Worksheet #1 Solutions

1. The SI units of volume are cubic meters (m^3). If we evaluate an expression and we do not get an answer with units of m^3, then the expression must not be a valid expression for the volume of an object.

Now, suppose we are given the expression $\pi r^3 h$. We know that both r and h have units of meters, and that π is unitless. Therefore, if we were to plug in actual numbers and evaluate the expression, we would get an answer with units of $m^3 \cdot m$, or m^4. This is **not** the same as m^3, so the expression must **not** be a valid expression for volume.

2. To solve this problem, we need to first determine the speed of light in feet per second. We can then use this to calculate how many nanoseconds it takes light to travel a distance of 1.00 feet.

First, we convert the speed of light from meters per second into feet per second. There are approximately 3.28 feet in 1 meter, so the conversion looks like:

$$\frac{2.9979 \times 10^8 \cancel{meters}}{1\ second} \cdot \frac{3.28\ feet}{1\ \cancel{meter}} = 9.83 \times 10^8 ft./s$$

Notice that the units cancel out just like numbers or variables would, leaving us with units of feet per second.

Next, we use our converted speed to calculate the number of seconds it takes light to travel one foot. To do this, we use the definition of speed, but we solve for time instead:

$$speed = \frac{distance}{time} \quad \rightarrow \quad time = \frac{distance}{speed}$$

Dividing 1 foot by our converted speed, we get 1.02×10^{-9} seconds. However, the question wants the answer in nanoseconds, so we do one more unit conversion:

$$1.02 \times 10^{-9}\ seconds \cdot \frac{1 \times 10^9 nanoseconds}{1\ second} = 1.02\ nanoseconds$$

3. Suppose we have two vectors, $\vec{A}$ and $\vec{B}$. Their scalar (or dot) product can be defined algebraically as

$$\vec{A} \cdot \vec{B} = ABcos\delta$$

where A and B are the magnitudes of the two vectors and δ is the angle between them. Now, let's suppose that the scalar product is negative. The magnitude of a vector is always positive, so the only possible way that the scalar product could be negative is if the cosine term is negative. Because cosine is negative between $\frac{\pi}{2}$ and $\frac{3\pi}{2}$ radians, we can conclude that a negative scalar product indicates that the angle between the vectors is between π and 2π radians.

Graphically, we can say that a negative scalar product indicates that the vectors are pointing in opposite directions.

4. Now we are looking at the vector (cross) product. Suppose we have the same vectors, $\vec{A}$ and $\vec{B}$. Their vector product can be defined algebraically as

$$\vec{A} \cdot \vec{B} = ABsin\delta$$

where A and B are the magnitudes of the two vectors and δ is the angle between them. If the vector product is negative then the sine term must be negative. This indicates that the angle between the vectors must be between π and 2π radians.

5. Suppose we have a rectangular room. If we define $\vec{L}$ to be the vector running along the length of the room and $\vec{W}$ to be the vector running along the width of the room, then the magnitude of their vector product will be the same as the area of the room.

Now, we define $\vec{H}$ to be the vector running along the height of the room. If we calculate the scalar triple product of the length, height, and width vectors ($(\vec{L} \times \vec{W}) \cdot \vec{H}$), we will find that it is equal to the volume of the room.