## ASTR 2320 General Astronomy II - Spring 2020, Brotherton Homework 4, due Tuesday March 3, 2020

Instructions: This homework covers topics featured in chapters 1-3 of Dobson's text, as well as associated labs, lectures, homeworks, and discussions. Calculators are allowed (and likely needed!). Some constants and equations you may need are provided on the last pages, as well as an explicit list of some things you're expected to have memorized. Please write all answers clearly and use good English. Box final answers when calculations are required. Use units when appropriate. Show your work. Draw pictures when helpful.

Write your name on every page in case pages become separated!
Part 1. Multiple choice. Please give the best answer. ( 6 Problems, 5 points each $=30$ )

1. The moon is overhead at 6AM. What phase is it in?
A) New
B) First quarter
C) Full
D) Third quarter
E) None of the above
2. The moon is first quarter and rising. What time is it?
A) Noon
B) 6 PM
C) 9 PM
D) Midnight
E) 6 AM
3. The moon is a waxing crescent phase and it's dusk. Where is the moon?
A) The moon is not visible in the sky.
B) The moon is on the eastern horizon.
C) The moon is on the western horizon.
D) The moon is at an intermediate angle in the eastern sky.
E) The moon is at an intermediate angle in the western sky.
4. Laramie is at 41 degrees north latitude. What are the coordinates of an object overhead at midnight around March 21?
A) RA 0 hours, declination 41 degrees
B) RA 12 hours, declination 41 degrees
C) RA 0 hours, declination 49 degrees
D) RA 12 hours, declination 49 degrees
E) None of the above are close
5. For a comet with an elliptical orbit with a period of 100 years, what is the semi-major axis?
A) 4.6 AU
B) 10 AU
C) 21.5 AU
D) 100 AU
E) 1000 AU
6. What are Lagrangian points?
A) Rewards you earn on your physics department credit card.
B) Talking points favoring a heliocentric solar system over a Earth-centered solar system.
C) Center of mass in multi-body gravitational systems.
D) Locations of balanced gravity in 2-body systems.
E) Locations to measure angles to solar system objects in order to measure relative distances.

Part 2. Short answers (please explain as appropriate, and draw figures if that will help illustrate your answer). ( 5 questions, 5 points each $=25$ )
7. Why are there more lunar eclipses than solar eclipses? Note that the angular sizes of the sun and moon are nearly the same.
8. Is the Moon's orbital radius changing, and if so, why?
9. What is the astrophysical process that eliminates the smallest dust grains from the solar system and how does it work? Don't just give a name, but describe the physics.
10. What is the difference between sidereal and synodic periods?
11. What is the virial theorem? Express in words and give the formula.

Part 3. Quantitative problems. (3 problems, 15 points each $=45$ ). Give answers to three significant figures in each case.
12. A) Assuming a circular orbit (not quite true!), what is the velocity of Mercury around the sun in km/s? B) What is the maximum angle of the aberration of starlight at Mercury? Please give your answer in arcseconds.
13. A comet is observed to return to the inner solar system every 64 years. A) What is the semimajor axis of its orbit in astronomical units? B) If the eccentricity $\mathrm{e}=0.9$, what is the perhelion distance from the sun in AUs?
14. Antiope is a binary asteroid that has two components of very nearly the same size in nearly circular orbits around their common center of mass. The orbital period is 16.5 hours. The separation between the centers of the asteroids is 171 kilometers. A) What is the mass of each of the asteroids in kg ? B) If the resolution of your eyes is 1 arcminute, how close would you have to be in kilometers to separate the two objects into one? (Assume the most favorable geometry.)

Equations or other information that may be useful:

$$
\begin{aligned}
& P^{2}=\frac{4 \pi^{2}}{G\left(m_{1}+m_{2}\right)} a^{3} . \\
& \mathrm{v}^{2}=\mathrm{GM}(2 / \mathrm{r}-1 / \mathrm{a})
\end{aligned}
$$

You should know Newton's Law of Gravitation, Newton's second law of motion, and the virial theorem. You should also know or be able to quickly derive escape velocity and circular velocity.
$\mathrm{G}=6.67 \mathrm{x}^{10-11} \mathrm{~N} \mathrm{x} \mathrm{m}^{2} \mathrm{x} \mathrm{kg}^{-2}$.
Mass of the sun is $2 \times 10^{30} \mathrm{~kg}$.
1 AU is 150 million km .
$\mathrm{c}=3.00 \times 10^{8} \mathrm{~m} / \mathrm{s}$.
You should know equations for circumference and area for circles, and surface area and volume for spheres. Also know your right triangle trigonometry.
You should know the small angle formula and conversions between units of angles.
Tidal forces:

$$
\vec{a}_{t, \text { axial }} \approx \pm \hat{r} 2 \Delta r G \frac{M}{R^{3}}
$$

## Properties of an Ellipse



Table 4.1: Summary of orbital and physical properties of selected solar system objects

| Object | semi-major axis (AU or km) | revolution period (days or years) | orbit eccentricity | \& inclination to ecliptic or planet equator ( ${ }^{\circ}$ ) | rotation period (hours or days) | obliquity ( ${ }^{\circ}$ ) |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Sun | - | - | - | - | $25-34 \mathrm{~d}$ | 7.25 |
| Mercury | 0.387 | 87.97 d | 0.206 | 7.0 | 58.65 d | 0.003 |
| Venus | 0.723 | 0.615 | 0.007 | 3.39 | -243 d | 177.4 |
| Earth | 1.0 | 1.0 | 0.017 | 7.2 to Sun eq. | 23.93 | 23.44 |
| Moon | $384,400 \mathrm{~km}$ | 27.32 d | 0.055 | 5.15 to ecl. | 27.32 d | 6.69 |
| Mars | 1.52 | 1.88 | 0.094 | 1.85 | 1.026 d | 25.19 |
| Phobos | $9,376 \mathrm{~km}$ | 0.32 d | 0.015 | 1.09 | 0.32 d | 0 |
| Deimos | 23,463 km | 1.26 d | $\sim 0$ | 0.93 | 1.26 d |  |
| 4 Vesta | 2.36 | 3.63 | 0.09 | 7.14 | 5.34 |  |
| Ceres | 2.77 | 4.60 | 0.08 | 10.59 | 9.07 | $\sim 3$ |
| 2 Pallas | 2.77 | 4.61 | 0.23 | 34.84 | 7.81 | $\sim 78$ ? |
| 10 Hygiea | 3.14 | 5.56 | 0.12 | 3.84 | 27.62 |  |
| Jupiter | 5.20 | 11.86 | 0.05 | 1.31 | 9.93 | 3.13 |
| Io | $421,700 \mathrm{~km}$ | 1.77 d | 0.004 | 0.05 | 1.77 d |  |
| Europa | 670,900 | 3.55 d | 0.009 | 0.47 | 3.55 d | 0.1 |
| Ganymede | $1.070 \cdot 10^{6} \mathrm{~km}$ | 7.15 d | 0.0013 | 0.20 | 7.15 d |  |
| Callisto | $1.883 \cdot 10^{6} \mathrm{~km}$ | 16.69 d | 0.0074 | $\sim 1$ | 16.69 d | 0 |
| Saturn | 9.58 | 29.46 | 0.056 | 2.49 | 10.55 | 26.73 |
| Mimas | $185,539 \mathrm{~km}$ | 0.94 d | 0.02 | 1.57 | 0.94 d | 0 |
| Enceladus | $237,948 \mathrm{~km}$ | 1.37 d | 0.005 | 0.019 | 1.37 d | 0 |
| Tethys | $294,619 \mathrm{~km}$ | 1.89 d | $\sim 0$ | 1.12 | 1.89 d | 0 |
| Dione | $377,396 \mathrm{~km}$ | 2.74 d | 0.002 | 0.019 | 2.74 d | 0 |
| Rhea | $527,108 \mathrm{~km}$ | 4.52 d | 0.001 | 0.345 | 4.52 d | 0 |
| Titan | $1.222 \cdot 10^{6} \mathrm{~km}$ | 15.95 d | 0.029 | 0.35 | 15.95 d | 0 |
| Iapetus | $3.561 \cdot 10^{6} \mathrm{~km}$ | 79.32 d | 0.029 | 15.47 | 79.32 d | 0 |
| 2060 Chiron | 13.71 | 50.76 | 0.38 | 6.93 | 5.92 |  |
| Uranus | 19.19 | 84.02 | 0.05 | 0.77 | -17.24 | 97.8 |
| Miranda | $129,390 \mathrm{~km}$ | 1.41 d | 0.0013 | 4.23 | 1.41 d | 0 |
| Ariel | $191,020 \mathrm{~km}$ | 2.52 d | 0.0012 | 0.26 | 2.52 d | ? |
| Umbriel | $266,300 \mathrm{~km}$ | 4.14 d | 0.0039 | 0.21 | 4.14 d | 0 ? |
| Titania | $435,910 \mathrm{~km}$ | 8.71 d | 0.0011 | 0.34 | 8.71 d | ? |
| Oberon | $583,520 \mathrm{~km}$ | 13.46 d | 0.0014 | 0.06 | 13.46 d | ? |
| 5145 Pholus | 20.36 | 91.85 | 0.57 | 24.65 | 9.98 | ? |
| Neptune | 30.07 | 164.8 | 0.0087 | 1.77 | 16.11 | 28.32 |
| Proteus | $117,647 \mathrm{~km}$ | 1.122 d | $\sim 0$ | 0.52 | 1.122 d | $\sim 0$ |
| Triton | $354,759 \mathrm{~km}$ | $-5.88 \mathrm{~d}$ | $\sim 0$ | 157 | 5.88 | 0 |
| Pluto | 39.26 | 247.7 | 0.25 | 17.16 | 6.387 d | 122.5 |
| Charon | $19,591 \mathrm{~km}$ | 6.387 d | 0 | 0 | 6.387 d |  |
| Haumea | 43.22 | 284 | 0.19 | 28.19 | 3.92 | ? |
| Makemake | 45.72 | 309 | 0.16 | 29.00 | 7.8 | ? |
| Eris | 67.78 | 558 | 0.44 | 44.04 | 25.9 | ? |
| 90377 Sedna | 524.4 | $\sim 11,400$ | 0.85 | 11.93 | 10.3 | ? |

Table 4.1: Summary of orbital and physical properties of selected solar system objects

| Object | mass (kg) | diameter (km) | $\begin{aligned} & \text { ave. density } \\ & \quad\left(\mathrm{g} / \mathrm{cm}^{3}\right) \end{aligned}$ | albedo (Bond or geom.) | surface temp. <br> (K) |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Sun | $1.99 \cdot 10^{30}$ | $1.39 \cdot 10^{6}$ (eq.) | 1.41 | - | 5780 |
| Mercury | $3.30 \cdot 10^{23}$ | 4,879 | 5.43 | 0.068 Bond | 80-700 |
| Venus | $4.87 \cdot 10^{24}$ | 12,104 | 5.24 | 0.90 Bond | 737 |
| Earth | $5.97 \cdot 10^{24}$ | 12,742 | 5.51 | 0.31 Bond | 184-330 |
| Moon | $7.35 \cdot 10^{22}$ | 3,474 | 3.35 | 0.12 geom | $70-390$ |
| Mars | $6.42 \cdot 10^{23}$ | 6,779 | 3.93 | 0.25 Bond | 130-308 |
| Phobos | $1.07 \cdot 10^{16}$ | $27 \times 22 \times 18$ | 1.88 | 0.071 geom | $\sim 233$ |
| Deimos | $\cdot 10^{20}$ | $15 \times 12 \times 11$ | 1.47 | 0.068 geom | $\sim 233$ |
| 4 Vesta | $2.59 \cdot 10^{20}$ | $\sim 525$ | 3.46 | 0.42 geom | 85-270 |
| Ceres | $9.39 \cdot 10^{20}$ | 938 | 2.17 | 0.09 geom | 168-235 |
| 2 Pallas | $2.11 \cdot 10^{20}$ | 544 | $\sim 2.8$ | 0.16 geom | $\sim 164$ |
| 10 Hygiea | $8.67 \cdot 10^{19}$ | $\sim 431$ | 2.08 | 0.07 geom | $\sim 164$ |
| Jupiter | $1.90 \cdot 10^{27}$ | 139,822 | 1.33 | 0.34 Bond | 165 @ 1 bar |
| Io | $8.93 \cdot 10^{22}$ | 3,643 | 3.53 | 0.63 geom | 110 |
| Europa | $4.80 \cdot 10^{22}$ | 3,122 | 3.01 | 0.67 geom | 102 |
| Ganymede | $1.48 \cdot 10^{23}$ | 5,268 | 1.94 | 0.43 geom | 110 |
| Callisto | $1.08 \cdot 10^{23}$ | 4.821 | 1.83 | 0.2 geom | 134 |
| Saturn | $5.68 \cdot 10^{26}$ | 116,464 | 0.69 | 0.34 Bond | 134 @ 1 bar |
| Mimas | $3.75 \cdot 10^{19}$ | 396 | 1.15 | 0.86 geom | $\sim 64$ |
| Enceladus | $1.08 \cdot 10^{20}$ | 504 | 1.61 | 0.99 Bond | 75 (ave) |
| Tethys | $6.17 \cdot 10^{20}$ | 1,062 | 0.98 | 0.80 Bond | 86 |
| Dione | $1.10 \cdot 10^{21}$ | 1,123 | 1.48 | 0.99 geom | 87 |
| Rhea | $2.31 \cdot 10^{21}$ | 1,527 | 1.24 | 0.95 geom | 53-99 |
| Titan | $1.345 \cdot 10^{23}$ | 5,150 | 1.88 | 0.2 geom | 93.7 |
| Iapetus | $6.5 \mathrm{xx} \cdot 10^{19}$ | 1,470 | 1.09 | $\sim 0.6$ geom | 90-130 |
| 2060 Chiron | ? | $\sim 166 \mathrm{~km}$ | ? | $\sim 0.15$ geom | $\sim 75$ |
| Uranus | $8.68 \cdot 10^{25}$ | 50,724 | 1.27 | 0.30 Bond | 76 K @ 1 bar |
| Miranda | $6.59 \cdot 10^{19}$ | 471 | 1.20 | 0.32 geom | $\sim 60$ |
| Ariel | $1.35 \cdot 10^{21}$ | 1,158 | 1.59 | 0.23 Bond | $\sim 60$ |
| Umbriel | $1.17 \cdot 10^{21}$ | 1,169 | 1.39 | 0.10 Bond | $\sim 75$ |
| Titania | $3.53 \cdot 10^{21}$ | 1,577 | 1.71 | 0.17 Bond | 70 |
| Oberon | $3.01 \cdot 10^{21}$ | 1,523 | 1.63 | 0.14 Bond | 70-80 |
| 5145 Pholus | $?$ | 185 | ? | 0.046 | $\sim 62$ |
| Neptune | $1.02 \cdot 10^{26}$ | 49,244 | 1.64 | 0.29 Bond | 72 K @ 1 bar |
| Proteus | $4.4 \cdot 10^{19}$ | $\sim 420$ | $\sim 1.3$ | 0.096 geom | $\sim 51$ |
| Triton | $2.14 \cdot 10^{22}$ | 2,706 | 2.06 | 0.719 geom | 38 |
| Pluto | $1.303 \cdot 10^{22}$ | 2,377 | 1.86 | $0.49-0.66$ geo | 33-55 |
| Charon | $1.586 \cdot 10^{21}$ | 1,212 | 1.66 | 0.37 geom | 53 |

