# Physics 2310 Lab 2: The Dispersion of Optical Glass

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**Based on a lab by Dr. M. Kruger (Univ. of Missouri, Kansas City)**

**Purpose:** The purpose of this lab is to introduce students to the dispersion of light in optical materials. That is, to the dependence of the index of refraction upon wavelength. This property must be considered during the design of optical instruments.

**Theoretical Basis:** We all know that a prism disperses light due to the change in the index of refraction with wavelength. As a result the refracted angle of light through a prism depends on wavelength. Specifically:

$n(\lambda) = c/v(\lambda)$.

No optical system is perfect and so a designer must carefully weigh cost vs. complexity in designing a lens or system of lenses for a particular application. This naturally results in compromises in performance. One example is the residual color seen around the images produced by your binoculars, riflescope, or telescope. In this case the designer choses different types of glass for the lenses in order to balance and partially cancel the residual color that results from the dispersion of light. A plot of the refractive indices of some representative materials vs. wavelength is shown here:

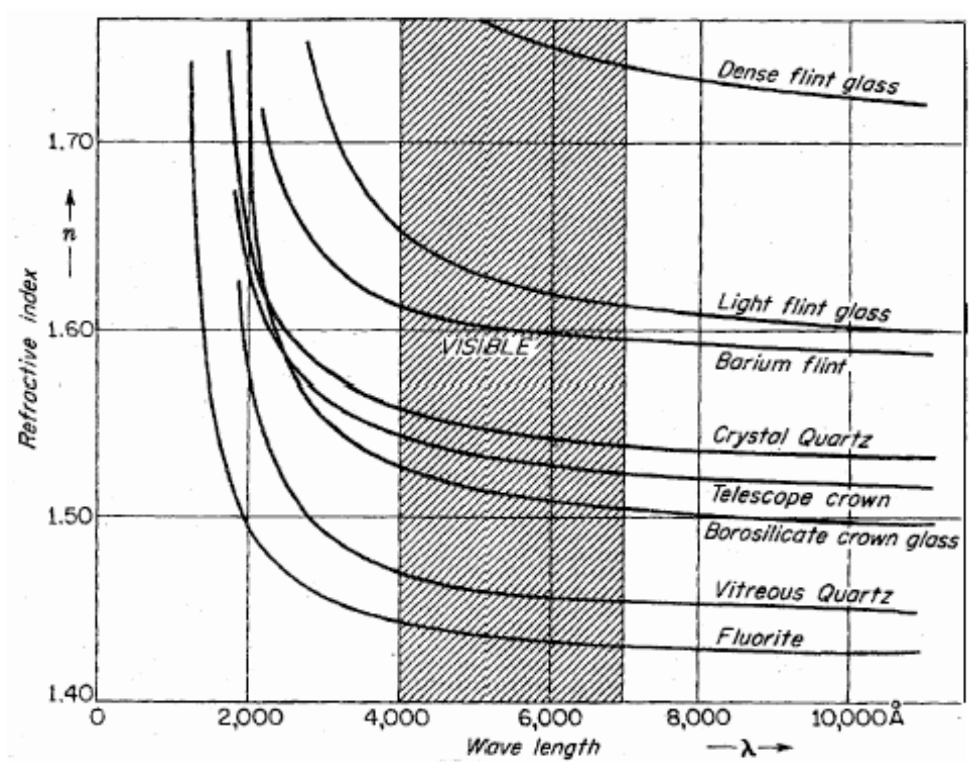


**Figure 1: Dispersion curves for some common optical glass types.**

Note that the index is generally higher at shorter wavelengths and so a prism refracts (bends) blue light more than red. It is also apparent that some materials disperse light more (flint) or less (crown) than others.

Cauchy modeled the dispersion of optical glass as a simple power series in wavelength ($\lambda$). Specifically, he used:

$$n(\lambda) = A + \frac{B}{\lambda^2} + \frac{C}{\lambda^4}$$

but our data are probably only sufficient to determine the first two terms. That is:

$$n(\lambda) = A + \frac{B}{\lambda^2}$$

Another measure of the dispersion that is much more useful in the design of optical instruments is the Abbe number. Abbe defined the dispersion of an optical glass in terms of the indices measured at certain wavelengths. Specifically as:

$$V_d = \frac{n_d - 1}{n_F - n_C}$$

where d, F, and C designate the Fraunhoffer lines seen in the spectrum of the Sun. The F and C lines are from Hydrogen and the d line is the Sodium doublet. However, a strong Helium line is found at nearly the same wavelength and, since Helium also has several other lines we could use to measure $n(\lambda)$, we will simply use the yellow Helium line for $n_d$ instead. Note that the Abbe number is a measure of the dispersion ($dn/d\lambda$) from blue to red scaled by the index of refraction in the yellow. We will discuss the Abbe number in more detail when we discuss the design of achromatic lenses.

**Minimum Deviation Angle for a Prism.** At a given (fixed) wavelength a prism will refract light through a range of angles depending on the angle of the light incident on the prism. In fact there is an angle of minimum deviation ($\delta_m$) as a prism is rotated. This condition, derived in lecture, is satisfied in the symmetric case shown below. Specifically, when $\theta_1 = \theta_2$, and $\theta_2 = \theta_1$ and thus $\theta_2 = \alpha/2$ and $\theta_1 = \theta_2 + \delta_m/2 = (\alpha + \delta_m)/2$.

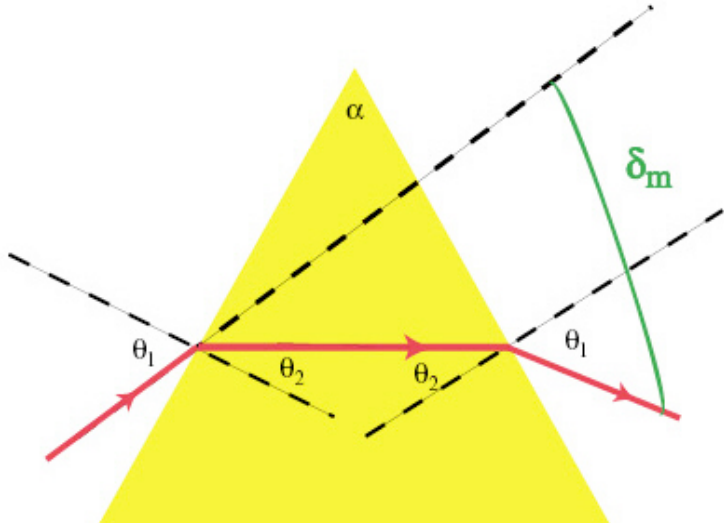


**Figure 2: Geometry for an equilateral prism orientated for minimum deviation.**

When combined with Snell's law we can derive a relationship between the index of refraction of the prism and the angle of minimum deviation (see the lecture). Namely:

$$n(\lambda) = \frac{\sin\theta_1}{\sin\theta_2} = \frac{\sin\left[\frac{1}{2}[\alpha + \delta_m(\lambda)]\right]}{\sin\left(\frac{\alpha}{2}\right)}$$

**So, this relationship can be used to compute the index of refraction at any wavelength given the angle of minimum deviation measured for that wavelength.**

(1) **Procedure**. Begin by reviewing the equipment on the laboratory optical bench. Take a picture with your smart phone or draw a quick sketch of the apparatus and label each component or make notes on the picture on the following page.

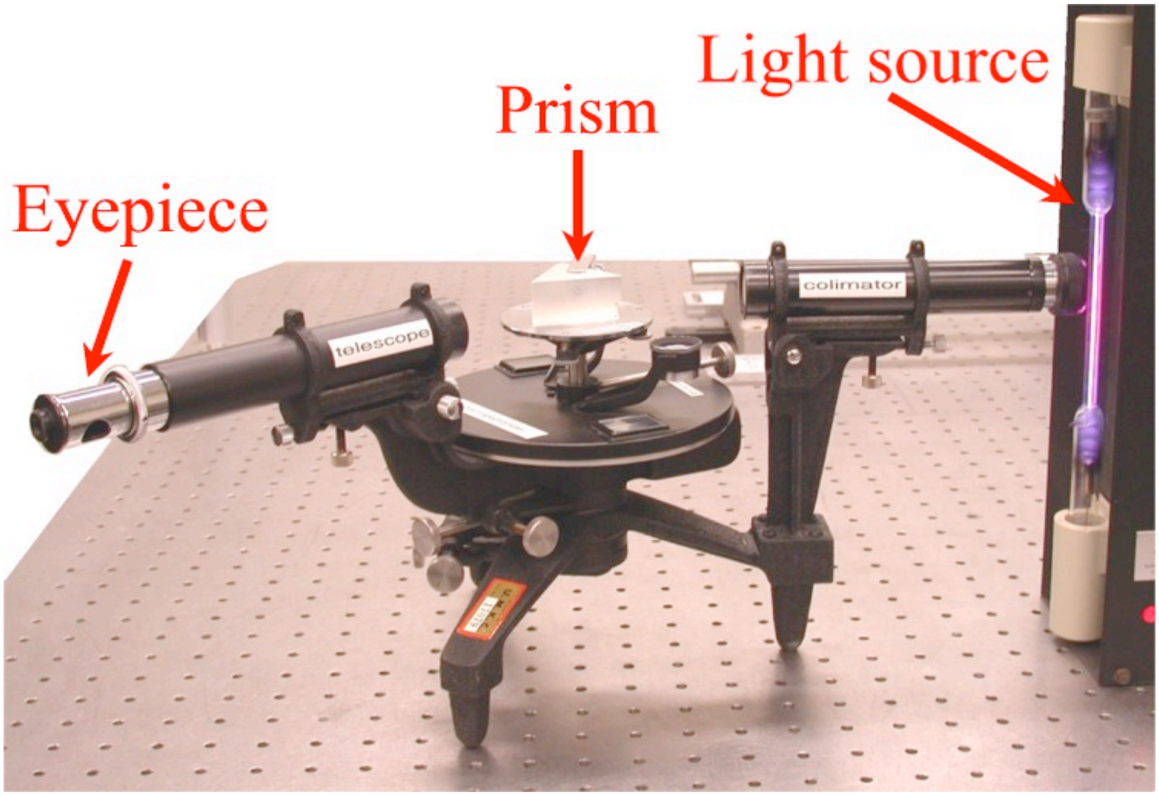


**Figure 3: Components of the prism spectrograph.**

a) The light source is a high voltage tube that ionizes a gas (Hydrogen) by stripping it of its electron. Upon recombination the electron cascades to its lowest level releasing photons corresponding to each of its allowed energy. This results in a series of discrete spectral lines or colors.
b) The collimator consists of a slit at the focus of a lens that collimates the light like a flashlight beam so that all light rays will enter the prism at the same angle.
c) The prism and rotational stage. The prism is centered on a silver rotational stage so that careful rotation of the stage will rotate the prism and change the angle of incidence. The dispersed light from the prism emerges at different angles according to its wavelength and this is the deviation angle.
d) The telescope and eyepiece. The telescope focuses the dispersed light onto a crosshair and the eyepiece acts as a magnifying glass to enlarge the image and crosshair. The telescope can be moved to center the crosshair on any particular spectral line.

**(3) Verifying the Spectroscopic Alignment**:

In order to make measurements we will first need to set the protractor angle at a sensible for making measurements. Begin by:

a) Removing the prism and block from the smaller, silver rotational stage.
b) Loosening the two clamps on the lower part of the device, below the larger black rotational stage. This allows both the large rotational stage and the telescope to move.
c) Turn on the spectral lamp and move the telescope back and forth until you can see and image of the slit through which the spectral lamp is shining. It should look like a vertical line. If it is out of focus carefully rotate the eyepiece of the telescope back and forth while pushing it in and out until the slit image is in focus. When the line is close to the crosshair clamp the telescope by tightening the upper clamp. Use the fine adjustment knob to center the crosshair on the slit image.
d) Now rotate the large black stage until the inner vernier scales read 0-deg. and 180-deg. Use the little magnifiers to verify this. Clamp the lowest clamp to set the stage in place. Don't unclamp or move this the rest of the lab.
e) Now rotate the telescope until the outer scales read about 25-deg and 205-deg. The outer two measurement scales are offset by about 180-deg so when you take measurements be sure and subtract 180-deg. from the one that is now reading about 205-deg. Don't forget to do this. Clamp the telescope in place. We will use the fine adjustment screws from now on.
f) Finally have your TA put the prism in place on the silver stage. This is a little tricky so it's best that they do it. Verify that you can see the spectral emission lines through the telescope. Be sure that the uppermost silver stage is unclamped since you will need to be able to rotate it.

**(4) Measurement of the Angle of Minimum Deviation:**

Before you begin any measurements log into a computer and open an Excel spreadsheet. Alternatively, make a table by hand. Make a column for wavelengths and another three for two angle measurements and their average. Be sure that you are subtracting 180-deg. from the one side that reads about 205-deg or have Excel do it for you. Don't forget to do this. For each spectrum tube you use refer to the Appendix for a picture of its spectrum and a table of its spectral line wavelengths. Enter these wavelengths into the appropriate column of your data table.

You should see the emission line spectrum of hydrogen (see Appendix). When you look through the telescope you should be able to see the spectral lines. Now we are ready to start taking measurements.

a) Slowly rotate the silver stage back and forth and you will notice that there is a position where as the prism table is rotated in one direction a spectral line will actually move in one direction, stop and begin moving in the opposite direction. The angle at which this happens is called the angle of minimum deviation. Assuming you are using the Hydrogen tube watch the red line until it just stops moving right before it reverses direction. Stop rotating the silver stage.

b) Loosen the telescope clamp and look through the telescope as you move it until the crosshair is aligned with the spectral line. Clamp it an move the fine adjust knob if you like.

c) Slowly rotate the prism back and forth to be sure that it is set and the angle of minimum deviation. Adjust the telescope with the fine adjust if necessary. When you are satisfied record both vernier readings (see Appendix I). Don't forget to subtract 180-deg from the largest reading. Repeat the measurement by have each person in your group do this two times, averaging the results.

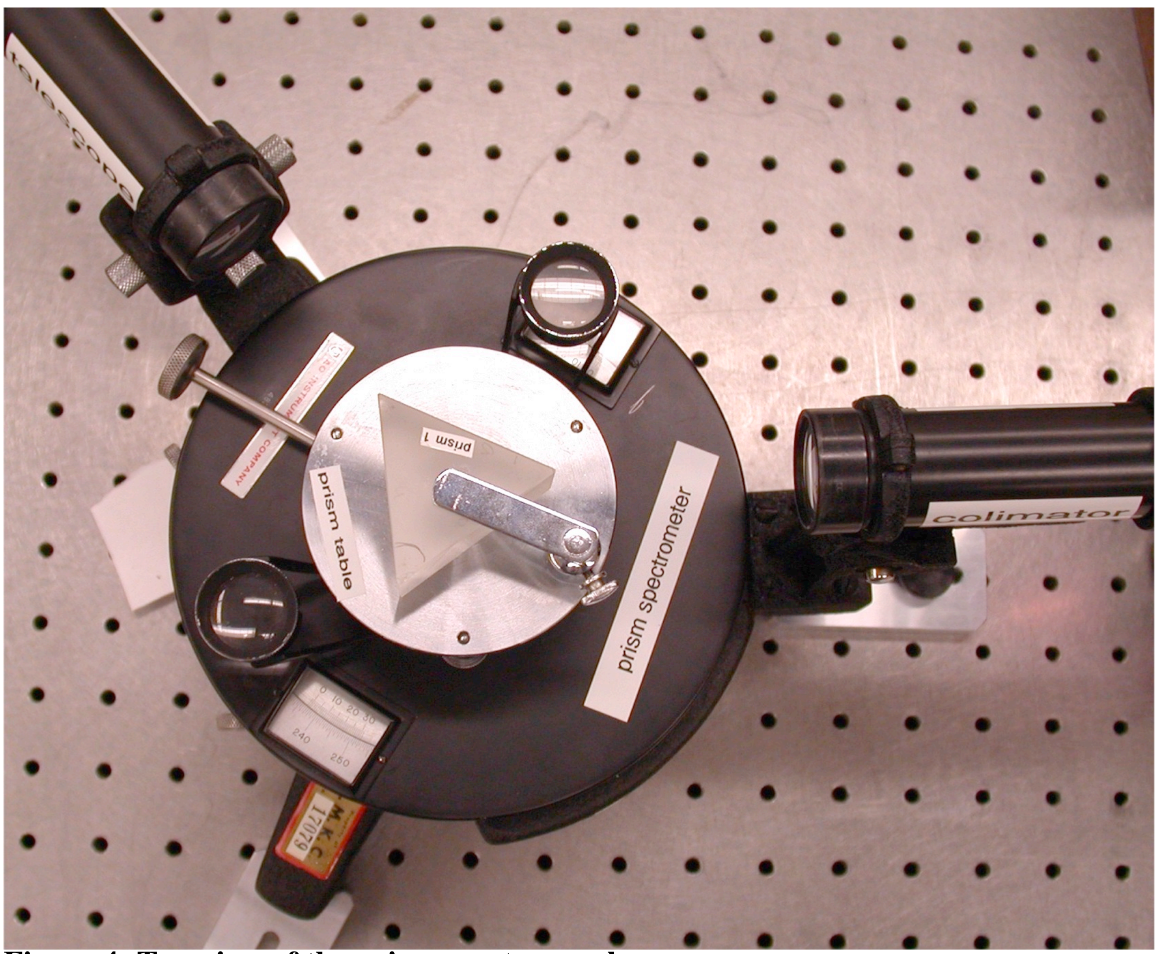


**Figure 4: Top view of the prism spectrograph.**

d) Repeat the above measurements for all of the lines. **Turn off the lamp power supply and wait for the hydrogen source to cool.** Replace the hydrogen source with the helium source, turn it on and repeat the measurements for all of the lines that you can see.

**(5) Data Analysis.**

1) Tabulate λ, and n for all of the lines. Use Excel to insert a column after your wavelengths and compute $1/\lambda^2$ . Also be sure to add a column to the right in which you compute the average of your vernier readings. Now add another column to the right in which you compute the angle of minimum deviation (see previous section) and another in which you solve the equation on page 3 for n. *Be sure to convert your angles to radians or Excel will do something unexpected!*

2) Make a plot of n vs. λ. Be sure to label your axes.

3) Using the first two terms of Cauchy's equation, determine A and B from your data. Plot n vs $1/\lambda^2$ and fit the best-fit trend line on the same graph. That is, the best fit to the equation: $n(\lambda) = A + B/\lambda^2$. Note that the equation will be linear and thus the values of A and B are easy to determine. Comment on your fit.

4) Using equation from part 3) and your value of B, determine $dn/d\lambda$ for $\lambda$=400, 500 and 600 nm.

5) The Abbe number is very useful for characterizing the dispersion of optical glass. What are the Abbe numbers for the prism glass?

**Before you leave the lab make sure that you have your lab partner's email so that the data can be shared within your lab group.**

**Summary**: In this lab we've measured the minimum deviation angle for a prism at a variety of wavelengths. The data have been plotted and analyzed using Excel in order to fit these data to the Cauchy model. We have also used these data to calculate the Abbe number for the prism glass. We will see later in lecture how the Abbe number is used in the design of Achromatic (color corrected) lenses.

**Appendix I: Reading verniers**

Verniers are read using both the lower and upper scale, the lower scale being in degrees and the upper scale in minutes. Find the largest number on the lower scale that is to the left of the "0" of the upper scale. For the photograph below, this is 184.5 degrees. Then find where the lines on the upper scale overlap those of the lower scale and read the number off of the upper scale. For the photograph, this is 10 minutes. Thus, this vernier reads: 184°.5 and 10'

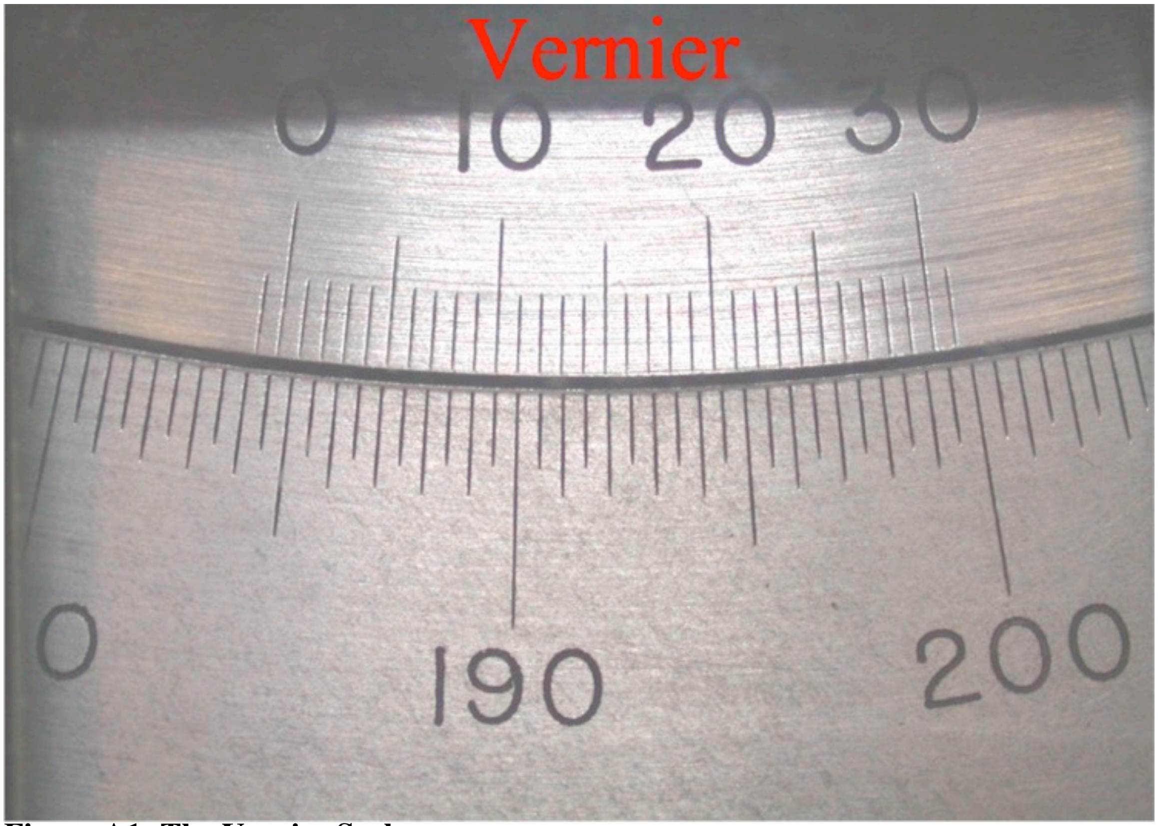

**Figure A1: The Vernier Scale.**

The prism spectrometer has 2 verniers. There would be a 180° difference in their readings if the system worked perfectly. This is not the case, so we will record the readings from both verniers, and in the end average our results using Excel.

**Appendix II: Emission Spectral Lines**

The hydrogen lines are at:

| Wavelength (nm) | Name | Color | Relative Intensity |
|---|---|---|---|
| 656.282 | Fraunhofer C | Red | 300 |
| 486.1327 | Fraunhofer F | Blue Green | 100 |
| 434.046 | | Violet | 30 |
| 410.174 | | Violet | 15 |

The helium lines are at:

| Wavelength (nm) | Name | Color | Relative Intensity |
|---|---|---|---|
| 706.519 | | Red | 180 |
| 667.815 | | Red | 200 |
| 587.5618 | | Yellow | 870 |
| 501.568 | | Lt. Green | 100 |
| 492.193 | | Dk. Green | 20 |
| 471.314 | | Blue Green | 34 |
| 447.148 | | Blue Violet | 225 |

The Hg lines are at:

| Wavelength (nm) | Name | Color |
|---|---|---|
| 365.4 | I-line | ultraviolet (UVA) |
| 404.7 | H-line | violet |
| 435.8 | G-line | blue |
| 546.1 | | green |
| 578.2 | | yellow-orange |

The Ne lines are at:

| Wavelength (nm) | Color |
|---|---|
| 585.2 | yellow |
| 594.5 | yellow-orange |
| 609.6 | orange |
| 616.6 | orange |
| 633.4 | orange |
| 640.2 | red |
| 650.6 | red |
| 659.9 | very red |
| 667.8 | very red |

The relative intensities are approximations. You may use the line spectra below to help you identify the wavelength of each lines. Please note that for the helium spectrum, the reddest line is missing from the following figure.

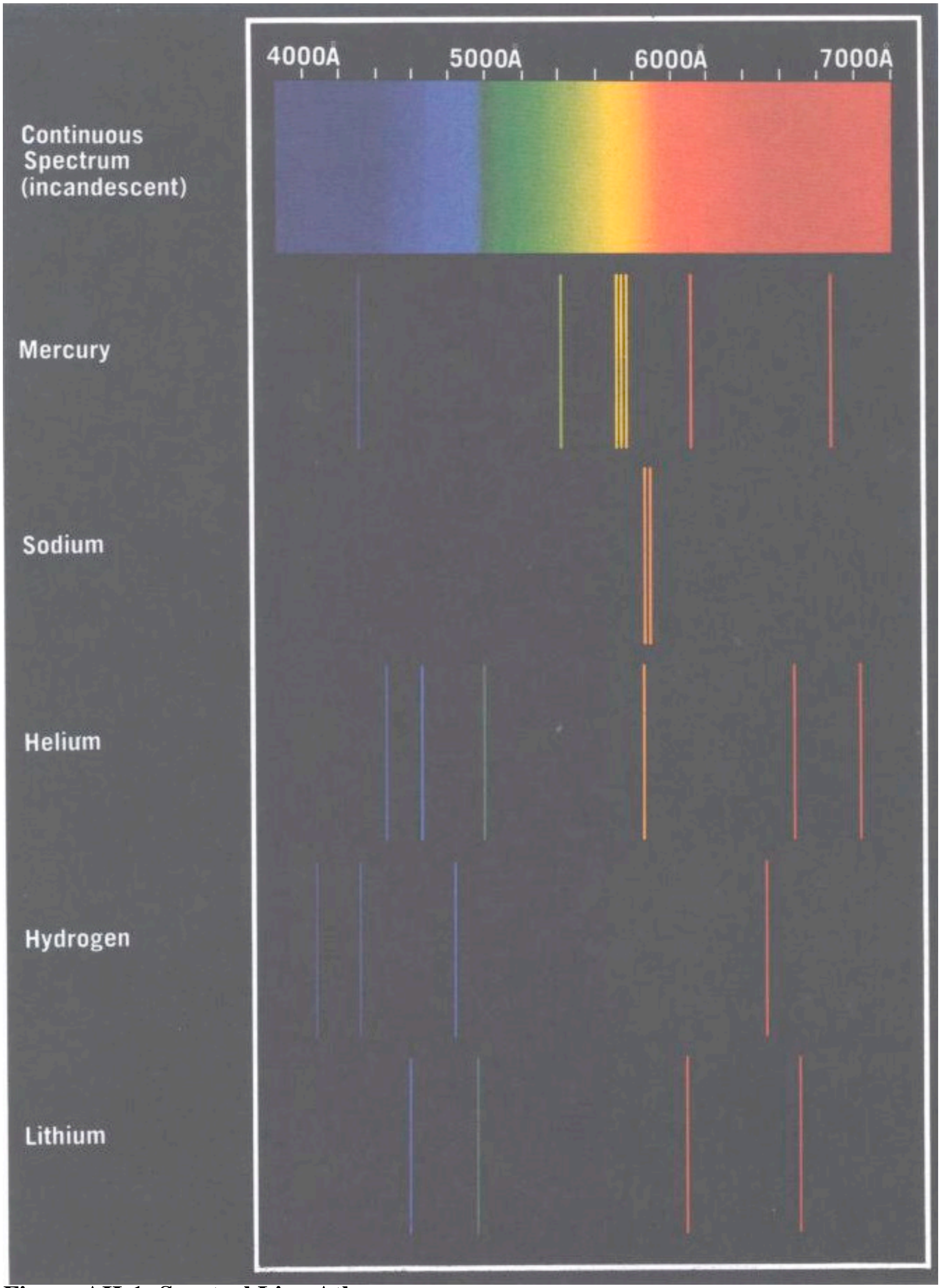


**Figure AII-1: Spectral Line Atlas.**