

Physics 2310: Waves & Optics

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- **Office Hours**

- **Tues:** 1:30 – 3:30 PM

- **Also by Appointment**

- **Lecture**

- **MWF 2:10 – 3:00 pm Stem 145**

- **Lab**

- **Combination of Lab & Discussion**

- **Wednesday 3:10pm-5:00 pm Stem 145**

Phys 2310 Wed. Aug. 30, 2016

Today's Topics

- **Class Overview**
 - How to do well in this class
 - Syllabus
 - Schedule
- **Introductions**
 - Who are you?
 - Who am I?
- **Course Prologue**
 - **Importance of Waves and Optics in Science**
 - Mechanical Oscillations,
 - Sound and Light Waves
 - Complex Phenomena

Today's Topics Continued

- **Review of Mechanical Waves**
 - **Simple Harmonic Motion**
 - **Waves in 1-d**
 - **Definitions**
- **Preview of Lab Next Week**
- **Homework**
- **Math Review Quiz Friday**
- **Reading for Next Time**

How to Do Well in this Class

- **Come to class**
- **Review your math! (partial derivatives, matrices)**
- **Read textbook chapters before class**
- **Print-out and read on-line notes (add to them during lecture)**
- **Ask questions in class**
- **Google topics/concepts as needed**
- **Turn in homework on time**
- **Prepare for tests**

Highlights of the Syllabus

- **Course Content: an Introduction to Waves & Optics**
 - Prerequisites: Calculus-I & II and Physics-I & II
- **Lectures: Reading done in advance, notes on the web**
 - One chapter covered each week!
 - We're using multiple textbooks so be sure to keep up.
- **Homework: Typically Assigned/Due on Wednesdays**
- **Exams: 2 mid-term exams + final**
- **Grading: Exams – 50%, Homework - 25%, Labs – 25%**

Who Are You?

- **Introductions**
 - **Majors**
 - **Math Background**
- **Please fill out the questions**
 - **I can make adjustments depending on math background.**
 - **Why are you taking the class?**
 - **What do you want to learn from it?**

Who am I?

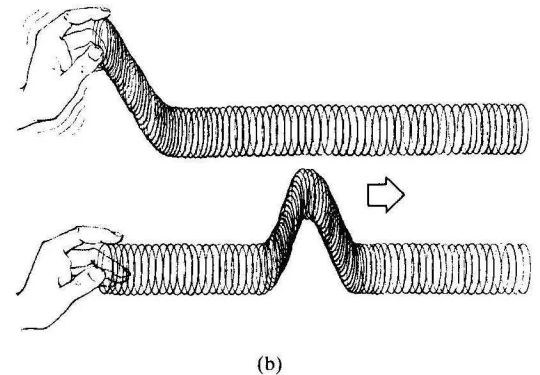
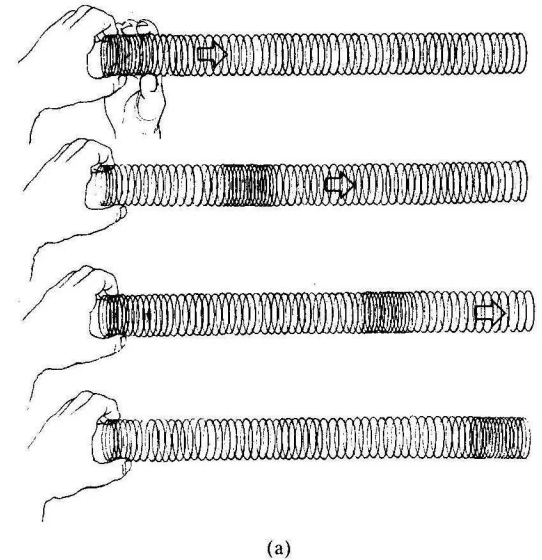
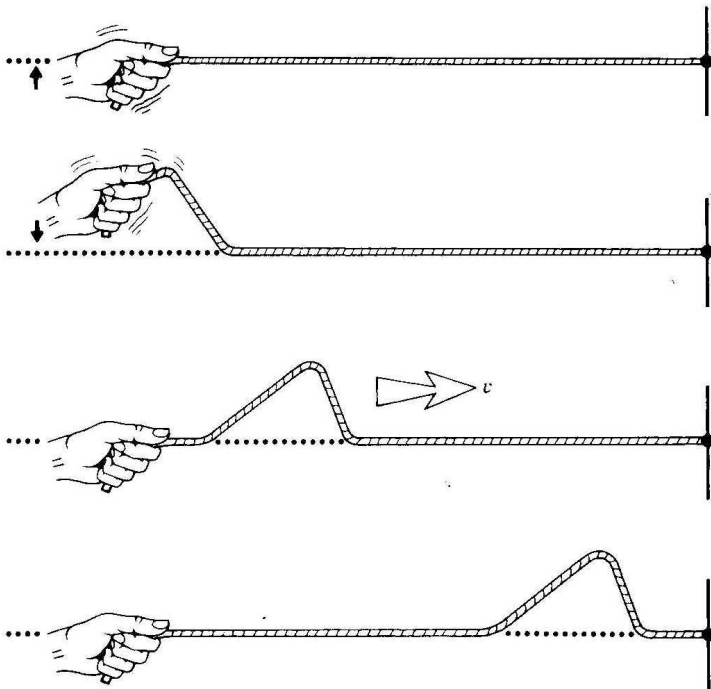
- **Background**
 - **Ph.D University of Hawaii**
 - **Measured Expansion of the Universe**
 - **Inferred Existence of Dark Energy**
 - **Plaskett Fellow, Herzberg Institute for Astrophysics (Victoria, BC)**
 - **Research Fellow, Kitt Peak National Obs. (Tucson, AZ)**
 - **Assistant Professor, Indiana University**
 - **Associate Professor, University of Wyoming**
- **Research Interests**
 - **Evolution of Galaxies**
 - **Cosmology**
 - **Astronomical Instrumentation**
 - **Come by and talk if you want to hear more or see some instruments**

Waves in Physics & Engineering

- **Importance of Waves & Optics in Physics and Engineering**
 - **Oscillatory phenomena are common in nature**
 - Mechanical vibration, pendulum, water sloshing, floating objects, vibrating string, sound, and light
 - (see SZ Chapter 14 for examples)
 - **Optics & Waves phenomena provide a transition between classical and modern physics**
 - **Optical instruments play a central role in measurement of physical systems (microscopes, telescopes, radar, spectroscopy)**
 - **Optical devices (displays, touch screens, sensors)**
 - **Optical phenomena can probe extreme environments (Remote Sensing)**
 - Regions of high and low temperatures and pressures
 - Quantum realm: atomic physics, solid state, quantum dots, etc.

Chapter 15 (SZ): Review of Mechanical Waves

- Longitudinal vs. Transverse Waves
- Propagation of a Disturbance
 - Wave is a profile (function) that propagates through medium (displaced)
 - Wave propagation
 - Longitudinal (sound) vs. Transverse (light)
 - Disturbance advances not medium



Chapter 15 (SZ): Review of Mechanical Waves

- We will use a mathematical function to describe wave shape.

$\psi(x,t) = f(x,t)$ where $f(x,t)$ is some specific function (wave shape)

If shape is constant then the profile "translates" with time:

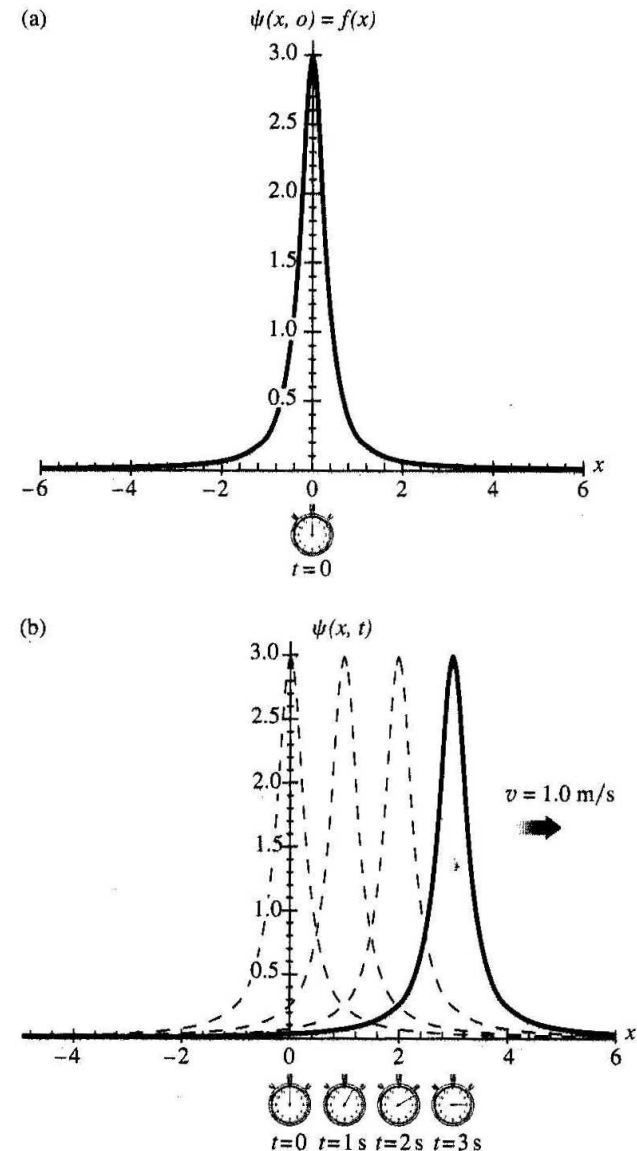
$x' = x - vt$ (equivalent to backward shift of coordinates) and so actually:

$$\psi(x,t) = f(x - vt)$$

Where $\psi(x,t)$ is called the "wavefunction". Example:

$$\psi(x,t) = e^{-a(x-vt)^2} \text{ see figure at right}$$

- Math. form must explicitly contain time dependence (motion at constant v) as well.
- Note that each term in () has the same units and thus similar.
- If $t = 0$ the wavefunction doesn't move
- If $x = \text{constant}$ then the function's motion is equivalent to shift of coordinates in $-x$ direction.



Chapter 15 (SZ): Review of Mechanical Waves

- **Importance of Partial Derivatives**
 - **Partials describe how a function changes when one quantity is held fixed.**
 - **Let's derive the wave equation**

$$\frac{\partial \psi}{\partial x} = \frac{\partial f}{\partial x} \quad (\text{at constant } t)$$

$$\frac{\partial \psi}{\partial x} = \frac{\partial f}{\partial x'} \frac{\partial x'}{\partial x} = \frac{\partial f}{\partial x'} \quad (\text{in terms of } x', \text{ since } \frac{\partial x'}{\partial x} = 1)$$

$$\frac{\partial \psi}{\partial t} = \frac{\partial f}{\partial x'} \frac{\partial x'}{\partial t} = \frac{\partial f}{\partial x'} (\mp v) = \mp v \frac{\partial f}{\partial x'}$$

combining yields :

$$\frac{\partial \psi}{\partial t} = \mp v \frac{\partial \psi}{\partial x}$$

The second derivatives are thus :

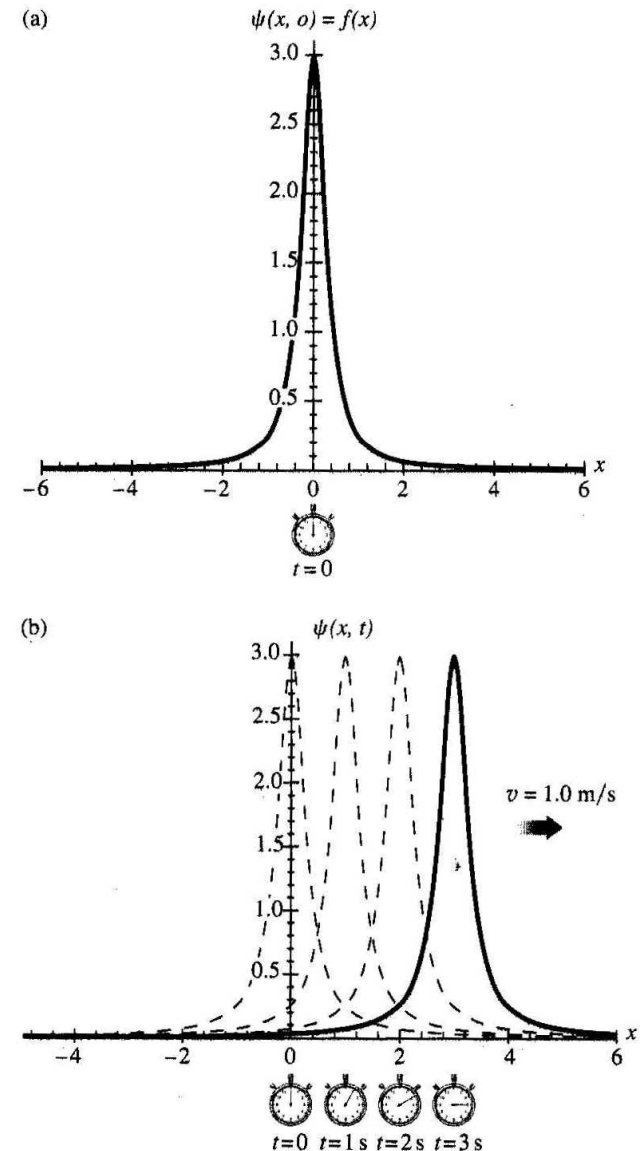
$$\frac{\partial^2 \psi}{\partial x^2} = \frac{\partial^2 f}{\partial x'^2} \quad \text{and}$$

$$\frac{\partial^2 \psi}{\partial t^2} = \frac{\partial}{\partial t} \left(\mp v \frac{\partial f}{\partial x'} \right) = \mp v \left(\frac{\partial f}{\partial t} \right) \quad \text{since } \frac{\partial \psi}{\partial t} = \frac{\partial f}{\partial t}$$

Thus :

$$\frac{\partial^2 \psi}{\partial t^2} = v^2 \frac{\partial^2 f}{\partial x'^2} \quad \text{and so we have :}$$

$$\frac{\partial^2 \psi}{\partial x^2} = \frac{1}{v^2} \frac{\partial^2 \psi}{\partial t^2} \quad (\text{1-d differential wave equation})$$



Chapter 15 (SZ): Review of Mechanical Waves

- Consider a string under tension:

The net force in the y direction is:

$$\sum F_y = T \sin \theta_B - T \sin \theta_A = T(\sin \theta_B - \sin \theta_A)$$

But if we approximate $\sin \theta \approx \tan \theta$ then:

$$\sum F_y = T(\tan \theta_B - \tan \theta_A)$$

But if the displacement is infinitesimal:

$\tan \theta = \frac{\partial y}{\partial x}$ and so we have:

$$\sum F_y \approx T \left[\left(\frac{\partial y}{\partial x} \right)_B - \left(\frac{\partial y}{\partial x} \right)_A \right]$$

If the mass/unit length is μ then $m = \mu \Delta x$ and so:

$$\sum F_y = m a_y = \mu \Delta x \frac{\partial^2 y}{\partial t^2} \text{ and combining gives:}$$

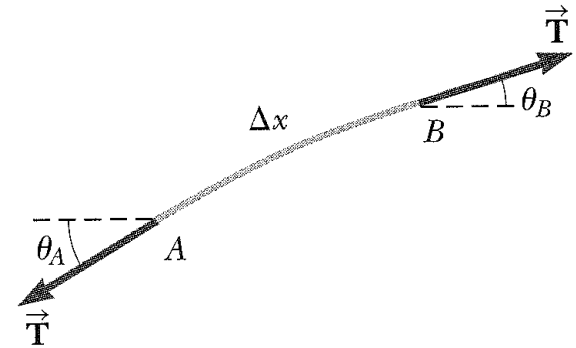
$$\mu \Delta x \frac{\partial^2 y}{\partial t^2} = T \left[\left(\frac{\partial y}{\partial x} \right)_B - \left(\frac{\partial y}{\partial x} \right)_A \right] \text{ or:}$$

$$\frac{\mu}{T} \frac{\partial^2 y}{\partial t^2} = \frac{\left[\left(\frac{\partial y}{\partial x} \right)_B - \left(\frac{\partial y}{\partial x} \right)_A \right]}{\Delta x}$$

But the right side is just the definition of a partial derivative derivative in the limit as $\Delta x \rightarrow 0$. So:

$$\frac{\mu}{T} \frac{\partial^2 y}{\partial t^2} = \frac{\partial^2 y}{\partial x^2} \text{ but the velocity } v = \sqrt{\frac{T}{\mu}} \text{ and so we have:}$$

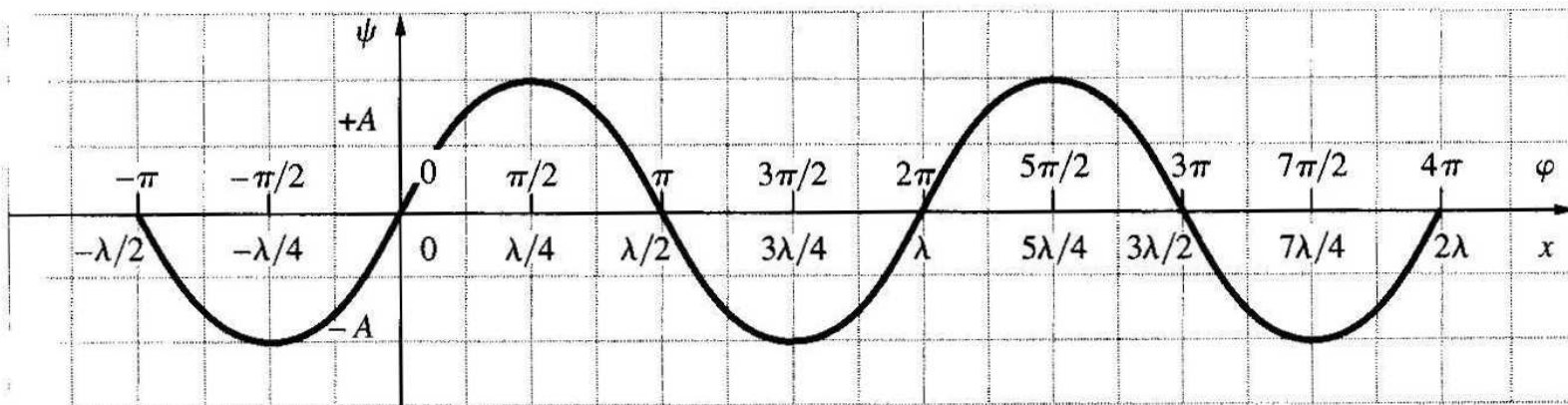
$$\frac{\partial^2 y}{\partial x^2} = \frac{1}{v^2} \frac{\partial^2 y}{\partial t^2}$$



Chapter 15 (SZ): Review of Mechanical Waves

- We need a waveform (function) whose second derivative is the same function multiplied by a constant (sine and cosine).
- Harmonic motion and waves (good model with simple mathematical form)
 - Amplitude: max strength of disturbance (pressure, E-field, etc.)
 - Wavelength (spatial period of wave: λ , distance between pattern repeating)
 - Frequency (number of waves passing fixed point per unit time: $\nu = v/\lambda$)
 - Period (temporal period τ , time for one wavelength to pass fixed point)
 - Propagation Number ($k = 2\pi/\lambda$: relates x to fractions of wavelength)
 - Wave Number (spatial frequency $\kappa = 1/\lambda$: number of waves per unit length)

$$\psi(x) = A \sin kx = A \sin 2\pi x/\lambda = A \sin \varphi$$



Chapter 15 (SZ): Review of Mechanical Waves

- Harmonic (sinusoidal) motion:**

Consider the projection of a rotating vector (**A**) onto the x-axis.

Result is:

$\Psi = A \sin k(x - vt)$ (note the units of k are $1/x$ since kx and kvt are then unit-less) or:

$$\Psi = A \sin 2\pi/\lambda(x - vt)$$

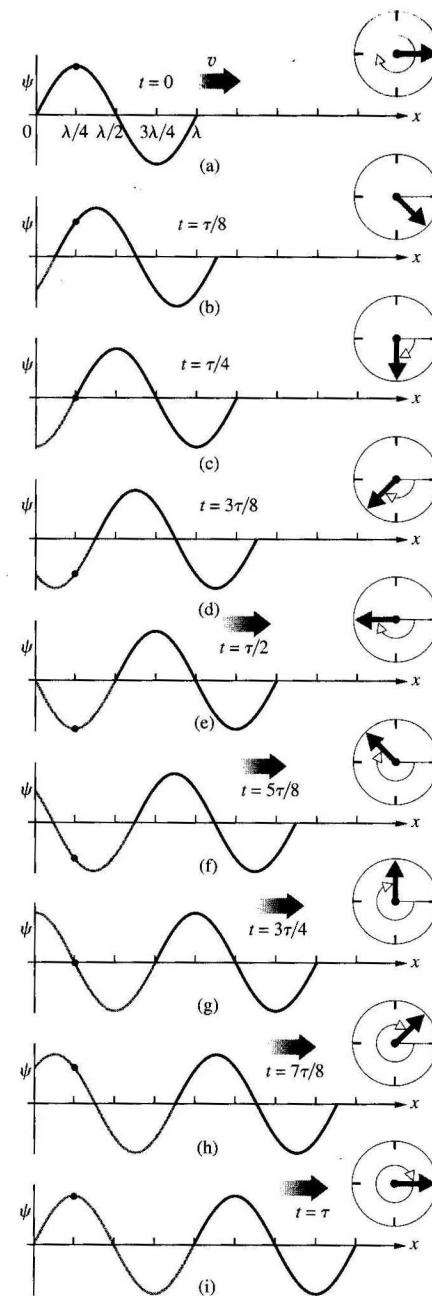
$$\Psi = A \sin (kx - \omega t) \text{ (where } \omega \text{ is angular velocity)}$$

Note that strictly speaking we consider these waves to go on forever, there is no limit to their extent in space or time.

Consider the phase:

$$\phi = (kx - \omega t)$$

It describes the fraction of the period at a given point in time so the initial phase need not be zero.



Chapter 15 (SZ): Review of Mechanical Waves

- **Change in Phase vs. Time**
 - **Phase:** $\phi = (kx - \omega t)$
 - **But ϕ might not be 0 at x and $t = 0$.**
 - **So introduce an initial phase (ϵ):**
 $\phi = (kx - \omega t + \epsilon)$

- **Phase Velocity:**

$$\left| \left(\frac{\partial \phi}{\partial t} \right)_x \right| = \omega \text{ and similarly:}$$

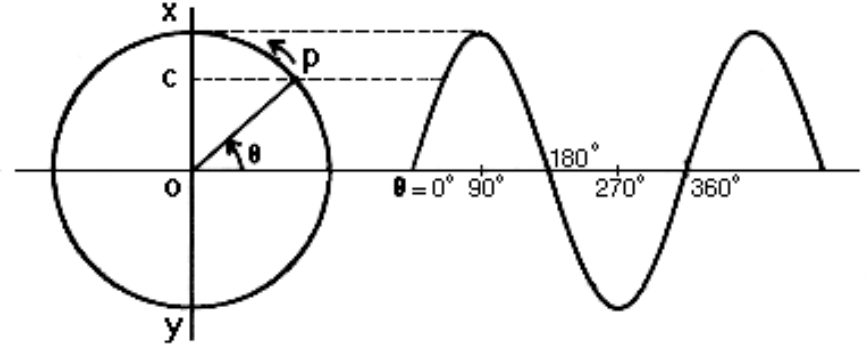
$$\left| \left(\frac{\partial \phi}{\partial x} \right)_t \right| = k \text{ and so since:}$$

$$\left(\frac{\partial x}{\partial t} \right)_\phi = \frac{- \left(\frac{\partial \phi}{\partial t} \right)_x}{\left(\frac{\partial \phi}{\partial x} \right)_t} \text{ then:}$$

$$\left(\frac{\partial x}{\partial t} \right)_\phi = \pm \frac{\omega}{k} = \pm v \text{ (phase velocity)}$$

Chapter 14 (SZ) Review: Mechanical Waves

- **Circular Motion & Simple Harmonic Motion**
 - **Amplitude, Period & Frequency**
 - **Mathematical Description**
 - **Energy in SHM**



Consider a mass hanging on a spring:

$$F = m \frac{d^2 y}{dt^2} = -ky + mg - T \quad \text{and so we can write:}$$

$$F_{net} = m \frac{d^2 y}{dt^2} = -ky \quad \text{or} \quad \frac{d^2 y}{dt^2} = -\left(\frac{k}{m}\right)y$$

The periodic motion of a rotating vector implies:

$$f(y) = C_1 \cos(\omega t) + C_2 \sin(\omega t) = A \cos(\omega t - \phi)$$

where $\omega = \left(\frac{k}{m}\right)^{1/2} = 2\pi f$ (frequency) with the period:

$$P = 1/f = 2\pi \sqrt{\frac{m}{k}}, \quad \text{so } f(y) \text{ is a suitable functional form.}$$

The initial conditions determine C_1, C_2 and thus ϕ .

Consider the velocity:

$$V(y) = \frac{dy}{dt} = -A\omega \sin(\omega t - \phi) \quad \text{with maximum: } V_{max} = A\omega$$

Consider the acceleration:

$$a(y) = \frac{d^2 y}{dt^2} = -A\omega^2 \cos(\omega t - \phi) \quad \text{with maximum: } a_{max} = A\omega^2$$

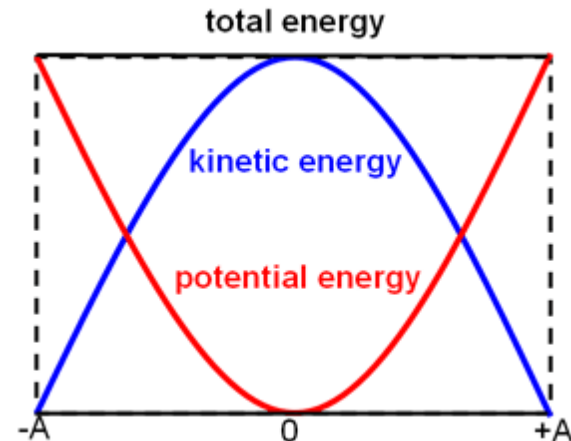
Now consider the kinetic energy:

$$K(t) = \frac{1}{2} m v^2(t) = \frac{1}{2} m \omega^2 A^2 \sin^2(\omega t - \phi) = \frac{1}{2} k A^2 \sin^2(\omega t - \phi)$$

with the potential energy of the system being:

$$U(t) = \frac{1}{2} k x^2(t) = \frac{1}{2} k A^2 \cos^2(\omega t - \phi) \quad \text{so the total energy is:}$$

$E = K + U = \frac{1}{2} k A^2$ (Note: K, U are 90° out of phase and are, along with E , always positive. Why?)



Supplement: Complex Representation

- **Harmonic waves can also be described by complex numbers.**
 - **Main motivation for this is that the math is easier.**
 - **Consider the Complex Plane (real numbers along x, imaginary numbers along y)**

$$\tilde{z} = x + iy \text{ where } i = \sqrt{-1}$$

x and y are the real and imaginary parts of \tilde{z}

amplitude (r) : $r = \sqrt{x^2 + y^2}$ and phase(θ)

Recall that x and y are the projections in polar coords. :

$x = r \cos \theta$ and $y = r \sin \theta$ and so :

$\tilde{z} = r(\cos \theta + i \sin \theta)$ with :

amplitude (r) : $r = \sqrt{x^2 + y^2}$ and phase(θ)

But since for $r = 1$:

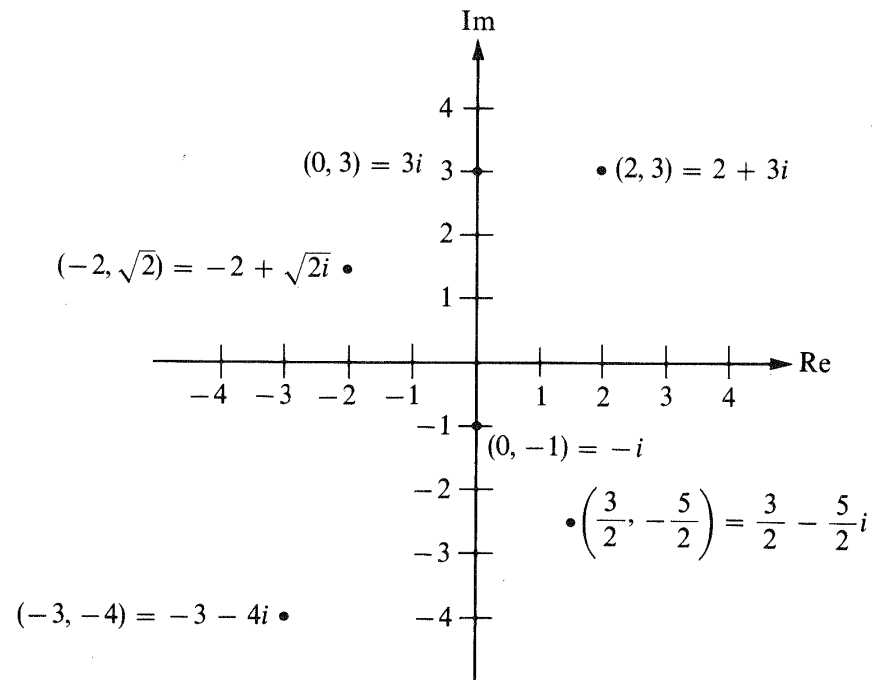
$d\tilde{z} = i\tilde{z}d\theta$ we can integrate this and see :

$\tilde{z} = e^{i\theta}$ and so :

$e^{i\theta} = \cos \theta + i \sin \theta$ and $e^{-i\theta} = \cos \theta - i \sin \theta$ so :

$$\cos \theta = \frac{e^{i\theta} + e^{-i\theta}}{2} \text{ and } \sin \theta = \frac{e^{i\theta} - e^{-i\theta}}{2i} \text{ and :}$$

$\tilde{z} = re^{i\theta}$ (r is amplitude and θ is the phase)



SZ Ch. 14, French Ch. 4: Damped Oscillations

Consider a horizontal mass and spring system with a frictional damping force. Empirically it can be shown that a good model is:

$$R(v) = b_1 v + b_2 v^2 \quad (\text{1-st two terms in a Taylor series in velocity})$$

For small v the first term is usually sufficient. Thus we have:

$$F_x = -kx - bv_x \quad \text{and so the equation of motion becomes:}$$

$$m \frac{d^2 x}{dt^2} + b \frac{dx}{dt} + kx = 0 \quad \text{or} \quad \frac{d^2 x}{dt^2} + \gamma \frac{dx}{dt} + \omega_0^2 x = 0 \quad (1) \quad \text{where:}$$

$$\gamma = \frac{b}{m} \quad \text{and} \quad \omega_0^2 = \frac{k}{m}. \quad \text{Let's assume a solution of the form:}$$

$x(t) = Ae^{j(pt+\alpha)}$ where p has units of frequency and α represents the initial phase. Substituting into (1) results in:

$$(-p^2 + jp\gamma + \omega_0^2)Ae^{j(pt+\alpha)} = 0. \quad (2) \quad \text{To be satisfied for all } t:$$

$$-p^2 + jp\gamma + \omega_0^2 = 0. \quad \text{Note that } p \text{ must be imaginary (2nd term)}$$

so $p = n + js$, where n and s are real numbers. Then:

$$p^2 = n^2 + 2jns - s^2 \quad \text{and substituting into (2) we have:}$$

$$-n^2 - 2jns + s^2 + jn\gamma - s\gamma + \omega_0^2 = 0. \quad \text{Separating real and imaginary:}$$

$$\text{Real parts:} \quad -n^2 + s^2 - s\gamma + \omega_0^2 = 0$$

$$\text{Imaginary parts:} \quad -2ns + n\gamma = 0 \quad \text{and so} \quad s = \frac{\gamma}{2} \quad \text{and:}$$

$$n^2 = \omega_0^2 - \frac{\gamma^2}{4} \quad \text{and substituting into our proposed functional form:}$$

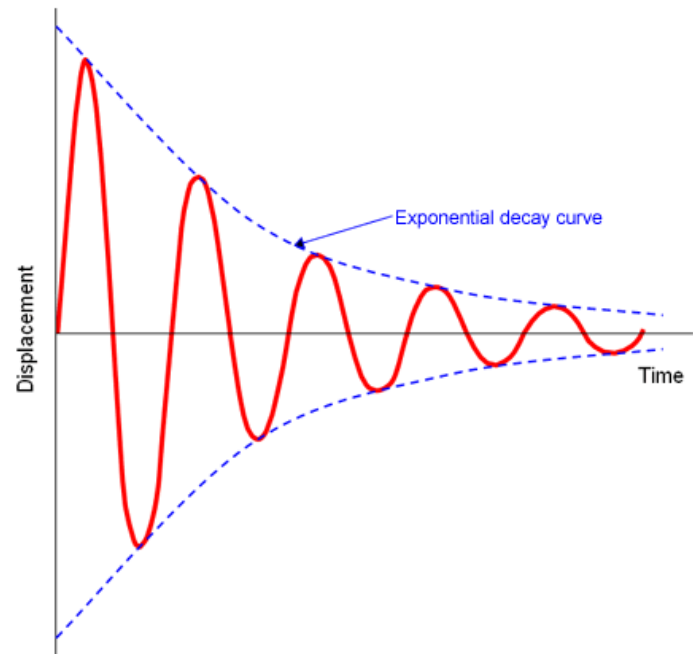
$$x(t) = Ae^{j(nt+js t+\alpha)} = Ae^{-st} e^{j(nt+\alpha)} = Ae^{-st} \cos(nt + \alpha)$$

or in terms of our original variables:

$$x(t) = Ae^{-\gamma/2 t} \cos(\omega t + \alpha) \quad \text{where:}$$

$$\omega^2 = \omega_0^2 - \frac{\gamma^2}{4} = \frac{k}{m} - \frac{b^2}{4m^2}$$

Note that the solution is comprised of SHM with an exponentially declining (damped) amplitude. Note further that the frequency is lower than the undamped case. Depending on the size of b we can describe system as being critically or overdamped (see text).



SZ Ch. 14 + French Ch. 4: Forced Oscillations

- Consider an Undamped Harmonic Oscillator with a Harmonic Driving Force

Summing the forces:

$$m \frac{d^2 x}{dt^2} = -kx + F_0 \cos \omega t$$

But recall that the natural frequency of the system is given by:

$$\omega_0 = \sqrt{\frac{k}{m}} \text{ so the system "wants" to oscillate at that frequency. The}$$

driving force will "want" to force the system to oscillate at a its frequency (ω). So that actual motion is some combination of those two frequencies (transient solution). In fact it will be a superposition of the two but friction will eventually result in a "steady state" oscillation at the driving frequency. To obtain let's try a solution of the form: $x = C \cos \omega t$ (note the driving frequency). Thus if we differentiate:

$$\frac{d^2 x}{dt^2} = -\omega^2 C \cos \omega t \text{ and so we have:}$$

$$-m\omega^2 C \cos \omega t + kC \cos \omega t = F_0 \cos \omega t \text{ and solving for the amplitude:}$$

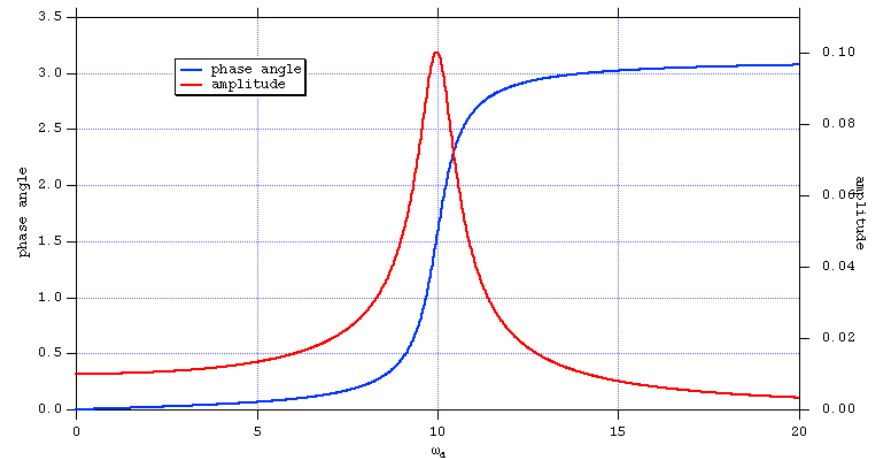
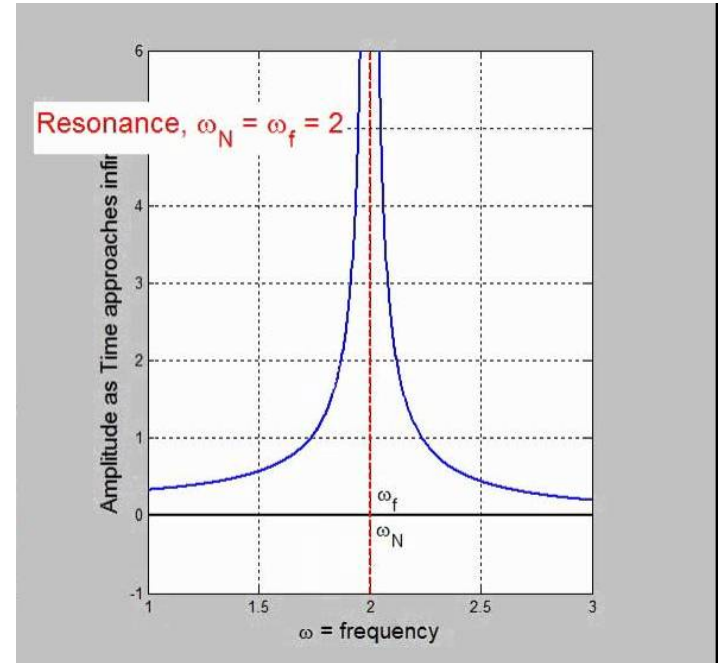
$$C = \frac{F_0}{k - m\omega^2} = \frac{F_0 / m}{\omega_0^2 - \omega^2} \text{ Note: as } \omega_0^2 - \omega^2 \rightarrow 0, C \rightarrow \infty! \text{ Now the phase:}$$

Let $x = A \cos(\omega t + \alpha)$ and we see that (try it via substitution!):

$$\omega < \omega_0 : \alpha = 0$$

$$\omega > \omega_0 : \alpha = \pi$$

With resistance the amplitude doesn't go to ∞ and the phase change not so abrupt.



French Ch. 4: Forced Oscillations with Damping

• Consider a Forced Harmonic Oscillator with Damping

Summing the forces as before (no damping) but using complex exponential:

$$m \frac{d^2 x}{dt^2} + kx = F_0 \cos \omega t \rightarrow F_0 e^{j\omega t} \text{ so let's try a solution like } x = A e^{j(\omega t + \alpha)}$$

Thus we have:

$$(-m\omega^2 A + kA)e^{j(\omega t + \alpha)} = F_0 e^{j\omega t} \text{ and we rewrite as:}$$

$$(\omega_0^2 - \omega^2)A = \frac{F_0}{m} e^{-j\alpha} = \frac{F_0}{m} \cos \alpha - j \frac{F_0}{m} \sin \alpha \text{ Separating real and imaginary:}$$

$$\omega_0^2 - \omega^2 A = \frac{F_0}{m} \cos \alpha \text{ and } 0 = -\frac{F_0}{m} \sin \alpha \text{ So now let's add damping:}$$

$$\frac{d^2 x}{dt^2} + \frac{b}{m} \frac{dx}{dt} + \frac{k}{m} x = \frac{F_0}{m} \cos \omega t = \frac{F_0}{m} e^{j\omega t} \text{ and letting } \frac{k}{m} = \omega_0^2 \text{ and } \frac{b}{m} = \gamma \text{ so:}$$

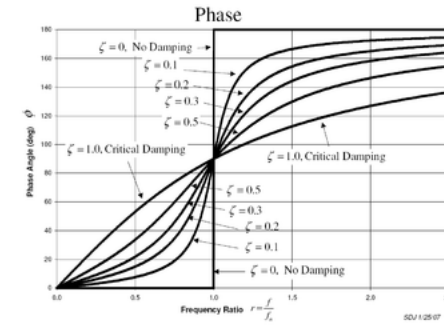
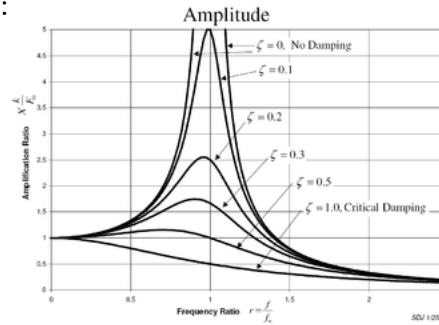
$$\frac{d^2 x}{dt^2} + \gamma \frac{dx}{dt} + \omega_0^2 x = \frac{F_0}{m} e^{j\omega t} \text{ Let's try: } x = A e^{j(\omega t - \delta)} \text{ and we get:}$$

$$(-\omega^2 A + j\gamma\omega A + \omega_0^2 A)e^{j(\omega t - \delta)} = \frac{F_0}{m} e^{j\omega t} = (\omega_0^2 - \omega^2)A + j\gamma\omega A \text{ (like a vector)}$$

The real and imaginary parts have to be equal separately and so:

$$(\omega_0^2 - \omega^2)A = \frac{F_0}{m} \cos \delta \text{ and } \gamma\omega A = \frac{F_0}{m} \sin \delta \text{ and so:}$$

$$A(\omega) = \frac{F_0 / m}{\left[(\omega_0^2 - \omega^2)^2 + (\gamma\omega)^2 \right]^{1/2}} \text{ and } \tan \delta = \frac{\gamma\omega}{\omega_0^2 - \omega^2} \text{ (see figure)}$$



To see what happens as the resistive term is varied let's define:

$Q = \frac{\omega_0}{\gamma}$ as a "quality factor". The less dissipative (γ) the higher the

amplitude. Substituting gives:

$$A(\omega) = \frac{F_0 / m}{\left[(\omega_0^2 - \omega^2)^2 + (\omega\omega_0 / Q)^2 \right]^{1/2}} \text{ with } \tan \delta(\omega) = \frac{\omega\omega_0 / Q}{\omega_0^2 - \omega^2}$$

Rewriting using ω / ω_0 (factoring out $\omega\omega_0$ from denominator) gives:

$$A(\omega) = \frac{F_0}{m\omega_0^2} \frac{\omega_0 / \omega}{\left[\left(\frac{\omega_0}{\omega} - \frac{\omega}{\omega_0} \right)^2 + \frac{1}{Q^2} \right]^{1/2}} = \frac{F_0}{k} \frac{\omega_0 / \omega}{\left[\left(\frac{\omega_0}{\omega} - \frac{\omega}{\omega_0} \right)^2 + \frac{1}{Q^2} \right]^{1/2}}$$

$$\omega_{\max} = \omega_0 \left(1 - \frac{1}{2Q^2} \right) \text{ and } A_{\max} = A_0 \frac{Q}{\left(1 - \frac{1}{4Q^2} \right)^{1/2}}$$

Summary and Key Concepts

- **Simple Harmonic Motion:**
 - Wavelength, Frequency, Speed, Energy
 - Damped Harmonic Motion
- **Mechanical Waves:**
 - Wave Functions, Definition & Propagation
 - Symmetry of Space & Time
 - Displacement, Velocity & Acceleration
 - Wave Equation

Math Quiz Next Week

We will have a math quiz on Wednesday!

Integration

Partial derivatives

Matrix multiplication

Review and Study using the handout and links on the web page.

Homework this Week (Due Fri. Sept. 8)

Review SZ Chapter 14 (SHM) & 15 (Mechanical Waves)

Review your math for the quiz on Wednesday

SZ Chapter 14: #4, 9, 11, 26

SZ Chapter 15: #6, 7, 12, 13, 17, 24

Reading this Week

By Wednesday:

Review SZ Chapter 14 & 15

Begin Math Review

- Calculus I
- Calculus II

By Friday:

Finish Math Review

- Calculus I
- Calculus II

For Good Math Reviews:

<http://www.sosmath.com>

<https://www.khanacademy.org/>