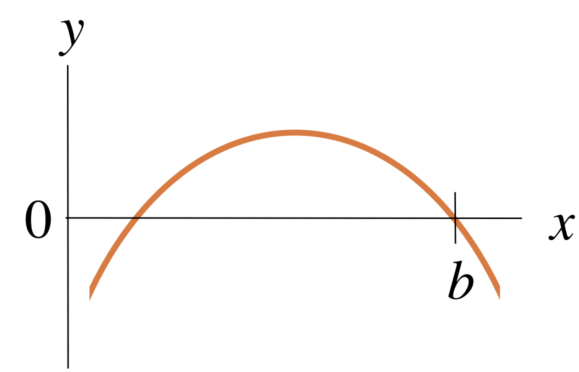
Q15.5

A wave on a string is moving to the right. This graph of $y(x, t)$ versus coordinate x for a specific time t shows the shape of part of the string at that time.



At this time, what is the *velocity* of a particle of the string at $x = b$?

A. The velocity is upward.

B. The velocity is downward.

C. The velocity is zero.

D. Not enough information given to decide.

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Announcements

- Homework 13 (next week) is extra credit
- Lab 9 this week (in your lab manual)
- Lab B next week (handout)
- Final Exam: Friday, May 13, 10:15am-12:15pm **CR 306**

Ch 15.3-5
Sinusoidal Waves

PHYS 1210 -- Prof. Jang-Condell

Goals for Chapter 15

- To study the properties and varieties of mechanical waves
- To relate the speed, frequency, and wavelength of periodic waves
- To interpret periodic waves mathematically
- To calculate the speed of a wave on a string
- To calculate the energy of mechanical waves
- To understand the interference of mechanical waves
- To analyze standing waves on a string
- To investigate the sound produced by stringed instruments

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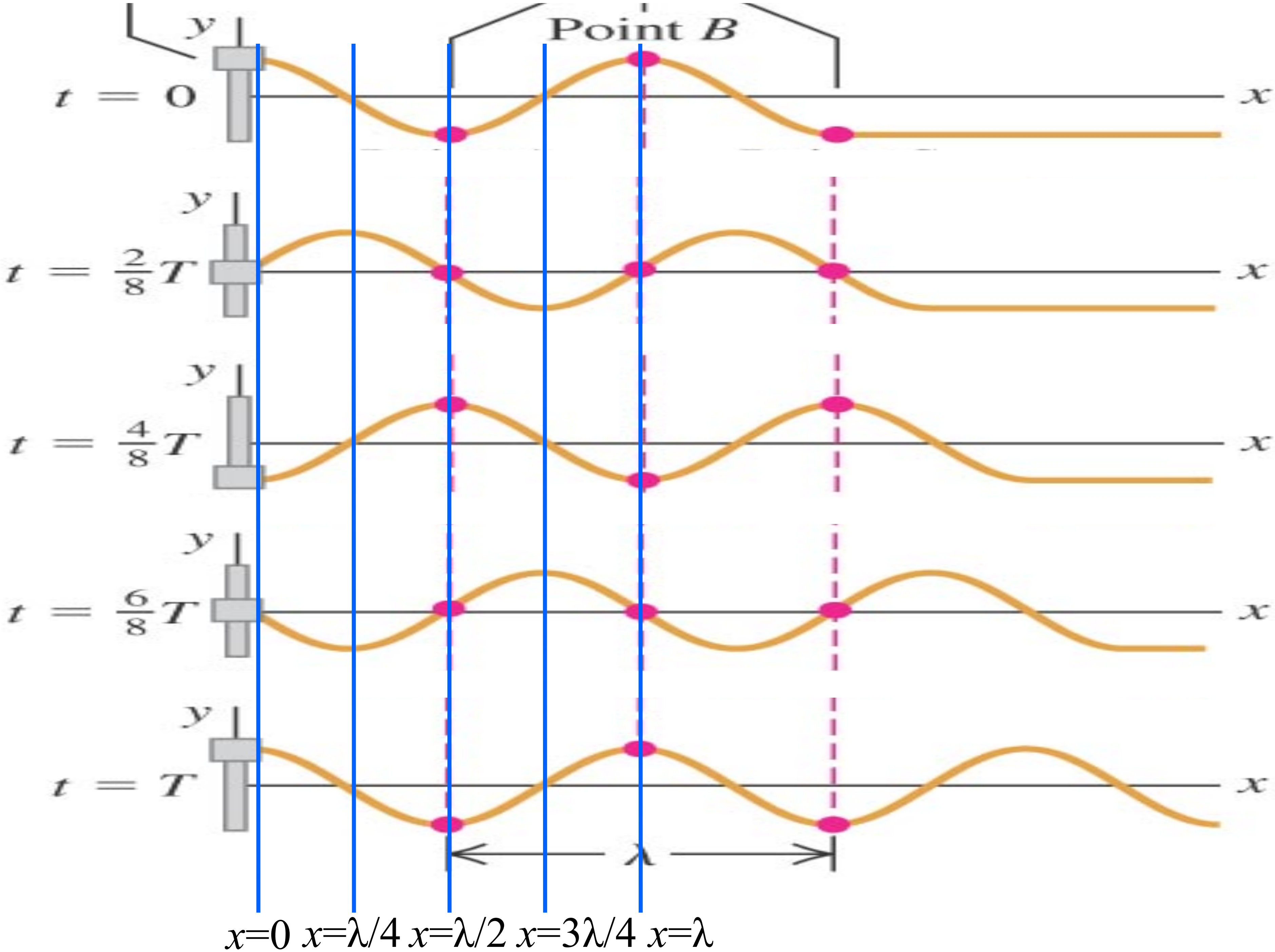


Point B
y
t = 0
x
t = 2/8 T
t = 4/8 T
t = 6/8 T
t = T
λ
x=0 x=λ/4 x=λ/2 x=3λ/4 x=λ

Graphing the wave function

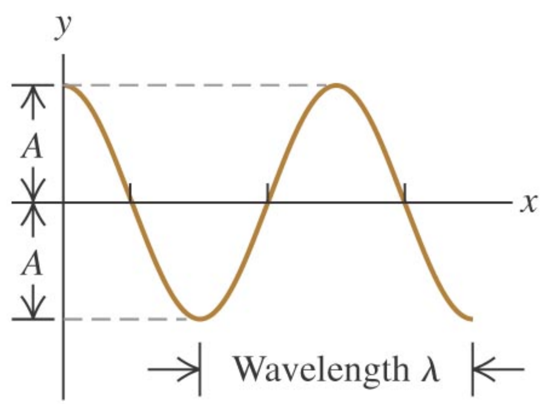
- The graphs to the right look similar, but they are *not* identical.
- Graph (a) shows the *shape* of the string at $t = 0$

$$y = A\cos(kx)$$

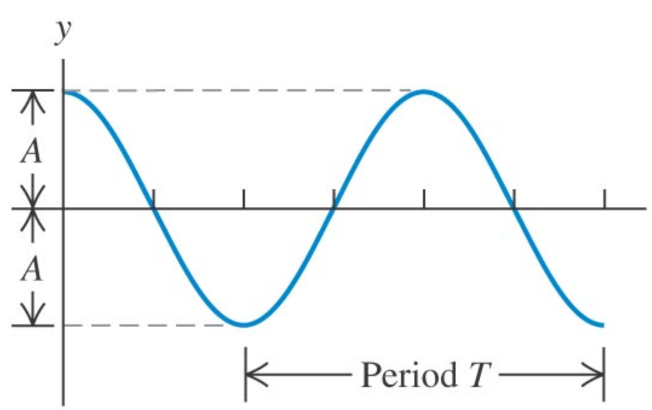
- Graph (b) shows the *displacement* y as a function of time at $x = 0$.

$$y = A\cos(-\omega t)$$

(a) If we use Eq. (15.7) to plot y as a function of x for time $t = 0$, the curve shows the *shape* of the string at $t = 0$.



(b) If we use Eq. (15.7) to plot y as a function of t for position $x = 0$, the curve shows the *displacement* y of the particle at $x = 0$ as a function of time.



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Mathematical description of a wave

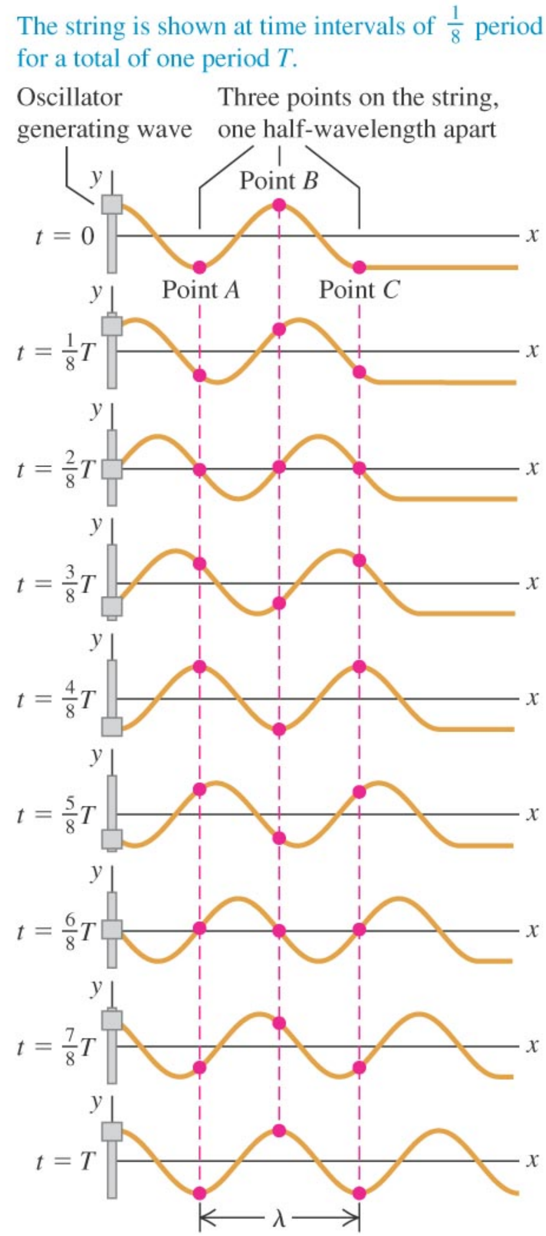
- The *wave function, y*(x,t), gives a mathematical description of a wave. In this function, y is the displacement of a particle at time t and position x.
- The wave function for a sinusoidal wave moving in the +x-direction is

$$y(x,t) = A\cos(kx - \omega t),$$

where $k = 2\pi/\lambda$ is called the *wave number*.

- In the −x-direction,

$$y(x,t) = A\cos(kx + \omega t)$$



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Sinusoidal wave moving in +x direction

$$y(x, t) = A\cos(kx - \omega t)$$

$$y(x, t) = A\cos\left[2\pi\left(\frac{x}{\lambda} - \frac{t}{T}\right)\right]$$

$$y(x, t) = A\cos\left[\omega\left(\frac{x}{v} - t\right)\right] = A\cos\left[2\pi f\left(\frac{x}{v} - t\right)\right]$$

Sinusoidal wave moving in −x direction

$$y(x, t) = A\cos(kx + \omega t)$$

$$y(x, t) = A\cos\left[2\pi\left(\frac{x}{\lambda} + \frac{t}{T}\right)\right]$$

$$y(x, t) = A\cos\left[\omega\left(\frac{x}{v} + t\right)\right] = A\cos\left[2\pi f\left(\frac{x}{v} + t\right)\right]$$

Q15.2

Which of the following wave functions describe a wave that moves in the –x-direction?

F. $y(x,t) = A \sin(-kx - \omega t)$

G. $y(x,t) = A \sin(kx + \omega t)$

H. $y(x,t) = A \cos(kx + \omega t)$

I. two of the above

J. all of the above

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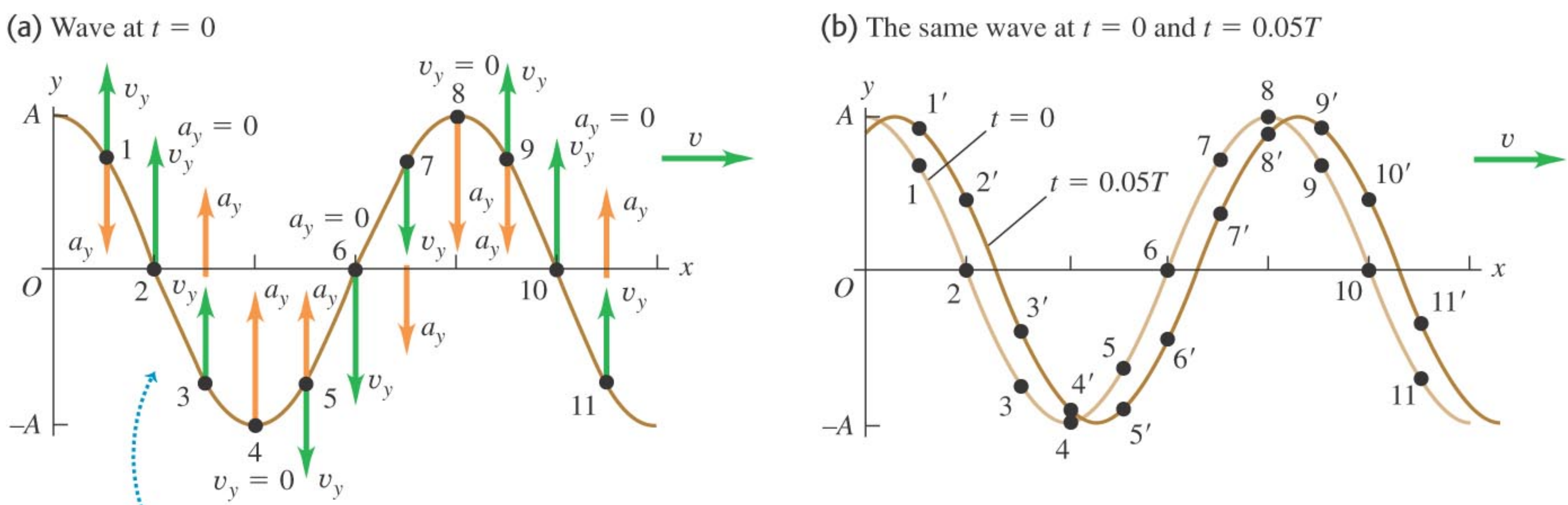
Phase speed

$$v = \frac{dx}{dt} = \frac{\omega}{k}$$

- Speed of propagation of the wave

Particle velocity and acceleration in a sinusoidal wave

- The graphs in Figure 15.10 below show the velocity and acceleration of particles of a string carrying a transverse wave.



- Acceleration a_y at each point on the string is proportional to displacement y at that point.
- Acceleration is upward where string curves upward, downward where string curves downward.

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Particle speed and acceleration

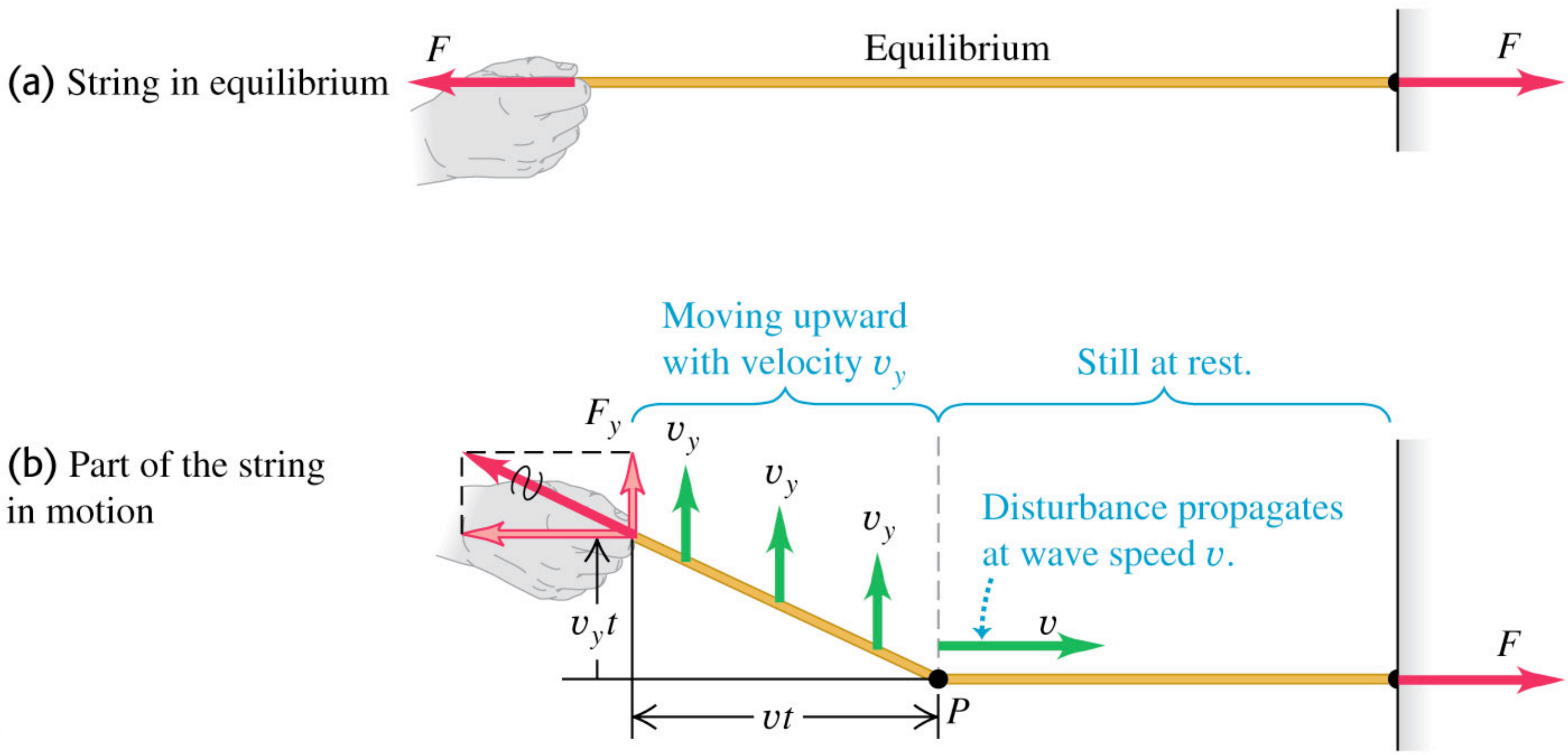
$$v_y(x, t) = \frac{\partial y(x, t)}{\partial t} = \omega A \sin(kx - \omega t)$$

$$a_y(x, t) = \frac{\partial^2 y(x, t)}{\partial t^2} = -\omega^2 A \cos(kx - \omega t) = -\omega^2 y(x, t)$$

The wave equation

$$\frac{\partial^2 y(x,t)}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 y(x,t)}{\partial t^2}$$

- All waves satisfy the wave equation, regardless of shape

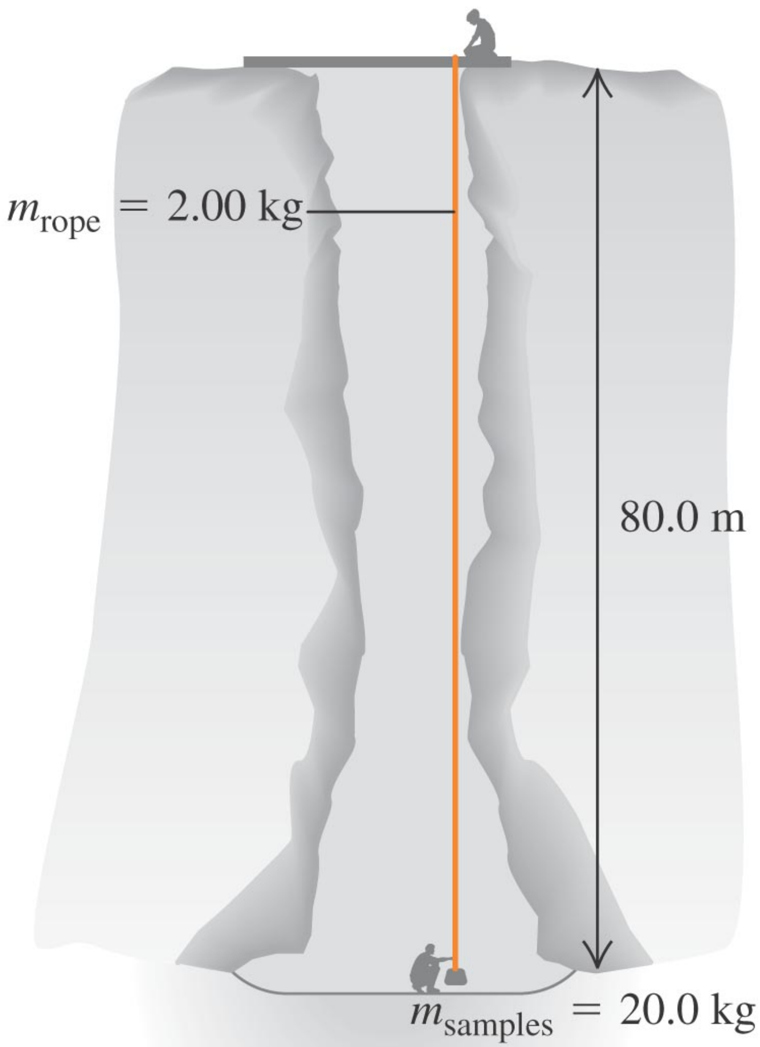

(a) String in equilibrium
F
Equilibrium
F
(b) Part of the string in motion
Moving upward with velocity v_y
Still at rest.
F_y
v_y
v_y
v_y
Disturbance propagates at wave speed v.
v
$v_y t$
vt
P
F

Speed of a transverse wave on a string

$$v = \sqrt{\frac{F}{\mu}}$$

- F = tension on string
- μ = linear mass density

Calculating wave speed



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Q15.8

The four strings of a musical instrument are all made of the same material and are under the same tension, but have different thicknesses. Waves travel

Q. fastest on the thickest string.

R. fastest on the thinnest string.

S. at the same speed on all strings.

T. not enough information given to decide

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Power in a wave

- A wave transfers power along a string because it transfers energy.

$$P(x, t) = \sqrt{\mu F}\,\omega^2 A^2 \sin^2(kx - \omega t)$$

$$P_{av} = \tfrac{1}{2}\sqrt{\mu F}\,\omega^2 A^2 \qquad \text{(average power, sinusoidal wave on a string)}$$

- The average power is proportional to the *square* of the amplitude and to the *square* of the frequency. This result is true for all waves.