

Announcements

- Homework 13 (next week) is extra credit
- Lab B next week
- Final Exam: Friday, May 13,
10:15am-12:15pm **CR 306**

Ch 15.6-8

Standing Waves

PHYS 1210 -- Prof. Jang-Condell

Goals for Chapter 15

- To study the properties and varieties of mechanical waves
- To relate the speed, frequency, and wavelength of periodic waves
- To interpret periodic waves mathematically
- To calculate the speed of a wave on a string
- To calculate the energy of mechanical waves
- To understand the interference of mechanical waves
- To analyze standing waves on a string
- To investigate the sound produced by stringed instruments

Using the following equations, answer the questions on the next slide.

$$v = \frac{dx}{dt} = \frac{\omega}{k}$$

$$f = 1/T$$

$$v = \sqrt{\frac{F}{\mu}}$$

$$F_x = -kx$$

$$\omega = \sqrt{\frac{k}{m}}$$

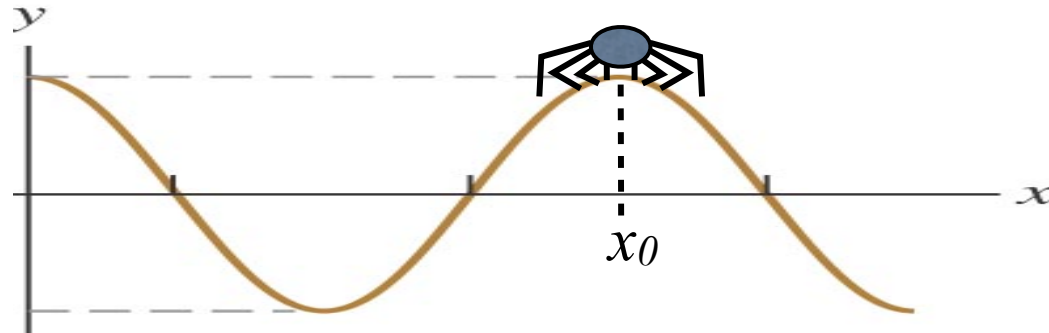
$$v = \lambda f$$

$$k = 2\pi/\lambda$$

$$\omega = 2\pi f$$

A spider sits on strand of webbing at $x=x_0$. The wind blows and causes a periodic wave to travel along the strand:

$$y(x,t) = A\cos(kx - \omega t)$$



- At $t=0$, the spider is at maximum height. At what time will the spider again be at maximum height?
- How far does the wave travel in that time?
- How far does the spider travel between maximum and minimum heights?
- What is the speed of the wave?
- What is the maximum speed of the spider?
- What is the average speed of the spider?

Speed of a transverse wave on a string

$$v = \sqrt{\frac{F}{\mu}}$$

- F = tension on string
- μ = linear mass density

Power in a wave

- A wave transfers power along a string because it transfers energy.

$$P(x, t) = \sqrt{\mu F} \omega^2 A^2 \sin^2(kx - \omega t)$$

$$P_{\text{av}} = \frac{1}{2} \sqrt{\mu F} \omega^2 A^2 \quad (\text{average power, sinusoidal wave on a string})$$

- The average power is proportional to the *square* of the amplitude and to the *square* of the frequency. This result is true for all waves.

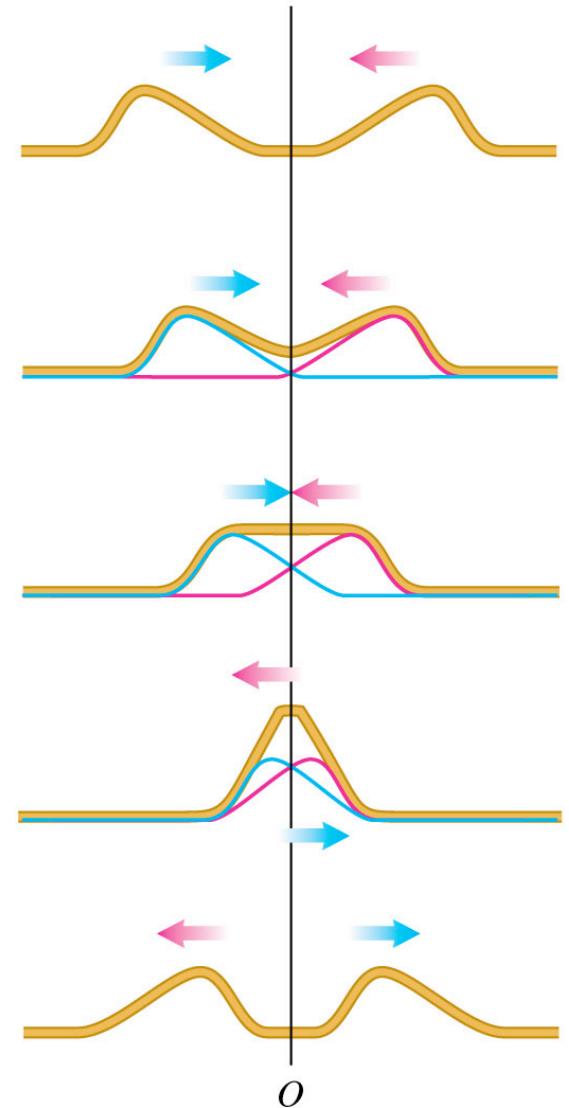
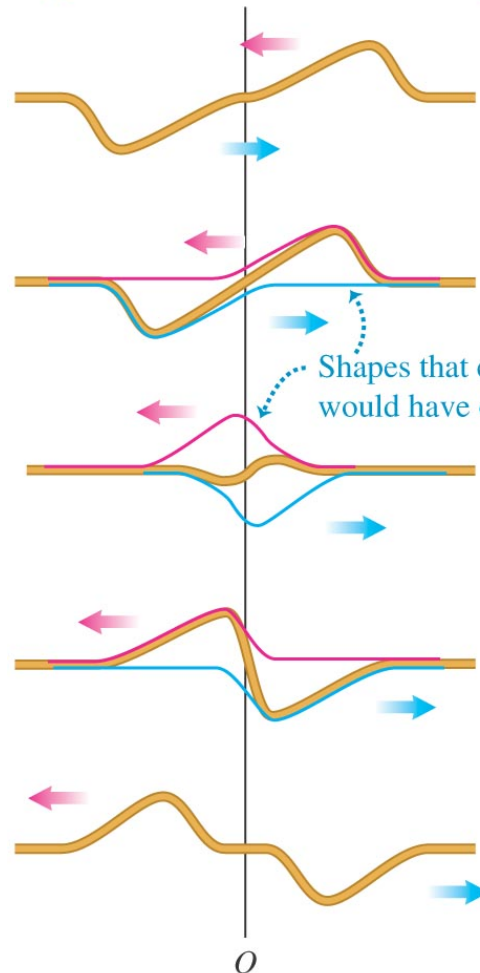
Interference & Reflection Demo

Wave interference and superposition

- *Interference* is the result of overlapping waves.
- *Principle of superposition*: When two or more waves overlap, the total displacement is the sum of the displacements of the individual waves.

$$y(x,t) = y_1(x,t) + y_2(x,t)$$

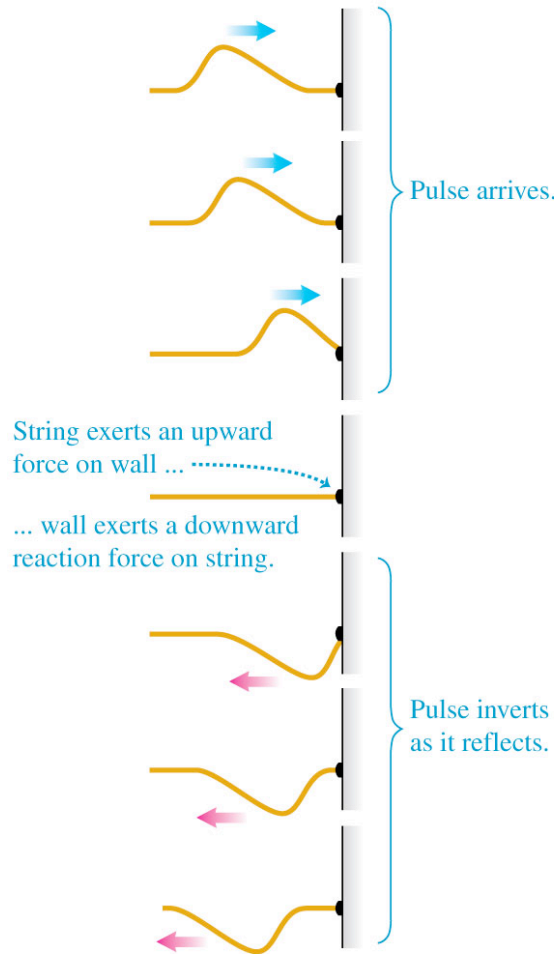
As the pulses overlap, the displacement of the string at any point is the algebraic sum of the displacements due to the individual pulses.



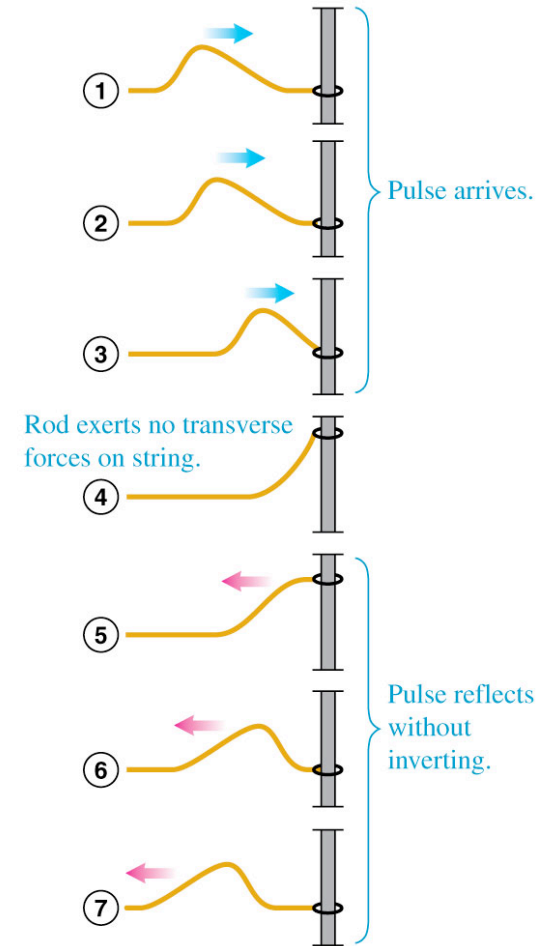
Boundary conditions

- When a wave reflects from a *fixed end*, the pulse *inverts* as it reflects. See Figure 15.19(a) at the right.
- When a wave reflects from a *free end*, the pulse reflects *without inverting*. See Figure 15.19(b) at the right.

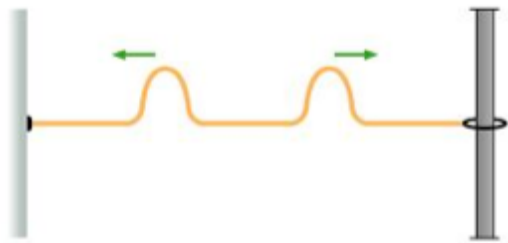
(a) Wave reflects from a fixed end.



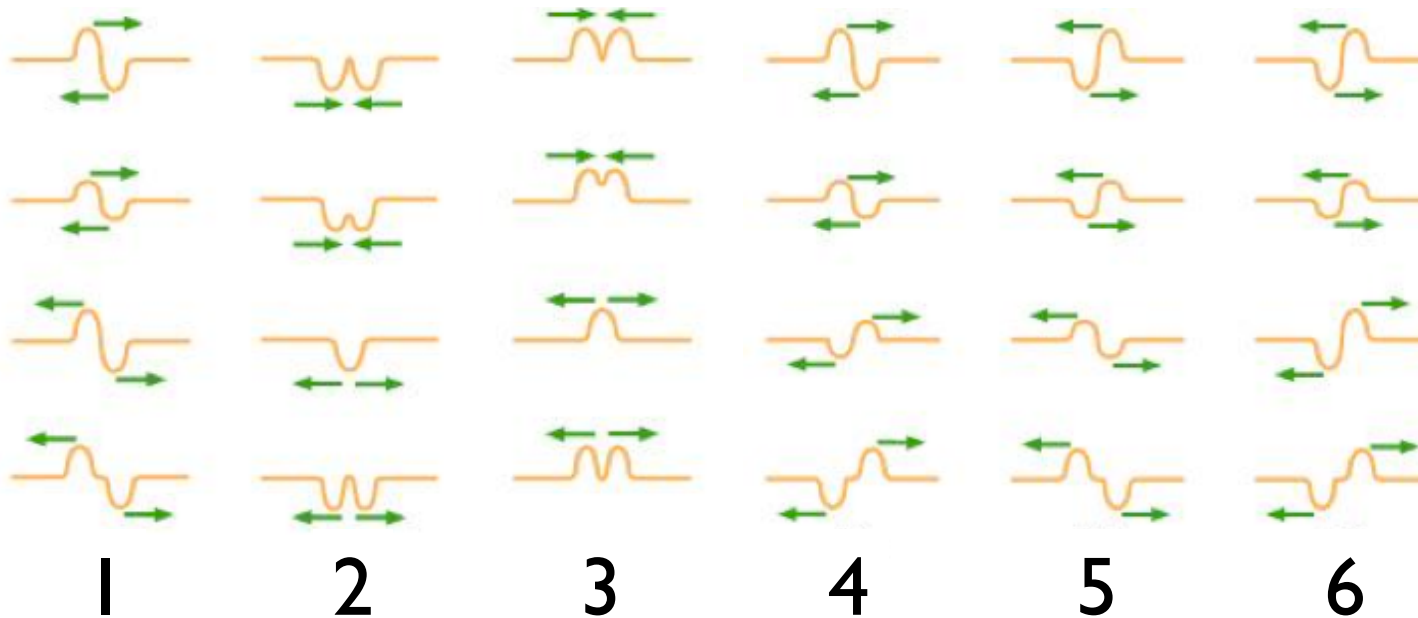
(b) Wave reflects from a free end.



Two identical pulses are moving in opposite directions along a stretched string that has one fixed end and the other movable, as shown below. Initially, the pulses are moving away from each other from each other.

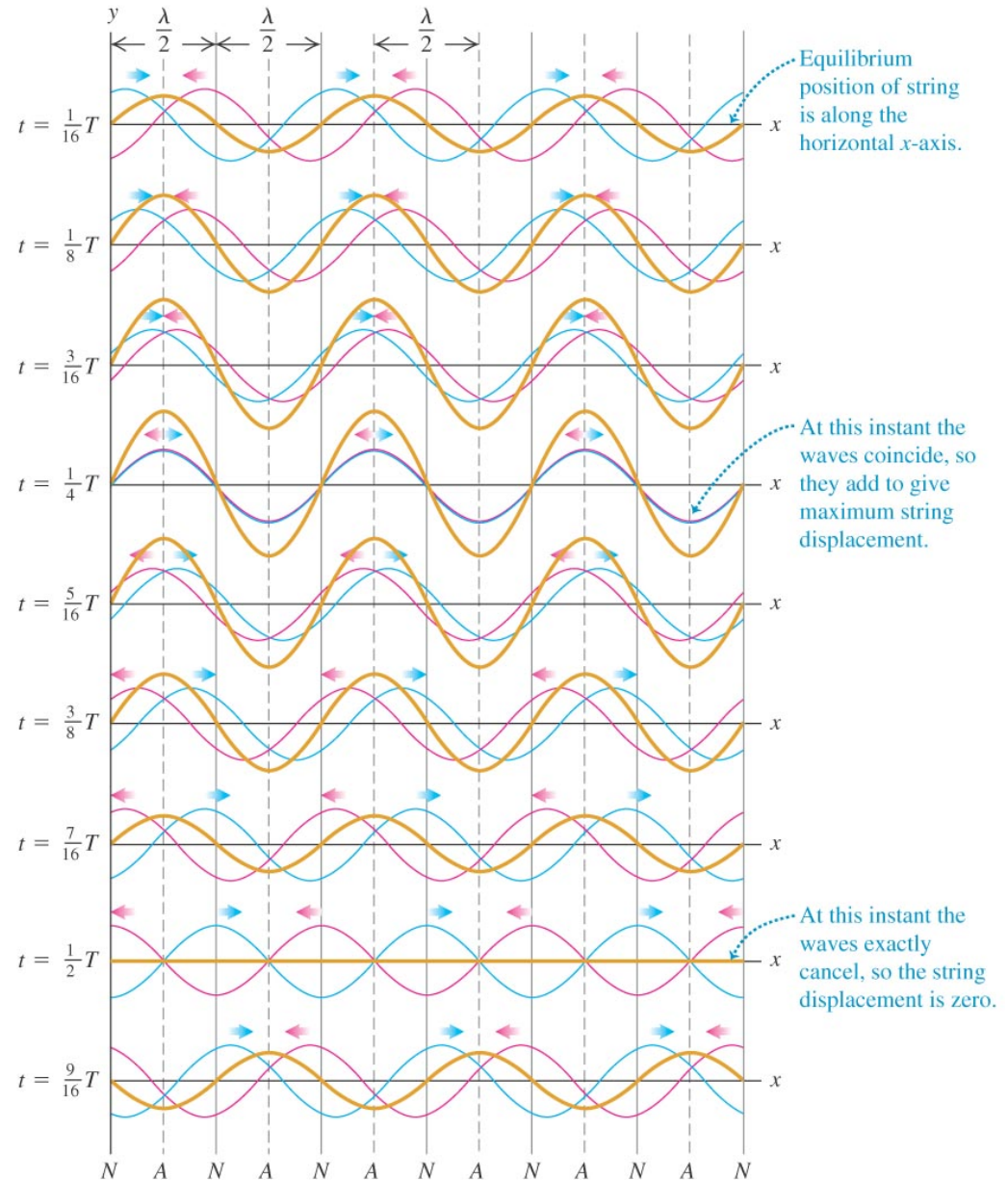


The two pulses reflect off the boundaries of the string. Which sequence pictured below correctly represents the displacement of the string as the pulses interfere?



The formation of a standing wave

- In Figure 15.24, a wave to the left combines with a wave to the right to form a standing wave.

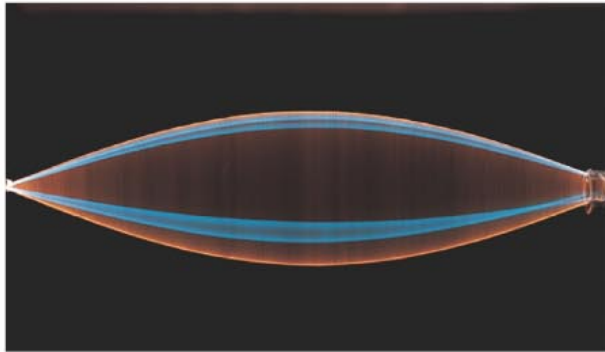


Standing Wave Demo

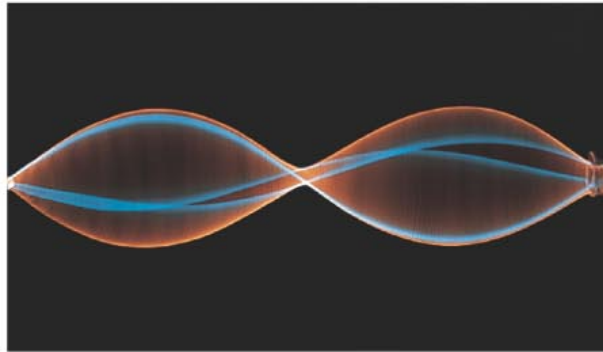
Photos of standing waves on a string

- Some time exposures of standing waves on a stretched string.

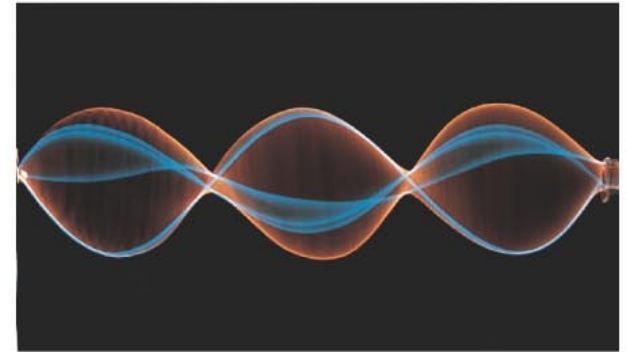
(a) String is one-half wavelength long.



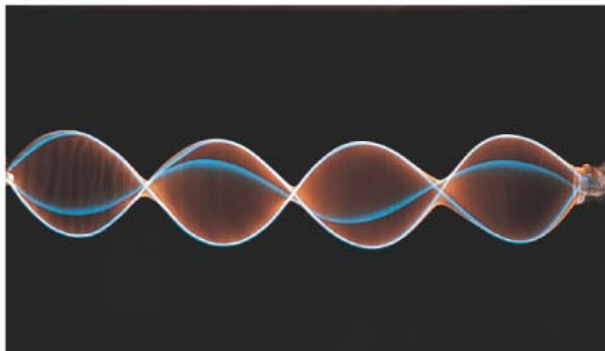
(b) String is one wavelength long.



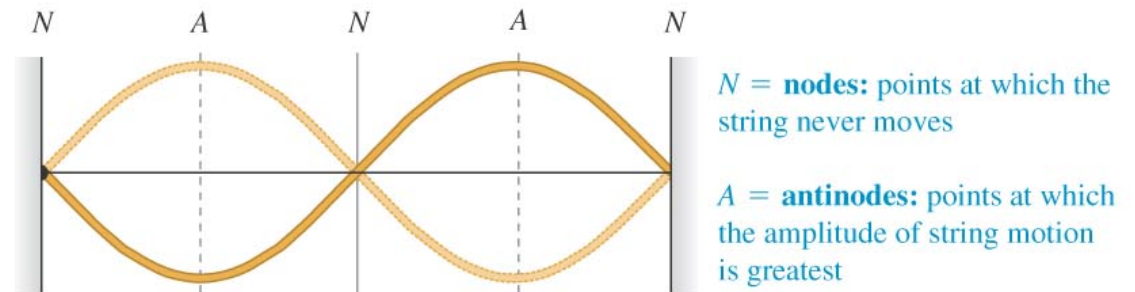
(c) String is one and a half wavelengths long.



(d) String is two wavelengths long.



(e) The shape of the string in (b) at two different instants



Standing Wave

$$y(x, t) = (A_{SW} \sin kx) \sin \omega t$$

(standing wave on a string, fixed end at $x = 0$)

Normal modes of a string

- For a taut string fixed at both ends, the possible wavelengths are

$$\lambda_n = 2L/n$$

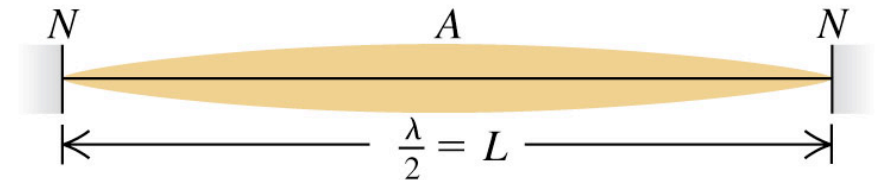
and the possible frequencies are

$$f_n = n v/2L = nf_1,$$

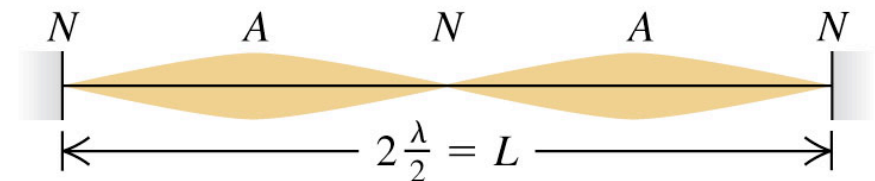
where $n = 1, 2, 3, \dots$

- f_1 is the *fundamental frequency*, f_2 is the second harmonic (first overtone), f_3 is the third harmonic (second overtone), etc.
- Figure 15.26 illustrates the first four harmonics.

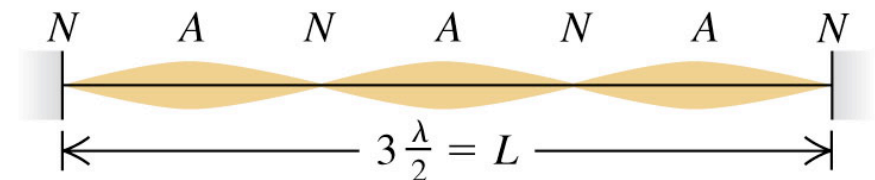
- (a) $n = 1$: fundamental frequency, f_1



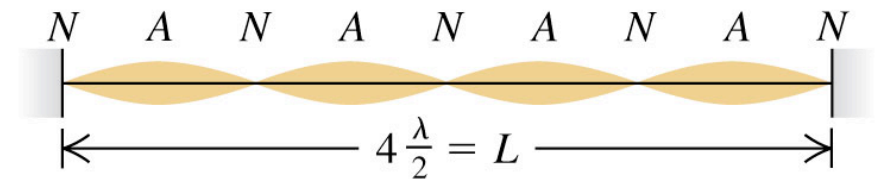
- (b) $n = 2$: second harmonic, f_2 (first overtone)



- (c) $n = 3$: third harmonic, f_3 (second overtone)

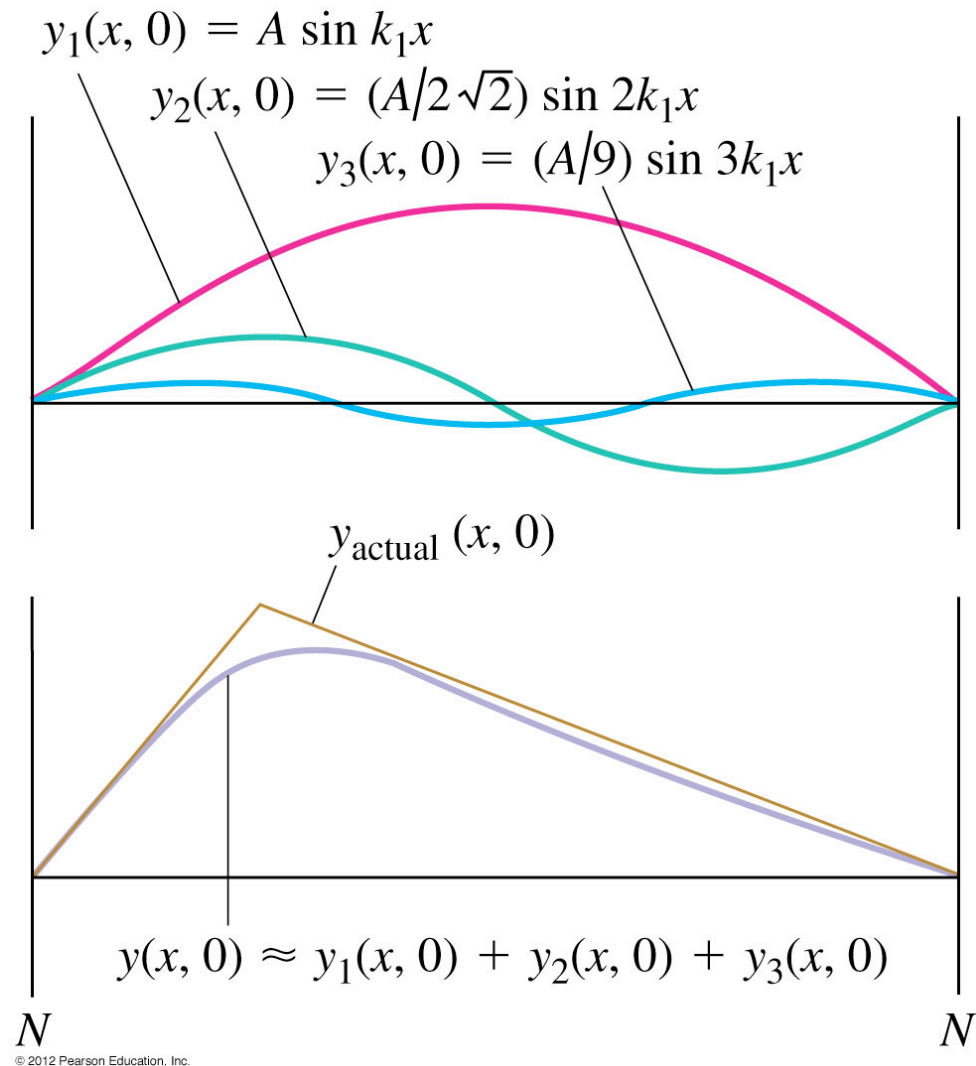


- (d) $n = 4$: fourth harmonic, f_4 (third overtone)



Plucking a String

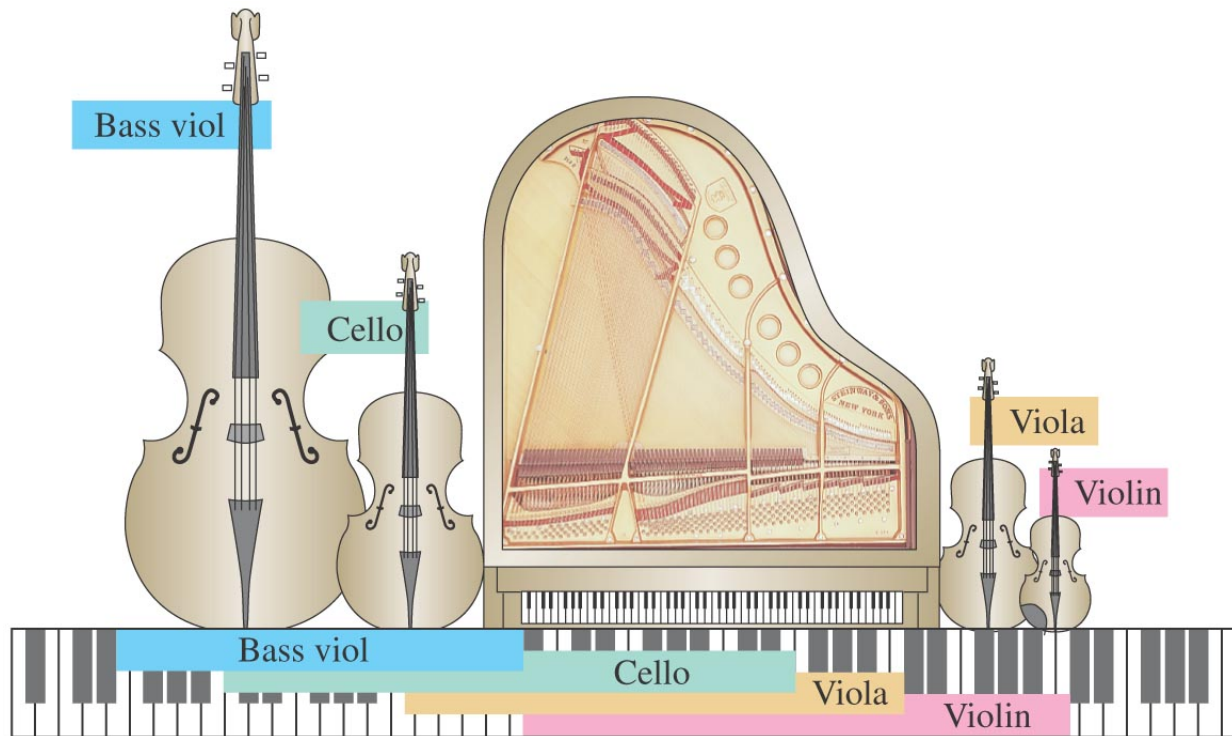
When the string is released, standing waves are formed



Standing waves and musical instruments

- A stringed instrument is tuned to the correct frequency (pitch) by varying the tension. Longer and thicker strings produce bass notes and shorter and thinner strings produce treble notes.

$$f_1 = \frac{1}{2L} \sqrt{\frac{F}{\mu}} \quad (\text{string fixed at both ends})$$



Q15.9



While a guitar string is vibrating, you gently touch the midpoint of the string to ensure that the string does not vibrate at that point.

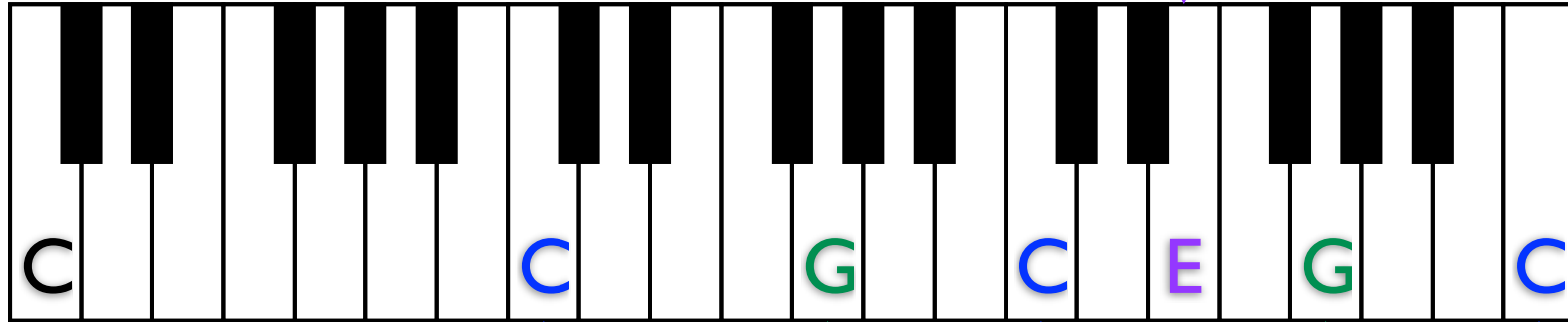
The lowest-frequency standing wave that could be present on the string

- A. vibrates at the fundamental frequency.
- B. vibrates at twice the fundamental frequency.
- C. vibrates at 3 times the fundamental frequency.
- D. vibrates at 4 times the fundamental frequency.
- E. not enough information given to decide

Normal Modes Demo

5th Harmonic,
4th Overtone

$$f = 5f_1$$



Fundamental,
1st Harmonic
 $f = f_1$

2nd Harmonic,
1st Overtone
 $f = 2f_1$

3rd Harmonic,
2nd Overtone
 $f = 3f_1$

4th Harmonic,
3rd Overtone
 $f = 4f_1$

6th Harmonic,
5th Overtone
 $f = 6f_1$

8th Harmonic,
7th Overtone
 $f = 8f_1$