Chapter 20: **Optics, Telescopes, & Detectors**

- electromagnetic radiation; polarization
- optics; lenses, mirrors, focal lengths
- optical telescopes
- characterizing telescope function; mounts
- spectrographs
- CCD (charge-coupled device) cameras
- high-energy astronomy
- radio telescopes
- sample problems

**Electromagnetic radiation**

Visible light is a subset of electromagnetic radiation, which also includes, on the short-wavelength side, ultraviolet, x-rays, and gamma rays, and, on the long-wavelength side, infrared, microwave, and radio. We describe electromagnetic radiation as a wave, with a wavelength $\lambda$ and a frequency $\nu$ (or $f$), or as a photon, a packet of energy. In a vacuum the wave travels with speed $c = \frac{\lambda}{\nu}$; we may convert between the wave and photon pictures by means of Planck’s constant $h$: $E = h\nu$. The photon also has a momentum $p = E / c$.

The more fundamental quantities are the electric and magnetic fields. Time-varying electric fields produce time-varying (“waving”) magnetic fields (in a perpendicular direction) and vice versa; the electromagnetic wave propagates in the direction perpendicular to the orientation of the electric and magnetic waves:

![Electromagnetic waves](image)

Figure 20.1: electromagnetic waves

If, as shown in the figure above, the fields continue to oscillate in the same planes, the light is said to be “plane-polarized”. If the planes vary, rotating the planes of the $\vec{E}$ and $\vec{B}$ fields, the light is elliptically polarized. We see polarization in several contexts in astronomy. For instance, some types of dust grains in the interstellar medium induce polarization, synchrotron radiation is polarized, and, closer to home, scattered sunlight is polarized (which you may have noticed if you have polarized sunglasses).

**Optics**

Lenses have been known for many thousands of years; there are mentions of magnifying lenses in ancient Egypt and Greece. Early use seems to have focused on igniting fires. By 1300 Europeans were making spectacles. Mirrors are also known in antiquity; early mirrors were made of polished obsidian or metals such as copper or bronze. By the 16th century glassmakers in Venice had adopted the modern technique of coating a glass substrate with a reflective layer to make a sturdy mirror.

Light traveling through a lens will be refracted. Light only travels at $c$, the “speed of light”, in a vacuum; the speed (properly, the phase velocity) of light is less in a medium, and how much less is described by the index of
refraction, \( n \), of the medium. For air \( n \) is about 1.0003 and for water about 1.3330. How strongly the light is bent depends on the index of refraction of the lens material and the curvature of the lens. Here is a sketch of refraction at a surface:

![Figure 20.2: Refraction at a surface](image)

Two different indices of refraction, \( n \), and two different speeds, \( v \); \( n = c / v \).

Snell’s law states:

\[
n_1 \sin \theta_1 = n_2 \sin \theta_2.
\]

here, \( n_1 < n_2 \).

Refraction is wavelength dependent, which gives us the both the positive capacity to use prisms to spread light out into a spectrum and the negative irritation of chromatic aberration, the fact that lenses bring different colors to a focus at different distances from the lens.

![Figure 20.3: refraction](image)

![Figure 20.4: chromatic aberration](image)

One or both sides of a lens is curved. Convex is the term for a lens that’s thicker in the middle than at the edges and concave for one that’s thinner. The following sketch illustrates biconvex and biconcave lenses:

![Figure 20.5: Biconvex lens, left; biconcave lens, right.](image)
Convex lenses and concave mirrors are converging and, for both, where the object is with respect to the focal length determines whether the image will be real or virtual, upright or inverted, magnified or not. Usually in astronomy our objects are approximately infinitely far away. The following sketch illustrates the fact that distant objects observed using a concave mirror produce inverted and reduced images. As the object distance approaches infinity the image location approaches the focal point of the mirror.

Optical Telescopes.

With an optical telescope we have two main possibilities, refracting or reflecting, depending on whether the principal optical element (the primary) is a lens or a mirror. The first telescope is usually attributed to a Dutch lens maker, Hans Lippershey, in 1608. He combined a long focal length converging lens with a short focal length diverging lens for the eyepiece. Because this is the design subsequently used, and made famous, by Galileo, it’s often called a Galilean telescope. If you make a simple Galilean telescope you will see that it’s not great. Twenty years later Kepler had improved on this design by replacing the concave eyepiece lens with a convex lens. The image flips over but the field of view is larger and as a whole the Keplerian telescope is easier to use.

There are limits to how large a lens can practically be — think about the fact that the lens is only supported around the edges and that glass will eventually sag and lose its original shape. Large telescopes today are reflectors rather than refractors. Isaac Newton was one of the first to build a reflecting telescope, using a parabolic mirror for the primary and a flat mirror for the secondary, directing light out the side of the telescope tube. Here is a picture of a replica of Newton’s second reflecting telescope, one “that he presented to the Royal Society in 1672” and next to it, a sketch of the light path in a Newtonian telescope:
Parallel light rays hitting different locations on a parabolic mirror come to a focus at a point. Parallel light rays hitting a spherical mirror don’t all come together at one point (‘spherical aberration’), so even though it’s harder to shape a parabolic mirror correctly, the shape is preferred over a sphere.

Large Newtonian telescopes can get quite long and unwieldy. Several alternative designs employ curved secondaries that redirect the light back through a central hole in the primary mirror. One of the most common is called a Cassegrain. The light path in a Cassegrain is shown in the following sketch.

Some designs take into account the type of instrumentation that’s likely to be used with the telescope. Telescopes are mounted, no shock, so that their weight is balanced and they’ll take less effort to move. It may make sense to mount a very heavy instrument along the axis of the telescope’s center of mass and to directed the light out along that axis. (E.g., by adding a flat tertiary mirror between the primary and secondary mirrors in the basic Cassegrain design.)

**Characterizing telescope functions.**

Telescopes accomplish several things to allow us to obtain more information than we could with our eyes while observing distant objects. To characterize telescopes we will consider:

- light-gathering power
- field of view
- angular magnification and image scale
- angular resolution

**Light-gathering power**

The diameter of your pupil is about 5-6 mm in the dark and you can see to about 6th magnitude. The larger the *area* of the telescope aperture, the higher the light-gathering power. Usually this property is described as a ratio, comparing the light-gathering power of two telescopes, or a telescope and an eye.
Example: a telescope with a 25 cm diameter aperture has about 2,500 times the area of your pupil: \((250 \text{ mm} / 5 \text{ mm})^2 = 2,500\).

In terms of magnitudes, recall that \(\frac{f_1}{f_2} = 10^{\frac{1}{2}(m_2-m_1)}\). Theoretically, larger area means proportionally larger flux. In practice, the area of the aperture is not exactly proportional to the received flux. Some light is lost at every optical surface, in addition to the fact that in reflectors the secondary is blocking some of the incoming light. Nonetheless, we could ignore these factors and ask what the theoretical best limiting magnitude would be for a telescope of a given size. Assume that your eye can, in fact, see to 6th magnitude; using a 25-cm diameter telescope, from the same observing location, you could see to a magnitude of 14.5:

\[
2500 = 10^\frac{1}{2}\Delta m.
\]

Take the log of both sides and solve for \(\Delta m\):

\[
\log(2500) = \frac{1}{2}(\Delta m)
\]
\[
\frac{1}{2}(3.398) = 8.5 = \Delta m.
\]

You can see 8.5 magnitudes fainter with the 25-cm telescope than with your eyes, or to a magnitude of 14.5.

**Field of view**

The field of view of a telescope describes the angular width of the patch of sky that you can see through the telescope. That angular extent is going to be inversely related to the focal length of the objective.

\[
\text{Field of view} = \frac{D}{f_{\text{obj}}},
\]

where \(D\) is the detector diameter and \(f_{\text{obj}}\) is the focal length of the objective.

Example: if we had an eyepiece with a usable diameter of 25 mm at a telescope with a 250 cm focal length, we would have a field of view of \((25 \text{ mm} / 2,500 \text{ mm}) = 1/100 \text{ radian}\). At 206,265 arcsec per radian, that’s ~2,000 arcsec or about ½ degree (i.e., about the size of the full moon).

If you have done some photography you may be familiar with the *f-number*. The f-number describes how rapidly an incoming bundle of parallel light rays comes to a focus by giving the ratio of the focal length of the optical element divided by the diameter of that bundle of light rays. For example, if we had a 25-cm diameter telescope with a focal length of 250 cm, we would describe it as being an f/10. In terms of f-values for simple telescopes, for rough comparison, an f/13 is a narrow field of view telescope and an f/3 is a wide field of view. Amateur telescopes are often ~ f/8.

**Angular magnification and image scale**

If we are observing with an eyepiece, we are often interested in answering the question, how many times more magnified is the image than what I can see with my eyes alone? The short answer is \(\text{Mag} = \frac{f_{\text{obj}}}{f_{\text{eyepiece}}.}\) Longer objectives give higher magnification, as do shorter eyepieces. Here is a sketch showing the difference:
Example: suppose we were using our 250-cm focal length telescope with a 40-mm focal length eyepiece. Our magnification would be \((2500 \text{ mm} / 40 \text{ mm}) = 62.5\). If we changed eyepieces, to one with a 10-mm focal length, we would increase the magnification to \((2500 \text{ mm} / 10 \text{ mm}) = 250\).

Note that while up to a point magnification can make it easier for you to see detail that’s in the image produced by the objective, it doesn’t increase the amount of detail in the image. You don’t get better resolution by increasing magnification.

If we are using a detector instead of an eyepiece, then rather than magnification the quantity we are interested in is called image (or plate) scale. It is usually given as the ratio of arcseconds on the sky to mm on the image. It only depends on the effective focal length of the telescope. Note that with multiple mirrors and/or lenses, the effective focal length of the entire optical system may not simply be the focal length of the objective. Image scale \(I\) is calculated as

\[
I = \frac{206,265}{f_{\text{objective}}},
\]

where the factor of 206,265 converts from radian to arcsec; \(f_{\text{obj}}\) would normally be in mm.

Example: our telescope with a focal length of 2500 mm has an image scale of \((206,265 / 2,500 \text{ mm}) = 82.5\) arcsec/mm.

To continue this example, if you were attaching to this telescope a digital camera with a detector 25 mm wide, your detector would see \((25 \text{ mm} \times 82.5 \text{ arcsec/mm}) = \sim 2,000 \text{ arcsec or } \frac{1}{2} \text{ a degree.}\)

Angular resolution

Even under ideal conditions, light coming through an aperture will undergo diffraction. The extent to which light from a point source is spread out is characterized by the point spread function. Real conditions – aberrations in the optics or turbulence in the atmosphere – will cause the point spread function to broaden. Under (unrealistically) ideal conditions, with no aberrations or atmosphere, the telescope would be said to be diffraction limited. In that case the point spread function looks like this, where the vertical axis represents image intensity:

For the round aperture of an optical telescope, this pattern is called an Airy disc (after Sir George Airy, who served for quite some time in the 1800’s as Britain’s Astronomer Royal).

What we are interested in is how close together on the sky two stars (which are pretty nearly point sources of light) could be such that we could still distinguish – resolve – one from the other. One way to estimate this is to
say that we need the peak of one star’s point spread function to be at least as far away as the first minimum in the second star’s point spread function. This is called the Rayleigh criterion. Mathematically it is given by

\[ \theta_{\text{min}} = 2.5 \cdot 10^5 \lambda / D \]

where \( \lambda \) is a representative wavelength being used for the observation and \( D \) is the diameter of the objective.

Here’s an illustration of stars that are well separated, just barely resolved, and not resolved:

![Illustration of stars](http://en.wikipedia.org/wiki/Angular_resolution)

Why does diffraction depend on aperture diameter and wavelength? Here is an illustration showing diffraction by a single slit:

![Single-slit diffraction](following Carroll & Ostlie, fig. 6.7)

Assume that light is incident from the left; the top ray is at the edge of the slit and the lower one is halfway down the slit. Let the wavefronts be coherent, meaning that if we are thinking about looking at a star, we are looking at light from one instant expanding outward from the star together. If the two rays arrive at point \( y \) exactly one half wavelength out of phase, then they will destructively interfere. In other words, with our angles in radians,

\[ \frac{D}{2} \sin \theta = \frac{1}{2} \lambda. \]

We can generalize this to consider all possible points of destructive interference:

\[ \sin \theta = m \frac{\lambda}{D}. \]

On the detector screen we get a central maximum with dark fringes at angles specified by the slit diameter and wavelength (for \( m = 1, 2, 3, \) etc.). Here’s a plot of what that looks like for our single slit:
Here is an image of monochromatic light (633 nm) through a grating with 150 slits, 0.0625 mm each, 0.25 mm slit separation.

In refraction, e.g. with prisms, short wavelengths of light are bent more than long wavelengths. With diffraction, short wavelengths are bent \textit{less}.

When you generalize from a slit to a circular aperture, you get an extra factor of 1.22;
\[ 1.22 \cdot 206.265 = 2.5 \cdot 10^4, \] giving us the appropriate factor in the equation above for the angular distance in arcsec to the first minimum in the diffraction limited point spread function.

Example: let’s return to our telescope with the 25-cm diameter. Assume that we are using it to observe visually, so a good wavelength for the problem would be something green, say 500 nm. The theoretical best resolution we could hope to achieve with this telescope would be
\[ \theta_m = 2.5 \cdot 10^{-4} \cdot (500 \text{ nm} \cdot 10^{-9} \text{ m/nm}) / 0.25 \text{ m} = 0.5 \text{ arcsec}. \]

In practice, it’s unlikely that we will get resolutions below a few arcsec with this telescope, in large part because of the motion of the air. Still, that’s a lot better than you can do with your eyes alone. If we assume a pupil diameter of 5 mm, our best resolution would be a factor of \((0.25 \text{ m} / 0.005 \text{ m})\) worse than this, or 50 arcsec. Note that for angular resolution, smaller angles are better.

\textbf{Telescope mountings}

Broadly speaking there are two usual ways of mounting telescopes, based on how they move to acquire or track objects around the sky.

The easiest to construct is the alt-az mounting. Altitude is the up-down direction, i.e., how high above the horizon something is; azimuth is the around direction, starting at 0° in the north and increasing toward the east. If you want to track an object while the Earth rotates, you have to drive an alt-az telescope in both axes.

The easiest for tracking is an equatorial mount. In this case one axis is aligned with the axis of the Earth. The altitude of the celestial pole is equal to your latitude, so your equatorial telescope mount needs to be tilted at an
angle that depends on your location. The telescope moves north-south (Declination) and east-west (Right Ascension). To track stars or galaxies, you only need to drive the east-west axis, since that will move the telescope parallel to the celestial equator, keeping it pointed at whatever you are observing. (Tracking solar system objects for long periods of time might require a bit more work.)

Spectrographs

Many astronomical observations fall into two broad categories, photometry and spectroscopy. In photometry we are asking how bright an object is over a broad range of wavelength; in spectroscopy we are asking what happens when we spread the light out into a spectrum and consider how the light varies over relatively small ranges in wavelength. Let’s look here at the process of obtaining that spectrum.

Astronomers are usually going to use a reflective diffraction grating (instead of a prism) to spread incoming light into a spectrum. An example of this principle with which you may be already familiar is to consider what the grooves on a CD will do to incident white light:

The grooves in a spectrograph are often very tightly spaced, perhaps 1000 grooves per mm. The top surface is reflective. Light interacts with each tiny wedge in a fashion that is very similar to light hitting a single slit, as described above. As with the single-slit diffraction pattern, points of constructive interference repeat, getting further and further spread out — $\theta$ increases with $m$, and we say that higher order spectra have a greater dispersion. Higher order spectra are also not a bright. Obtaining the best spectra is a balancing act between spreading the wavelengths out as much as possible while still having a spectrum that contains enough photons.

The remainder of this section is a Physics note: The following is a bit esoteric for our purposes but for those of you who are interested, let’s take a sketch of several grooves and examine what’s happening to incident light. The ray of light on the right will have to travel an extra distance ($d \sin \alpha$) compared to the ray on the left.

Similarly, light being reflected from the grating at an angle $\beta$ means that the light rays shown on the right will travel an extra ($d \sin \beta$).
The light comes in and goes back out again, so the total path difference for light reflecting off the grating is $d(\sin \alpha + \sin \beta)$. If that path difference is an integer number of wavelengths then the light will experience constructive interference. The grating equation is

$$d(\sin \alpha + \sin \beta) = n\lambda,$$

where $n$ is called the spectral order. A feature of this equation that you may already know, for instance if you’ve seen a laser beam directed through a transmission diffraction grating, is that a given incident wavelength will be diffracted at multiple different angles. The grating will be manufactured with a specific groove spacing, $d$, and chances are good that the spectrograph design will ensure that all wavelengths of light will enter at the same incident angle. That means that the angle $\beta$ is dependent on wavelength. In other words, different wavelengths will be reflected at different angles and incident light will be spread out into a spectrum.

If you do a few calculations you can see that higher order spectra are more spread out. Suppose we have a grating with 1000 grooves per mm, and incoming light with an incident angle of 19 degrees to the normal. Rearrange the grating equation as follows:

$$\sin \beta = n\lambda/d - \sin \alpha,$$

or,

$$\sin \beta = n\lambda \text{ (nm)}/(1000 \text{ nm}) - 0.326$$

<table>
<thead>
<tr>
<th>wavelength (nm)</th>
<th>$\beta$ for $n = 1$ (degrees)</th>
<th>$\beta$ for $n = 2$ (degrees)</th>
</tr>
</thead>
<tbody>
<tr>
<td>450</td>
<td>7.1</td>
<td>35.0</td>
</tr>
<tr>
<td>500</td>
<td>10.0</td>
<td>42.4</td>
</tr>
<tr>
<td>550</td>
<td>12.9</td>
<td>50.7</td>
</tr>
<tr>
<td>600</td>
<td>15.9</td>
<td>60.9</td>
</tr>
<tr>
<td>650</td>
<td>18.9</td>
<td>76.9</td>
</tr>
</tbody>
</table>

Higher orders are not as bright, but the fact that higher orders are more spread out can make it easier to distinguish spectral features that are closely spaced in wavelength. You can also see that 700 nm, in first order, would be at the same location as 350 nm in second order. That means that it is often necessary to filter out wavelengths that you aren’t interested in and that would otherwise be in the way of wavelengths you do want to study.

One variant is to use a pair of gratings at right angles, called an Echelle spectrograph. This results in a set of stacked spectra, each of a part of the full spectral range.
CCD cameras

The development, pun intended, of dry-plate photography in the latter part of the 19th century played a major role in astronomy in that now one could systematically record observations for future analysis. Photographic emulsions consist of tiny (on the order of 5µm) grains of light-sensitive molecules (such as silver halide) laid down on glass plates or, later, film. The grains don’t tend to be all that sensitive, however, responding to at best a few percent of the incident photons, making for long exposure times for faint objects. Long time exposures, though, did reveal objects not visible to the eye, even aided by the telescope. Emulsions sensitive to different wavelength regions made it possible to begin to catalogue the colors of stars. Images of astronomical objects could be made available to the general public.

Astronomical photography took a quantum leap forward in the 1970s and 80s with the development of the charge-coupled device, or CCD. The CCD chip consists of an array of tiny photodiodes called pixels (short for “picture elements). Photons above a threshold energy level strike the light-sensitive material and kick off electrons. This is an example of what’s called the photoelectric effect; Einstein’s 1905 paper explaining the photoelectric effect in terms of quanta of energy — photons — hitting a surface with enough energy to eject electrons, was the major piece of work for which he receive the 1921 Nobel Prize in Physics. More photons, more electrons. Pixels are like individual buckets or wells, in that they are somewhat insulated from each other, although at some point if too much light hits a pixel it will “saturate” and charge will start to bleed into adjacent pixels.

In the simplest CCD design, the CCD chip is exposed to light for some amount of time, charge builds up in the pixels, and then the camera shutter is closed and the pixel charges are read out and converted to an image. Charges are generally read out one at a time. There’s a spare row, or horizontal “register”, of pixels along one edge of the chip. An appropriately applied voltage encourages the charges in the row above to transfer vertically down to the vacant pixels and subsequently each row of charge is transferred down one into the now-vacant pixels below. The charge on the pixel at one end of the horizontal register is read out electronically and then the charges that have been transferred into the horizontal row of pixels are nudged sideways one by one and each charge read out as it reaches the end of the row. Once that row has been recorded, the charges are shifted down vertically by one more row, the refilled horizontal row is read out, and so on. Here’s a schematic to illustrate what’s happening.

![High-dispersion spectrum of the star Arcturus simulating an echelle spectrum.](https://www.noao.edu/image_gallery/html/im0609.html)
There are various designs for CCD chips and the above is only intended to give a general picture of how they work. Reality intrudes in several ways to complicate this picture. Here are two of the most important: First, electron “noise” exists, charge that has nothing to do with the astronomical objects whose light is being recorded. To account for these spurious electrons observers take “dark” images at the same temperature and exposure time as the actual program images and subtract these darks from the program images. Cooling the chip will reduce the noise. Second, pixels vary. Some are simply defective, but even if they aren’t, variation happens, due not just to the differing sensitivity of the pixels but to transient effects such as specks of dust in the optical path. To deal with this variation the observer records a “flat” image of a uniformly illuminated source, such as the twilight sky opposite the sun’s position, and divides the program images by this flat field.

Even after dealing with the complications, the CCD has one overwhelming advantage over photographic emulsions: the sensitivity (the “quantum efficiency”) is often greater than 50% and may be over 90% for some chips in particular wavelength ranges. Exposure times for astronomical observations dropped by approximately an order of magnitude once the CCD came online. The CCD has played such an important role in imaging, in many fields, that George E. Smith and Willard Boyle received the 2009 Nobel Prize in Physics for their role in the invention of the concept.

High-energy astronomy

The shortest-wavelength electromagnetic radiation — gamma rays and high-energy X-rays — are often best considered as particles rather than as waves. These, along with high-energy cosmic rays (mostly high-speed ions) and neutrinos, which are very light weight and travel at relativistic speeds, constitute a class of high-energy particles that are not going to be detectable using the optical techniques appropriate for visible and near-visible light. Neutrinos are tough to detect because they interact by the weak nuclear force but not electromagnetically. High-energy photons would just blast through a normal mirror. (There’s a reason the doctor is more likely to x-ray your broken arm than to shine a heat lamp on it. . .) The energies of the most extreme cosmic rays are roughly a million times higher than the highest-energy photons. Here we are going to look at a few examples of ways to detect high-energy particles.

Several places in this text you’ve seen a reference to or images from the Chandra X-ray Space Telescope, launched in 1999. X-rays will go through a mirror if they hit at or near normal incidence (i.e., along the mirror axis). Light, including x-rays, that hits a mirror at grazing incidence will have its path slightly bent. Chandra uses two sets of four nested mirrors to bend the incoming x-rays paths just enough to focus them on a detector. This process is shown in the following diagram:
Chandra has several instruments; one of the two at the focus is the High-Resolution Camera (HRC), so called because it has a resolution of approximately 0.5″. The HRC contains millions of tiny (10-micron diameter) glass tubes whose coating releases electrons when struck. A voltage accelerates the electrons which hit the sides of the tubes and release more electrons. Past the tubes, this electron cascade is detected by a fine grid of wires, allowing highly precise determinations of the positions of the original incident x-rays.

The Fermi Gamma-ray Space Telescope was launched in 2008. Fermi operates over a wide energy range, detecting gamma rays from ~8 keV to roughly 300 GeV with two instruments, a Large-Area (i.e., wide-field) Telescope (LAT) and a Gamma-ray Burst Monitor (GBM). Fermi gets hit by far more high-energy cosmic rays than gamma rays but fortunately the former are electrically charged which permits the instruments to discriminate between these two types of particles. The LAT is sensitive from ~20 MeV – ~300 GeV. Gamma rays enter the LAT, hitting thin tungsten sheets with enough energy to produce electron-positron pairs (recall that $E = mc^2$). Interleaved silicon strips record the gamma ray’s passage, producing small but detectable electric signals. Finally, cesium-iodide crystals scintillate in response to the passing gamma ray, giving a measure of its original energy. The GBM consists of a dozen scintillators pointing in different directions and sensitive to high-energy X-rays and low-energy gamma rays. When a gamma-ray burst is observed, the scintillator most directly pointing toward the source will record more photons, giving a rough directionality to the event.

In chapter 13 we looked at the Ray Davis / John Bahcall experiment detecting the first solar neutrinos using the weak interaction in which $^{37}\text{Cl}$ captures a neutrino and is converted into $^{37}\text{Ar} + e^-$. Another way to detect various high-speed particles is to make use of the fact that light doesn’t always travel at “the speed of light”. Most materials have an index of refraction greater than 1, meaning that light has a speed less than c. High-speed charged particles entering, for instance, the atmosphere or a tank of water or the Antarctic ice cap, may find themselves going faster than the local speed of light. Černkov radiation is kind of like the optical equivalent of a sonic boom, a blue glow emitted by these faster-than-light particles. The detected particles may be secondary particles; e.g., electrons may accelerated by interactions with neutrinos or an extremely high-energy cosmic ray may create a whole shower of high-speed particles as it collides with atoms high in our atmosphere. Photometers detecting the Černkov radiation are thus observing the path taken by particles such as neutrinos that might otherwise be incredibly hard to detect. Examples of experiments based on detecting Černkov radiation include Super-Kamiokande and IceCube neutrino detectors (in Japan and the Antarctic, respectively), and the Pierre Auger ultra-high-energy cosmic ray detector (in Argentina) and HAWC (the High-Altitude Water Černkov Observatory, in Mexico), intended to detect air showers.

To reiterate, these few examples are only to give you some sense of the methods used to detect high-energy particles.

Radio Telescopes

Optical wavelengths range from ~400 – 700 nm; radio covers everything from the microwave (mm – few cm) to 100s of km. It is tough to get images with good angular resolution in the radio part of the spectrum. As mentioned above, the theoretical best resolution for a telescope is given by this equation:

$$\theta_{\text{max}} = 2.5 \cdot 10^5 \frac{\lambda}{D}$$
although in practice the atmosphere won’t permit us to achieve this limit. Still, good resolution at longer wavelengths requires larger telescopes. That can get unwieldy. One solution is to use an array of radio telescopes, usually called “dishes”.

Radio waves reaching one radio dish from an astronomical source arrive slightly out of phase with radio waves reaching another dish. With a bit of mathematics the interference pattern produced by the electromagnetic waves arriving at all possible pairs of antennae can be converted into a pattern of source brightness on the sky.

Math and physic notes; again, what follows is a bit esoteric but may be of interest to some of you. Here’s a sketch of a basic two-element radio interferometer:

The time delay $\tau_g = (b \cdot s)/c$, where $b$ is the baseline vector and $s$ is a unit vector indicating the direction toward of the source. The signals from the two antennas will produce an interference pattern. The incoming wavefront represents light that left the object (star, galaxy, exoplanet) together, at the same time. The path delay can be thought of either as the extra time it takes the incoming wavefront to reach the left-hand telescope or as some number of extra wavelengths.

- If it’s an integer number of wavelengths, the signals received at the two telescopes will be in phase.
- If it were equal to $\frac{1}{2} \lambda$, the signals would be exactly out of phase and cancel out. As the Earth rotates, the direction toward the source changes, meaning that the length $\tau_g$ changes, and the signals received by the two telescopes will come in and out of phase, meaning that the interference pattern will change.

Aperture synthesis requires observing the amplitude and phase of the incoming signals from an array of telescopes and recording the signals along with an accurate time. If the telescopes can be moved, providing different baselines, and we wait while the Earth rotates, providing different delay times, we can synthesize, approximately, the observation we would get from one large telescope with a diameter equal to the longest baseline.
For example, the VLA (Very Large Array) in New Mexico, shown in the image above, is an array of twenty-seven 25-m antennas on a Y-shaped railroad track. The track arms are 21 km long, giving an effective diameter of 36 km when the antennas are in the widest possible configuration. The VLA can be used at several different frequencies, from 50 GHz to 74 MHz (0.7 – 410 cm) in eight bandpasses. The initial detector at the prime focus is a metal dipole antenna, whose length determines the wavelengths to which it is sensitive; the VLA has eight receivers. For the shortest wavelength bandpass, the resolution of the VLA at its widest configuration is about 0.05 arcsec! At the other end, though, at 400 cm, it’s only 24 arcsec. It’s really hard to get good resolution at long wavelengths.

You may or may not have encountered Fourier transforms before. A Fourier transform “expresses a function of time (or signal) in terms of the amplitude (and phase) of each of the frequencies that make it up” (http://en.wikipedia.org/wiki/Fourier_transform). For instance, if we’ve got a function \( f(x) \) that can be integrated, the Fourier transform and its inverse look like this:

\[
\hat{f}(\xi) = \int_{-\infty}^{\infty} f(x)e^{-2\pi i x \xi} \, dx; \\
\hat{f}(\xi) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(x)e^{2\pi i x \xi} \, dx
\]

where \( x \) and \( \xi \) are real numbers. Here, for instance, they might represent time and frequency. This is related to a Fourier series: periodic functions can be expressed as a sum of sines and cosines. The exponential comes about because

\[
e^{-2\pi i \theta} = \cos(2\pi \theta) + i \sin(2\pi \theta).
\]

A Fourier transform will measure how much of a particular frequency is present in a given function. In the case of the radio interferometry, we can measure the amplitude and phase of the electric field at each antenna and transform that to produce the brightness of the source as a function of location on the sky.

There is quite a lot to be learned observing at wavelengths beyond the visible. As an example, let’s look briefly at ALMA, the Atacama Large Millimeter/submillimeter Array. This 66-dish interferometer is located in Chile’s Atacama Desert, making it one of the highest (~5,000 m) and driest observatories on Earth. ALMA operates at wavelengths between 0.3 and 9.6 mm (or frequencies from 1000 to 31 GHz). Similar to the VLA, the individual radio antennas can be moved. ALMA is an international collaboration and played a role in the imaging of the shadow of the black hole in the galaxy M87 (see chapter 18). What else has it done? Among other things,

- observed the distribution of HCN (hydrogen cyanide) around the nucleus of comet 46P / Wirtanen;
- discovered aluminum oxide around a young star, relevant for understanding meteorites;
- observed a protoplanetary disk around HD169142 that provides evidence supporting the migration of young planets;
- observed the thermal emission from the cold particles in the rings of Uranus;
- detected the molecule C₂H₃CN (vinyl cyanide) in the atmosphere of Titan, which might suggest prebiotic chemistry on Saturn’s largest moon;
- participated in multi-wavelength observations of AT2018cow, a super-energetic explosion that appears to have occurred in a galaxy ~ 60 Mpc away and for which a single convincing explanation is still (as of early 2019) lacking;
- measured the abundance of ¹⁸O, produced by massive stars, and ¹³C, produced by intermediate mass stars, in distant starburst galaxies, relevant because of the relatively larger-than-expected number of massive stars forming;
- observed emission from oxygen, carbon, and dust from a galaxy merger 13 billion year ago.

In other words, ALMA is very useful for detecting molecules and seeing (or seeing better) through the dust, from young stellar systems nearby to the most distant galaxies.
Sample problems.

1. The Keck telescopes on Mauna Kea are a pair of telescopes, each 10 meters in diameter. Compare one of these telescopes to a 14-inch diameter reflector.
   a) How many times more light can one of the Keck 10-m telescopes gather than a 14-inch telescope?
   b) How many magnitudes fainter would this be, assuming equivalent atmospheric conditions?
   c) Calculate the theoretical best angular resolution that could be achieved with one of the Keck telescopes; use a wavelength of 500 nm.
   d) How many times better is the best resolution of a 10-m telescope than the 14-inch?

2. Suppose that we wanted a radio telescope with a theoretical best resolution of 0.5" for observations at 21 cm; how large would that telescope need to be?

3. Reading carefully? Briefly explain
   a) refractor vs reflector
   b) Cassegrain and Newtonian
   c) photometry vs spectroscopy
   d) grazing incidence mirrors
   e) equatorial vs alt-az mounting

Answers to selected problems are on the next page.
1. 
   a) ~ 790
   b) ~ 7 mags
   c) ~ 0.0125 arc seconds
   d) ~ 28 times better

2. ~ 105 km