ASTRO 1050 - Fall 2012
LAB #4: Planetary Orbits and Kepler's Laws

ABSTRACT

Johannes Kepler (1571-1630), a German mathematician and astronomer, was a man on a quest to discover order and harmony in the solar system. He lived at a time when the motions of the planets through space were not well understood: Nicolaus Copernicus had only recently (1543) published his claim that the Sun, not the Earth, was the center of the solar system, and the idea was not well accepted. With years of hard work, and without the benefit of calculus or the Universal Law of Gravitation (both were invented by Isaac Newton only in the latter half of the 17th century), Kepler was able to derive three laws that state how the planets move in their orbits around the Sun. In this lab, you will learn about Kepler's laws of planetary motion, and explore some of their consequences.

Materials

Keppler's First Law: two tacks, a loop of string, a protractor, a piece of cardboard and a piece of blank white paper

1. Exercises

Kepler's First Law: The planetary orbits are ellipses, with the Sun located at one focus.

For this part of the lab, you will work with your lab partners. Your instructor will give each of you two tacks, a loop of string, a piece of cardboard, and a piece of blank white paper. You will then use this material to draw an ellipse on one of the pieces of cardboard.

One person in your group should place his or her tacks close together. Another person should place the tacks very far apart (almost as far as the loop of string will allow).

1. Use the equipment provided to carefully draw an elliptical orbit on your piece of paper. (To assure an accurate ellipse, have the other students in your group hold down the pins while you draw.)

   The positions of your pins are called the foci (singular: focus) of the ellipse.

2. Label one focus as the sun, and the other as the secondary focus.

3. The point in the orbit furthest away from the Sun is called the "aphelion". The closest point is called the "perihelion". Label these positions in your orbit.

Now define the following terms:

- Major axis:
- Semimajor axis:
- Minor axis:
- Semiminor axis:

4. Label these parts of your ellipse.

5. Measure the major axis in centimeters and record the length here:__________

   An important parameter of an ellipse is the eccentricity (e). It can be found via the following formula:

   $$e = \frac{c}{2a} \quad (1)$$

   where c is the distance between the foci (here, between your tack marks), and a is the semimajor axis.

   Measure c (in cm) and record it here:__________

6. Find the eccentricity of your ellipse using the above equation. SHOW YOUR WORK. Compare your value of e with those obtained by the other students in your group. Comment on the relation between the eccentricity of an ellipse and its shape.

7. Let's consider an orbit with e = 0.

   What would the value of c be?

   How would you place the takes in order to draw such an orbit?

   What would the orbit look like?

   What would the semi major axis (a) be called?

8. Choose three points at random on the perimeter of your ellipse (which could be three possible locations of the planet during its orbit). Label these points A, B, and C. Fill in the following table.

	Distance from Sun to point (cm)	+	Distance from point to 2nd focus (cm)	=	Total Distance (cm)
Point A:		+		=	
Point B:		+		=	
Point C:		+		=	

What do you notice? Where have you seen the numbers in the last column before?

Most of the planetary orbits have quite small eccentricities, which is one of the reasons why they are often called circular. However, the orbits of comets may be much more eccentric.

Kepler's Second Law: The line between a planet and the Sun (called the "radius vector" sweeps out equal areas in equal times.

The orbit of the imaginary Comet Fluffy is shown on the last page of this lab. Comet Fluffy travels around the Sun once in exactly 100 years. It was at perihelion in the year 2000.

In this section of the lab, you will predict where in its orbit the comet will be at the following times: 2000, 2010, 2020, 2030, 2040, 2050, 2060, 2070, 2080, 2090, and 2100.

1. According to Kepler's Second Law, how will the area be swept out by Fluffy's radius vector between 2000 and 2010 compare to the area swept out…

    between 2010 and 2020?

    between 2050 and 2060?

    between any two of the adjacent years in the above list?

    Explain your reasoning in detail.

2. Fill in the second column (percent area) of the following table, using what you know about equal areas in equal times:

Year	Percentage of ellipse's total area swept out by Fluffy's radius vector since the year 2000	Position angle of Fluffy (measured conterclockwise from perihelion) - in degrees
2000		
2010		
2020		
2030		
2040		
2050		
2060		
2070		
2080		
2090		
2100		

The areas swept out by the radius vector fix the position of the comet in its orbit. But, how do we measure these areas? It's not as easy as you might think; there is no simple equation for the area of a swept-out elliptical sector.

In order to plot the position of the comet, we would like to know how these swept-out areas relate to the position angle of the comet (the angel perihelion - Sun - comet, measured counterclockwise from perihelion). The Comet Fluffy Table near the end of the Lab will tell you just that.

3. Using the Comet Fluffy Table at the end of the lab, fill in the third column (position angle) of the table in part (2). Then, with a protractor to measure the position angles, plot the position of the comet as it orbits the Sun. Label these positions with the corresponding years.

4. Does the comet always travel at the same speed?

   If not, at what position in its orbit does it travel fastest?

   What position does it travel the slowest?

Kepler's Third Law: The square of the orbital period is proportional to the cube of the semimajor axis:

$$P^2 = a^3 \tag{2}$$

(**where P is measured in years, and a is measured in AU!**).

Semimajor axis (a): This was defined at the beginning of this lab. Sometimes a planet is closer to the Sun than this distance, and sometimes it is farther away, but the semimajor axis is the average distance from the Sun. For the Earth, a = 1 AU by definition.

Orbital period (P): This is the amount of time that it takes for the planet to go once around the Sun.

You can see that for the Earth, Kepler's Third Law works (P=1, a=1, $1^2 = 1^3$).

1. If the Earth's orbit had a semimajor axis of 2 AU, how long would it take to go around the Sun? Use Kepler's Third Law to find out, and show your work.

2. Choose one planet other than the Earth and look up its orbital period. Using Kepler's Third Law, calculate what the semimajor axis should be. How does this compare to the value given in the table?

Halley's Comet travels around the Sun in a highly elliptical orbit (e = 0.967). It most recently passed through our neck of the woods in 1986, but it was not very bright, so you might have missed it. Fortunately, it will be back, although not until the year 2062.

3. What is the semimajor axis of Comet Halley's orbit, in AU?

4. How close to the Sun is Halley's Comet at perihelion (in AU)?

How far away from the Sun is it at aphelion (in AU)?

Explain in detail the reasoning by which you obtained these answers, and show all work.

[`Hint:` To solve this problem, you need both e and a. It might help to draw a sketch of Halley's orbit around the Sun. If you were going to draw this orbit with string, tacks, and cardboard, where would you place the tacks?]

Comet Fluffy Table

Position angle (measured from perihelion in degrees)	Percent of total area swept out by radius vector since year 2000
0	0.00%
5	0.05%
10	0.10%
15	0.15%
20	0.20%
25	0.26%
30	0.31%
35	0.37%
40	0.43%
45	0.50%
50	0.57%
55	0.64%
60	0.72%
65	0.81%
70	0.91%
75	1.02%
80	1.14%
85	1.28%
90	1.44%
95	1.63%
100	1.85%
105	2.11%
110	2.43%
115	2.82%
120	3.30%
125	3.91%
130	4.69%
135	5.71%
140	7.04%
145	8.82%
150	11.22%
155	14.46%
160	18.82%
165	24.55%
170	31.82%
175	40.48%
180	50.00%

Position angle (measured from perihelion in degrees)	Percent of total area swept out by radius vector since year 2000
180	50.00%
185	59.52%
190	68.18%
195	75.45%
200	81.18%
205	85.54%
210	88.78%
215	91.18%
220	92.96%
225	94.29%
230	95.31%
235	96.09%
240	96.70%
245	97.18%
250	97.57%
255	97.89%
260	98.15%
265	98.37%
270	98.56%
275	98.72%
280	98.86%
285	98.98%
290	99.09%
295	99.19%
300	99.28%
305	99.36%
310	99.43%
315	99.50%
320	99.57%
325	99.63%
330	99.69%
335	99.74%
340	99.80%
345	99.85%
350	99.90%
355	99.95%
360	100.00%

Orbit of Comet Fluffy
Scale: 1 cm = 2.5 AU