Astr 2310 Tues. Feb. 4, 2014
Today's Topics

- Celestial Mechanics cont.
  - Least Energy Orbits
  - Example of Least-Energy Orbit to Mars
- Chapter 2: Solar System Overview
  - Constituents
  - Discovery of Outer Planets
  - Fundamental Characteristics
    - Mass and Radius
    - Surface Temperature and Black Body Radiation
    - Planetary Atmospheres and Composition
  - Radioactivity and Half-Life
    - Nuclear Physics (see The Making of the Atomic Bomb by R. Rhodes)
    - Age Dating of Solar System

Chapter 2: Homework

- #2, 3, 6 + Kepler's Law Graph in Excel

- Due Thursday February 6

Newtonian Gravity Cont.

To reach low Earth orbit requires enormous energy

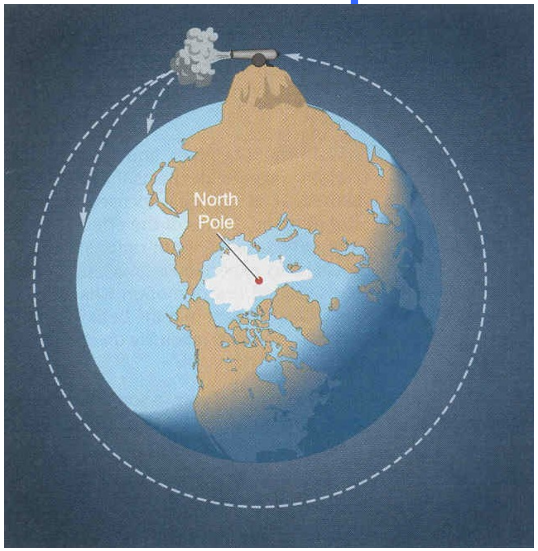
- **At Earth's equator: V_{rot} = 1000 mi/hr = 0.5 km/s**
- **Compare to velocity of circular orbit:**

$$V_c = \sqrt{\frac{GM}{r}}$$

= SQRT[$(6.67\text{x}10^{-11})(5.97\text{x}10^{24})/6.387 \text{ x } 10^{6}$)]

V_c = 7.9 km/sec

Consider the energy equation which will relate the shape of the orbit to its energy [$V(R_E)$]



<u>Consider 3 cases:</u>

$V = V_{cir}$ @ R_E
$V < V_{cir}$ (R_E = apogee)
$V > V_{cir}$ (R_E = perigee)

Note change in focus

Conservation of Energy Yields Energy Equation (object in orbit around the Earth or the Sun)

$TE = KE + PE$

$$= 1/2\ m_1 v_1^2 + 1/2\ m_2 v_2^2 - \frac{G m_1 m_2}{r}$$

Since $m_1 v_1 = m_2 v_2$ then $v_1 = m_2 v_2 / m_1$

Now consider the relative velocity:

$v = v_1 + v_2 = m_2 v_2 / m_1 + v_2$ so:

$v_2 = m_1 v / (m_1 + m_2)$ and $v_1 = m_2 v / (m_1 + m_2)$

In this case the total energy becomes:

$$TE = 1/2\ m_1 [m_2^2 v_2 / (m_1 + m_2)]^2 + 1/2\ m_2 [m_1^2 v_2 / (m_1 + m_2)]^2 - G m_1 m_2 / r$$

$$= v^2 / 2(m_1 + m_2)^2 [m_1 m_2^2 + m_2 m_1^2] - G m_1 m_2 / r$$

$$= v^2 m_1 m_2 / 2(m_1 + m_2) - G m_1 m_2 / r$$

$$TE = m_1 m_2 [v^2 / 2(m_1 + m_2) - G / r]$$

$TE = m_1 m_2[v^2 / 2(m_1 + m_2) - G / r]$

Now lets evaluate this at perihelion ($\theta = 0$):

First recall that $r = a(1 - e^2)/(1 + e\cos\theta)$ so $r_{peri} = a(1 - e^2)/(1 + e)$

And so:

$(TE)_{peri} = m_1 m_2[v^2 / 2(m_1 + m_2) - G(1 + e)/a(1 + e^2)]$ continuing:

Recall: $v_{peri} = (2\pi a / P)[(1 + e)/(1 - e)]^{1/2}$ thus:

$(TE)_{peri} = m_1 m_2\{[4\pi^2 a^2 (1 + e)]/[2\pi^2 (m_1 + m_2)(1 - e)] - G(1 + e)/a(1 + e^2)\}$

But $P^2 = 4\pi^2 a^3 / G(m_1 + m_2)$ *so* $G = 4\pi^2 a^3 / P^2 (m_1 + m_2)$

And: $(TE)_{peri} = m_1 m_2\{[G(1 + e)]/2a(1 - e)] - G / a(1 - e)\}$

$$= (Gm_1 m_2 / 2a)[(1 + e - 2)/(1 - e)]$$

so:

$$(TE)_{peri} = -Gm_1 m_2 / 2a$$

but energy is always conserved so this is also true in general. Thus:

$-Gm_1 m_2 / 2a = m_1 m_2[v^2 / 2(m_1 + m_2) - G / r]$ and so:

$G(1/r - 1/a) = v^2 / 2(m_1 + m_2)$ and solving for v gives us:

$v^2 = 2G(m_1 + m_2)[1/r - 1/2a]$ or:

$v^2 = G(m_1 + m_2)[2/r - 1/a]$ this is the energy equation

Example of a Least-Energy Orbit to Mars

We can rewrite the energy equation in terms of "Earth orbital units" (P in years, r and a in AU). In this case we have:

$V^2 = 2/r - 1/a$

Let $r = r_E = 1AU$, $r_m = 1.524$ AU

So $a = (r_m + r_E)/2 = 1.262$ AU

Thus: $V^2 = 2 - 1/1.262 = 1.206$

So: v = 1.098 (in terms of Earth's orbital velocity)

$V_c(E) = 2\pi r_E/(365X24X3600)$

$= 9.400x10^8$ km/$3.154x10^7$ sec

= 29.8km/sec

So V(E$\rightarrow$ M) = 32.7 km/sec

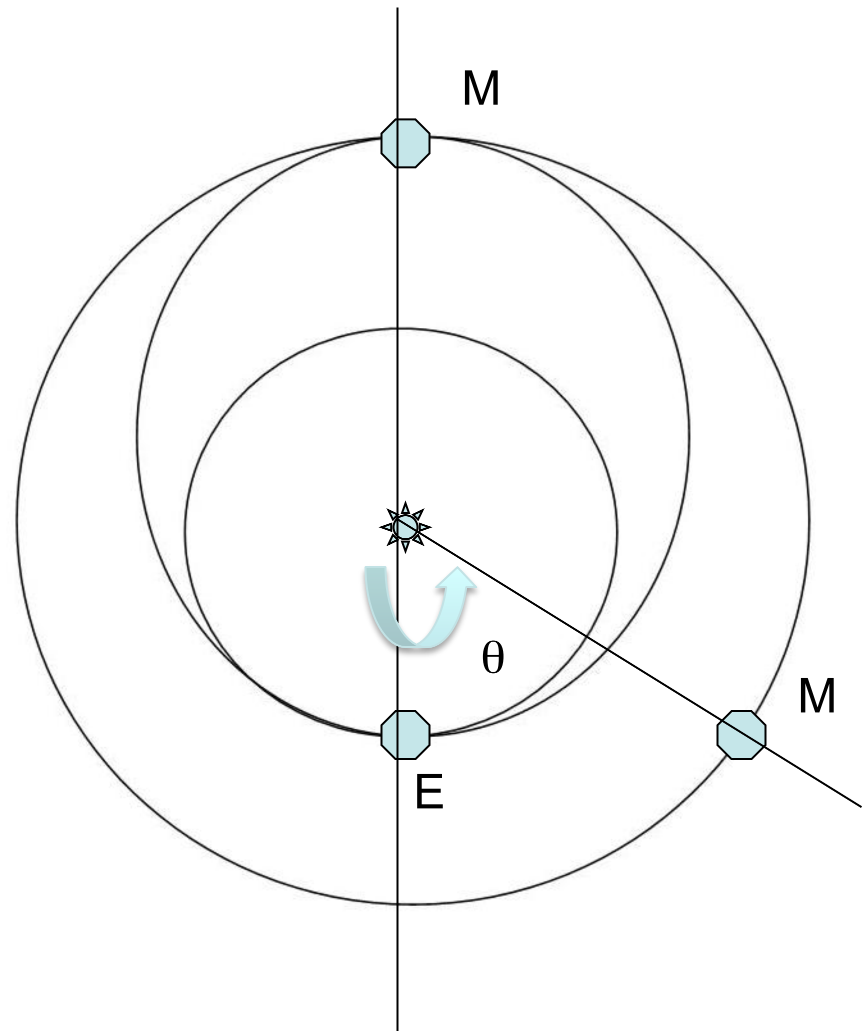
What is the time to reach Mars?

$P^2 = a^3$ so $P =(1.262)^{3/2} = 1.418$ yrs

Time to Mars = P/2 = 259 days

P(Mars) = 687 days so $\theta = 136^o$

(pos. of Earth & Mars at Launch)



Some Special Cases of the Energy Equation

Recall the energy equation:

$V^2 = G(m_1+m_2)[2/r – 1/a]$

(1) For a circular orbit about a mass ($m_2 << m_1$), $a = r$ so:

$V^2 = Gm_1[2/r – 1/r] = Gm_1/r \longrightarrow V = SQRT(GM/r)$

(2) For the escape velocity we want $a \longrightarrow$ infinity so:

$V^2 = Gm_1[2/r – 0] = 2Gm_1/r \longrightarrow V = SQRT(2Gm/r)$

And so $V_{esc} = SQRT(2)V_{cir}$