (SHM)

- When the restoring force is *directly proportional* to the displacement from equilibrium, the resulting motion is called *simple harmonic motion* (SHM).
- An ideal spring obeys Hooke's law, so the restoring force is $F_x = -kx$, which results in simple harmonic motion.



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Characteristics of SHM

- The displacement as a function of time for SHM with phase angle φ is

 $x = A\cos(\omega t + \phi).$

• For a body vibrating by an ideal spring:

$$\omega = \sqrt{\frac{k}{m}}$$

• The angular frequency, ω , is 2π times the frequency:

$$\omega = 2\pi f.$$

$$f = \frac{\omega}{2\pi} = \frac{1}{2\pi} \sqrt{\frac{k}{m}} \quad T = \frac{1}{f} = \frac{2\pi}{\omega} = 2\pi \sqrt{\frac{m}{k}}$$

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Simple Harmonic Motion Animation

A mass suspended from a spring is oscillating up and down as indicated. Consider two possibilities:

- (i) at some point during the oscillation the mass has zero velocity but is accelerating (positively or negatively);
 (ii) at some point during the oscillation the mass has zero velocity and zero acceleration.
 - A. Both occur sometime during the oscillation.
 - B. Neither occurs during the oscillation.
 - C. Only (i) occurs.
 - D. Only (ii) occurs.

Displacement as a function of time in SHM

- The displacement as a function of time for SHM with phase angle φ is x = Acos(ωt + φ). (See Figure 14.9 at the right.)
- Changing *m*, *A*, or *k* changes the graph of *x* versus *t*, as shown below.



(b) Increasing k; same A and m Force constant k increases from curve 1 to 2 to 3. Increasing k alone decreases the period. $3 \ 2 \ 1$ $0 \ t$



(c) Increasing A; same k and m



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 $x = A\cos(\omega t + \phi)$

$$\omega = \sqrt{\frac{k}{m}}$$

What happens if I change m, k, or A?

 $x = A\cos(\omega t + \phi)$

$$\omega = \sqrt{\frac{k}{m}}$$

(a) Increasing m; same A and k Mass *m* increases from curve 1 to 2 to 3. Increasing *m* alone increases the period. x 3 2 0

 $x = A\cos(\omega t + \phi)$

$$\omega = \sqrt{\frac{k}{m}}$$



 $x = A\cos(\omega t + \phi)$

$$\omega = \sqrt{\frac{k}{m}}$$

(c) Increasing A; same k and m

Amplitude A increases from curve 1 to 2 to 3. Changing A alone has x no effect on the period. 0

 $x = A\cos(\omega t + \phi)$

$$\omega = \sqrt{\frac{k}{m}}$$

Changing ϕ , keeping everything else same

These three curves show SHM with the same period *T* and amplitude *A* but with different phase angles ϕ .



(a) Displacement x as a function of time t



(b) Velocity v_x as a function of time t

$$v_{\max} = \omega A$$

$$\begin{array}{c} v_x & v_x = -\omega A \sin (\omega t + \phi) \\ 0 & -v_{\max} = -\omega A \end{array}$$

$$\begin{array}{c} v_x & v_x = -\omega A \sin (\omega t + \phi) \\ 0 & 1 & 1 & 1 \\ T & T & 2T \end{array}$$

$$\begin{array}{c} T & t \\ T & 2T \\ T & 2$$

(c) Acceleration a_x as a function of time t

$$a_{\max} = \omega^2 A \xrightarrow{O}_{A \max} a_x = -\omega^2 A \cos(\omega t + \phi)$$

$$a_{\max} = -\omega^2 A \xrightarrow{O}_{A \max} t$$
The *a* -t graph is shifted by $\frac{1}{2}$ cycle from the

The a_x -t graph is shifted by $\frac{1}{4}$ cycle from the v_x -t graph and by $\frac{1}{2}$ cycle from the x-t graph.

Energy in SHM

• The total mechanical energy E = K + U is conserved in SHM: $E = 1/2 mv_x^2 + 1/2 kx^2 = 1/2 kA^2 = \text{constant}$



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Q12.6

This is an *x*-*t* graph for an object connected to a spring and moving in simple harmonic motion.



At which of the following times is the *potential energy* of the spring the greatest?

- F. t = T/8G. t = T/4H. t = 3T/8
- I. t = T/2
- J. more than one of the above

Q12.7

This is an *x*-*t* graph for an object connected to a spring and moving in simple harmonic motion.



At which of the following times is the *kinetic energy* of the object the greatest?

K. t = T/8L. t = T/4M. t = 3T/8N. t = T/2

P. more than one of the above

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Energy diagrams for SHM

- Figure 14.15 below shows energy diagrams for SHM.
- Refer to Problem-Solving Strategy 14.2.
- Follow Example 14.4.

(a) The potential energy U and total mechanical energy E for a body in SHM as a function of displacement x







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Spring + Gravity

(b) A body is suspended from the spring. It is in equilibrium when the upward force exerted by the stretched spring equals the body's weight.

(c) If the body is displaced from equilibrium, the net force on the body is proportional to its displacement. The oscillations are SHM.



(a)







An object hangs motionless from a spring. When the object is pulled down, the sum of the elastic potential energy of the spring and the gravitational potential energy of the object and Earth

Q.increases.

- R. stays the same.
- S. decreases.

Angular SHM

- A coil spring (see Figure 14.19 below) exerts a restoring torque $\tau_z = -\kappa \theta$, where κ is called the *torsion constant* of the spring.
- The result is *angular* simple harmonic motion.



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