Exam Spring 2017 (Brotherton)

Phys 1210 (Ch. 6-8) ________________

your name

The exam consists of 6 problems. Each problem is of equal value.

You can skip one of the problems (best five will count if you do all problems). Calculators are allowed.

Tips for better exam grades:

Read all problems right away and ask questions as early as possible.

Make sure that you give at least a basic relevant equation or figure for each sub-problem.

Make use of the entire exam time. When you are done with solving the problems and there is some time left, read your answers over again and search for incomplete or wrong parts.

Show your work for full credit. The answer '42' only earns you any credit IF '42' is the right answer. We reserve points for 'steps in between', figures, units, etc.

No credit given for illegible handwriting or flawed logic in an argument.

Remember to give units on final answers.

Please box final answers so we don't miss them during grading.

Please use blank paper to write answers, starting each problem on a new page.

Please use 10 m/s^2 as the acceleration due to gravity on Earth.

'Nuff said.

1. **Spider-man stops a train.** In Spider-man 2, Spidey stops a runaway train with his webs. Assume the train has a mass of 320,000 kg and an initial velocity of 90 km/h (hint: convert to m/s). Ignore friction. Assume that he uses **two** web lines, which are initially unstretched, and we can treat them as springs. What is their spring constant k if the train is stopped after 250 meters?

2. **Powergirl helps Aquaman rescue a stranded whale.** A humpback whale, with a mass of 30,000 kg, is stranded on the beach ten meters from the water. Our heroes connect a heavy-duty chain to a harness around the whale, and Powergirl pulls it toward the ocean, keeping an angle of 30 degrees between the beach and the chain. If she pulls the chain with a force of 90,000 Newtons, and the coefficient of sliding friction is 0.30, how fast is the whale moving horizontally when it hits the water? Reminder: use $g=10\ m/s^2$ and reduce to two significant figures at the end.

3. **Catwoman Drags Batman.** Batman (100 kg) has caught up to Catwoman (60 kg) on top of a building with a rain-slick roof you can treat as frictionless. She is 5 meters away from Batman, catches him with her whip, then steps off the edge. She falls, pulling Batman toward the edge. How fast is he sliding when he gets to the edge? (Don't worry – he'll cut the whip with a batknife just before he falls!)

4. **Black Widow Shoots at Captain America!** It's ok! He has his shield, and she's only temporarily under Loki's mind control. The Black Widow uses a Glock 26. The mass of one type of 9mm bullet used in the Glock 26 is 7.5 grams. The velocity of this bullet shot from the Glock 26 is 370 m/s. If the bullet comes in horizontally, and bounces up and back off the shield at a 45 degree angle (from the horizontal), what is the magnitude and direction of the impulse of the shield on the bullet? Assume a totally elastic collision, such that the outgoing spped is also 370 m/s.

5. **Hulk Catches Ironman.** After saving New York from a nuclear weapon, Ironman (200 kg) is falling from the sky at 50 m/s straight down. Hulk (500 kg) is jumping horizontally at 80 m/s to catch him. Immediately after the catch before gravity has had time to change the velocity significantly, what is the speed of the pair? (No need to give direction, just speed for this question.)

6. **Wonder Woman has lassoed Captain Cold.** They are standing 10 meters apart on essentially frictionless ice. She masses 75 kilograms. He masses 90 kilograms. She starts pulling in the lasso. How far does she move before she has completely reeled him in?

Master Equations – Physics 1210

One-dimensional motion with constant acceleration:

① $x = x_0 + v_{0x}t + \frac{1}{2}a_x t^2$ find the other forms of master equation 1 by

(a) building the derivative of the equation
(b) solving the new equation for t and substituting it back into the master equation, and
(c) using the equation for average velocity times time

Two-dimensional motion for an object with initial velocity v_o at an angle α relative to the horizontal, with constant acceleration in the y direction:

② $x = x_0 + v_0 cos\alpha\, t$

③ $y = y_0 + v_0 sin\alpha\, t + \frac{1}{2}a_y t^2$ find the related velocities by building the derivatives of the equations

Newton's Laws

④ $\Sigma\vec{F} = 0,\ \Sigma\vec{F} = m\,\vec{a},\ \vec{F}_{A\to B} = -\vec{F}_{B\to A}$ find the related component equations by replacing all relevant properties by their component values

The quadratic equation and its solution:

$$a \cdot x^2 + b \cdot x + c = 0\,, then\ x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Table with some values for trig functions:

Degrees:	30	45	60	330	
sin	0.5	0.707	0.866	-0.5	
cos	0.866	0.707	0.5	0.866	
tan	0.577	1	1.732	-0.577	

Work and Power definitions:

Work $W = \vec{F} \cdot \vec{s} = Fs\cos\phi$
Power P = dW/dt

Hook's Law:

F = -kx (where k is the spring constant)

Kinetic Energy:

$K = \frac{1}{2} mv^2$ (linear)
$K = \frac{1}{2} I w^2$ (rotational)

Potential Energy:

U = mgh (gravitational)
$U = \frac{1}{2} kx^2$ (elastic for a spring constant k)

Work-energy with both kinetic and potential energy:

$K_1 + U_1 + W_{other} = K_2 + U_2$

Linear Momentum:

$$\vec{p} = m\vec{v} \text{ and } \vec{F} = d\vec{p}/dt$$

Impulse and Impulse-Momentum Theorem:

$$\vec{J} = \int_{t1}^{t2} \Sigma \vec{F}\, dt = \vec{p}_2 - \vec{p}_1$$

Angular-Linear Relationships:

$a = v^2/r$ (uniform circular motion)
$v = r\omega$, $a_{tan} = r\alpha$, and $a_{rad} = v^2/r = r\omega^2$

Parallel axis theorem for the moment of inertia I:

$I_p = I_{cm} + Md^2$

Angular dynamics:

Torque $\vec{\tau} = \vec{r} \, X \, \vec{F}$ and $\sum \tau_z = I\alpha_z$

Angular Momentum:

$$\vec{L} = \vec{r} \, X \, \vec{p} \text{ and } \vec{\tau} = d\vec{L}/dt$$

Center of Mass:

$$\vec{r}_{cm} = \frac{\sum_i m_i r_i}{\sum_i m_i}$$

Fluid Mechanics

$p = p_0 + \rho g h$ (pressure in an incompressible fluid of constant density)
$A_1 v_1 = A_2 v_2$ (continuity equation, incompressible fluid)
$dV/dt = Av$
$p_1 + \rho g y_1 + ½ \rho v_1^2 = p_2 + \rho g y_2 + ½ \rho v_2^2$ (steady flow, ideal fluid)

Gravity:

$F = Gm_1m_2/r^2$
$U = -Gm_Em/r$
T (orbital period) = $2 \pi r^{3/2}/\text{sqrt}(Gm_E)$
$G = 6.67 \times 10^{-11}\ N \cdot (m/kg)^2$

Periodic Motion

$f = 1/T$; $T = 1/f$
$\omega = 2\pi f = 2\pi/T$ (angular frequency here)
$\omega = \text{sqrt}(k/m)$ (k is spring constant)
$x = A \cos(\omega t + \Phi)$
$\omega = \text{sqrt}(\kappa/I)$ (angular harmonic motion)
$\omega = \text{sqrt}(g/L)$ (simple pendulum)
$\omega = \text{sqrt}(mgd/I)$ (physical pendulum)

Mechanical Waves in General

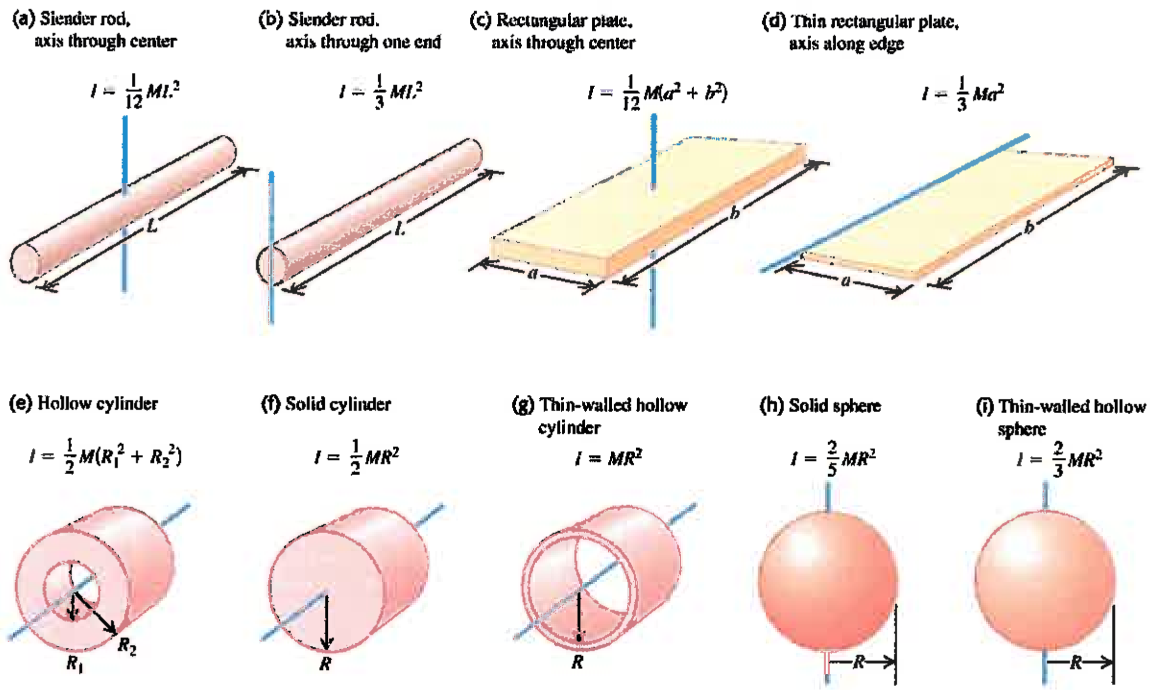
$V = \lambda f$
$Y(x,t) = A \cos(kx - \omega t)$ (k is wavenumber, $k = 2 \pi f$)
$V = \text{sqrt}(F/\mu)$
$P_{av} = ½\ \text{sqrt}(\mu F)\ \omega^2 A^2$
$I_1/I_2 = (r_2/r_1)^2$ (inverse square law for intensity)

Sound Waves

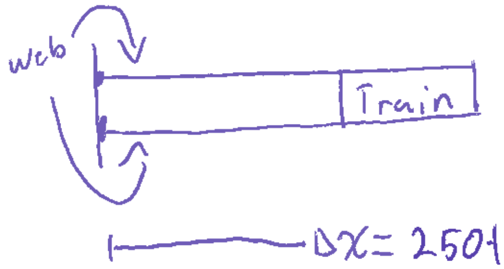
$P_{max} = BkA$ (B is bulk modulus)
$B = (10\ dB) \log(I/I_0)$ where $I_0 = 1 \times 10^{-12}\ W/m^2$
$f_L = f_s * (v+v_L)/(v+v_s)$ -- Doppler effect

Table 9.1 Comparison of Linear and Angular Motion with Constant Acceleration

Straight-Line Motion with Constant Linear Acceleration	Fixed-Axis Rotation with Constant Angular Acceleration
$a_x = \text{constant}$	$\alpha_z = \text{constant}$
$v_x = v_{0x} + a_x t$	$\omega_z = \omega_{0z} + \alpha_z t$
$x = x_0 + v_{0x}t + \frac{1}{2}a_x t^2$	$\theta = \theta_0 + \omega_{0z}t + \frac{1}{2}\alpha_z t^2$
$v_x^2 = v_{0x}^2 + 2a_x(x - x_0)$	$\omega_z^2 = \omega_{0z}^2 + 2\alpha_z(\theta - \theta_0)$
$x - x_0 = \frac{1}{2}(v_x + v_{0x})t$	$\theta - \theta_0 = \frac{1}{2}(\omega_z + \omega_{0z})t$



Spider-Man Saves a Train



$m_i = 3.2 \times 10^5$ [Kg]

$|\vec{V}_i| = 90$ [km/h]

Ignore Friction

<u>Spring Constant?</u>

$\Delta x = 250$ [m]

$$\Sigma E_i = \Sigma E_f$$

two webs

$$\frac{1}{2} m v_i^2 = 2\left(\frac{1}{2} k x^2\right)$$

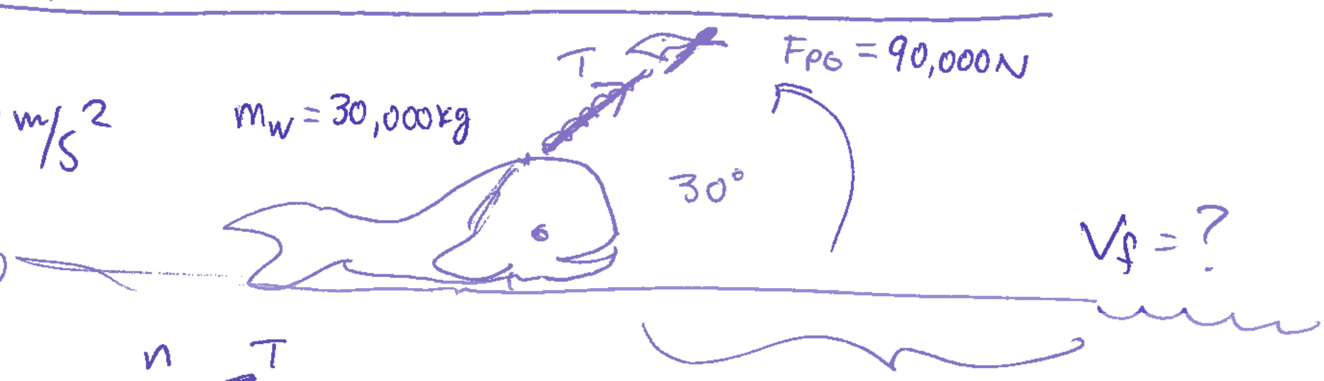
$$\Rightarrow k = \frac{m v_i^2}{2x^2}$$

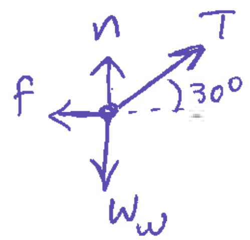
$$\therefore K \approx 1600 \left[\frac{N}{m}\right]$$

Powergirl & Aquaman Save a Whale

$g \downarrow 10\ m/s^2$ $\quad m_W = 30{,}000\,kg$

$V_i = 0$





$\mu_K = 0{,}3$

$$\cancel{K_1}^{\,0} + \cancel{U_1}^{\,0} + W_{other} = K_2 + \cancel{U_2}^{\,0}$$

$$W_{other} = \frac{1}{2} m_w V_f^2$$

$$W_{other} = \underbrace{\vec{T}\cdot\vec{S}}_{} - \underbrace{n \cdot \mu_K \cdot S}_{\text{friction}} = \frac{1}{2} m_w V_f^2$$

$T \cdot S \cdot \cos\theta = PG \text{ work}$

$$n + T\sin\theta = W_w \quad \text{so} \quad n = W_w - T\sin\theta$$

$$W_{other} = (90{,}000N)(10\,m)(0.866) - (300{,}000N - 90{,}000N(0.5))\,0{,}3 \cdot 10_m$$

so $$14{,}422.9\,J = \frac{1}{2}(30{,}000\,kg)\,V_f^2$$

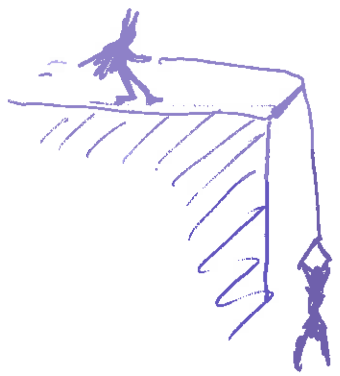
$$\boxed{V_2 = 0.98\ m/s}$$

Catwoman drags Batman to Edge

$m_B = 100\,kg.$

Rain-slick surface.
Assume frictionless.

$g \downarrow 10\,m/s^2$



$m_C = 60\,kg.$

$V_i = 0$ when Catwoman Steps off edge.

How fast in Batman Sliding after 5 m.
When he hits the edge?

(Wwhip cancels out)

$$\underbrace{K_1}_{\frac{1}{2}mv_i^2} + \underbrace{U_1}_{mgh} + \cancel{W_{other}}^{\,0} = K_2 + U_2$$

Initially all Zero on left.

$$0 = \underbrace{\tfrac{1}{2}m_B}_{50\,kg}V_f^2 + \underbrace{\tfrac{1}{2}m_C}_{30\,kg}V_f^2 - \underbrace{m_C g h}_{-3000\,kg\cdot m^2/s^2}$$

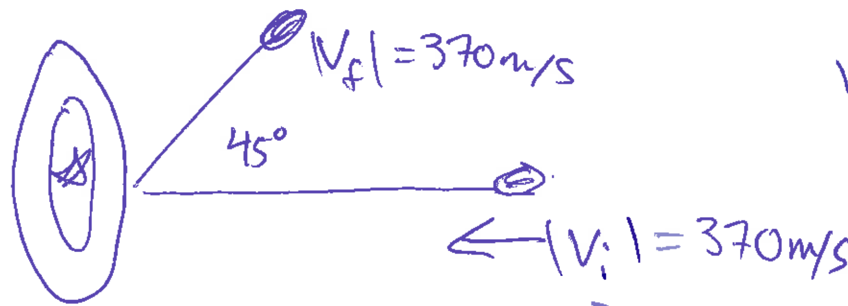
$h = -5\,m.$

$$3000\,kg\,m^2/s^2 = 80\,kg\,V_f^2$$

$$V_f^2 = (37.5)\,m^2/s^2$$

$$\boxed{V_f = 6.1\,m/s}$$

Black Widow Shoots at Captain America



$m_B = 7.5 g$
$= 0.0075 kg$

What is $\vec{J}$?

$\vec{J} = \vec{p}_2 - \vec{p}_1$

$\vec{p} = m\vec{v}$

$J_x = p_{2x} - p_{1x}$

$J_y = p_{2y} - \overset{0}{p_{1y}}$

$J_x = (0.0075 kg)(370 m/s)\cos\theta$
$\underbrace{- \; -}_{+} (0.0075 kg)(370 m/s)$

$J_y = (0.0075 kg)(370 m/s)\sin 45^\circ$

$J_x = 4.74 \; kg \cdot m/s$

$J_y = 1.96 \; kg \cdot m/s$

$$|J| = \sqrt{J_x^2 + J_y^2} = \boxed{5.1 \; kg \cdot m/s = 5.1 \cdot N \cdot s}$$

$$\theta = \tan^{-1}\left(\frac{1.96}{4.74}\right) = \boxed{22.5^\circ \text{ above horizontal}}$$

Hulk Saves Iron-Man

$\vec{g} = -10 \left[\frac{m}{s^2}\right] \hat{y}$

$\vec{V}_H = 80 \left[\frac{m}{s}\right] \hat{x}$

$m_H = 500 [Kg]$

$\vec{V}_I = -50 \left[\frac{m}{s}\right] \hat{y}$

$m_I = 200 [Kg]$

$|\vec{V}_f| = ?$

$$\Sigma P_i = \Sigma P_f$$

$$m_H \cdot \vec{V}_H + m_I \cdot \vec{V}_I = (m_H + m_I) \cdot \vec{V}_f$$

$$\Rightarrow |\vec{V}_f| = \left| \frac{m_H \cdot \vec{V}_H + m_I \cdot \vec{V}_I}{m_H + m_I} \right|$$

$$\cong \left| 57.14 \left[\frac{m}{s}\right] \hat{x} - 14.29 \left[\frac{m}{s}\right] \hat{y} \right|$$

$$= \sqrt{\left(57.14 \left[\frac{m}{s}\right]\right)^2 + \left(14.29 \left[\frac{m}{s}\right]\right)^2}$$

$$\therefore |\vec{V}_f| \approx 58.90 \left[\frac{m}{s}\right] \simeq 59 \, m/s$$

Wonder Woman Lassos Captain Cold

$M_{WW} = 75\text{ kg}$ $M_{CC} = 90\text{ kg}$

when WW pulls CC to her,
how much does she move?

→ C.O.M. constant (no external forces) ←

make $X_{WW} = 0$ initially ⇒ $\overset{10\text{ m}}{''}$ $\overset{90\text{ kg}}{''}$

$$X_{c.o.m.} = \frac{(m_{WW})(\cancel{X_{WW}}^{\,0}) + (X_{cc})(m_{cc})}{(75+90)\text{ kg}}$$

$$X_{c.o.m.} = 5.45\text{ m.}$$

After:

$$5.45\text{ m} = \frac{(\overset{75}{m_{WW}})(X_{final}) + (X_{final})\overset{90\text{ kg.}}{m_{cc}}}{165\text{ kg}}$$

$$5.45\text{ m} = \frac{165\text{ kg}}{165\text{ kg}} X_{final}$$

$$\boxed{X = 5.45\text{ m}}$$