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ASTR-2310 HW #1 Solutions

Problem 1)

a) vis-viva: $v^2 = G(m_1+m_2)\left(\frac{2}{r} - \frac{1}{a}\right)$

- your mass is insignificant $m_1+m_2 = M_{Earth}$
- circular orbit so $r = a$

$$v^2 = G M_{Earth}\left(\frac{1}{r}\right)$$

plug in $G = 6.67 \times 10^{-11}\, m^3 kg^{-1} s^{-2}$, $M_{Earth} = 6 \times 10^{24}$ kg

$r = 10{,}000$ km $= 10^7$ m

$$\boxed{v = 6{,}313 \text{ m/s}}$$

b) You enter the elliptical orbit at its apogee.

Use vis-viva to find a:

$$v^2 = GM_E\left(\frac{2}{r} - \frac{1}{a}\right)$$

$$\frac{v^2}{GM_E} = \frac{2}{r} - \frac{1}{a} \Rightarrow \frac{1}{a} = \frac{2}{r} - \frac{v^2}{GM_E} \Rightarrow a = \left(\frac{2}{r} - \frac{v^2}{GM_E}\right)^{-1}$$

- plug in $v = 6000$ m/s, $r = 10^7$ m

$\cancel{a = 9180 \text{ km}}$ $\boxed{a = 9180 \text{ km}}$

Then solve for eccentricity of orbit

apogee $= 10000$ km $= a(1+e) \Rightarrow \boxed{e = 0.09649}$

Finally perigee $= a(1-e) = \boxed{8240 \text{ km} = 8.24 \times 10^6 \text{ m}}$

c) vis-viva $v^2 = GM_E\left(\frac{2}{8240 \text{ km}} - \frac{1}{9120 \text{ km}}\right)$

$$\boxed{v = 7280 \text{ m/s}}$$

% faster $= \frac{|v_{peri} - v_{circ}|}{v_{circ}}$

15.3% faster.

d) period $P^2 = \frac{4\pi^2 a^3}{GM_E}$

if $a = 10000$ km, $P = 9950$ sec (circular)

if $a = 9120$ km, $P = 8666$ sec (elliptical)

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Problem 2

Start w/ known equations

$$m_p r_p = m_c r_c \qquad p^2 = \frac{4\pi^2 a^3}{G(m_p+m_c)}$$

$$r_c + r_p = 19{,}571 = R$$

$$m_c + m_p = M$$

Given $r_c = 8.2 r_p$ find mass ratio

$$\frac{r_c}{r_p} = \frac{m_p}{m_c} = 8.2 \Rightarrow 8.2 m_c = m_p$$

Find total mass

$$M = \frac{4\pi^2 a^3}{G p^2}$$

$$p = 153\,hr, \quad a = 19{,}571\,km$$

$$M = 1.462 * 10^{22}\,kg$$

Seperate into m_c, m_p

$$m_c + m_p = 1.462 * 10^{22}\,kg$$

$$9.2 m_c = 1.462 * 10^{22}\,kg$$

$$\boxed{\begin{aligned} m_c &= 1.589 * 10^{21}\,kg \\ m_p &= 1.303 * 10^{22}\,kg \end{aligned}}$$

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Problem 3)

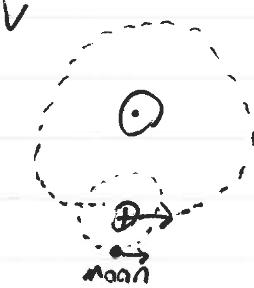
a) $v_{esc} = \sqrt{\frac{2GM}{r}}$

$M = 1M_\odot$, $r = 1\,AU$

$\boxed{v_{esc} = 42.119\ km/s}$

relative to Sun

b) Moons max speed in orbit will occur when its velocity is fully pointed away from Sun.



$v_{tot} = v_\oplus + v_{moon} = \sqrt{\frac{GM_\odot}{1au}} + \sqrt{\frac{GM_\oplus}{1ld}}$

l.d. = lunar distance = $3.84*10^5$ km

$\boxed{v_{tot} = 30.799\ km/s}$

c) $v_f = v_i + at$

$v_f = v_{esc} = 42.119\,km/s$

$v_i = 30.799\ km/s$

$t = 150\ s$

$a = 7.55*10^{-2}\ km/s^2 = 75.5\ m/s^2$

with $g = 9.8\ m/s^2$

$\boxed{a = \frac{75.5m/s^2}{9.8m/s^2} = 7.70g}$

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P4)

a) In a bound system the kinetic energy is half the potential energy.

b) The Tidal force from the moon causes a "high Tide" on both the near and far side of the Earth. The Earth's daily rotation means every location will be both the near and far side over the course of a day.

c) Perihelion - closest point in orbit around Sun.
Aphelion - furthest point in orbit around Sun.

d) Apogee - furthest point in orbit around Earth.
Aphelion - furthest point in orbit around Sun.

e) $1\,AU = 1.5 * 10^{11}\,m$, $1pm = 1*10^{15}\,m$

$$1pm = 1*10^{15}\,m * \frac{1\,Au}{1.5*10^{11}\,m} = 6.67*10^{3}\,AU$$

f) $e = \frac{q-Q}{q+Q}$ — q - distance at apoapse, Q - distance at periapse

runs from 0 to 1

g) Lagrange points are stable or equilibrium points in an ~~[illegible]~~ the potential formed by two orbiting objects.