Phys 2310 Wed. Sept. 13, 2017
Today's Topics

- Supplemental: Oscillations in 2-d
  - 2-d Oscillations with the Same Frequency
  - 2-d Oscillations with Different Frequencies
- Supplemental: Forced Oscillations
  - Harmonic Forcing with No Damping
    - Resonance
  - Harmonic Forcing with Damping
  - Transient Phenomena
- Reading for Next Time

Homework this Week

SZ Chapter 15: #22, 32, 35, 39, 40, 42, 46, 47

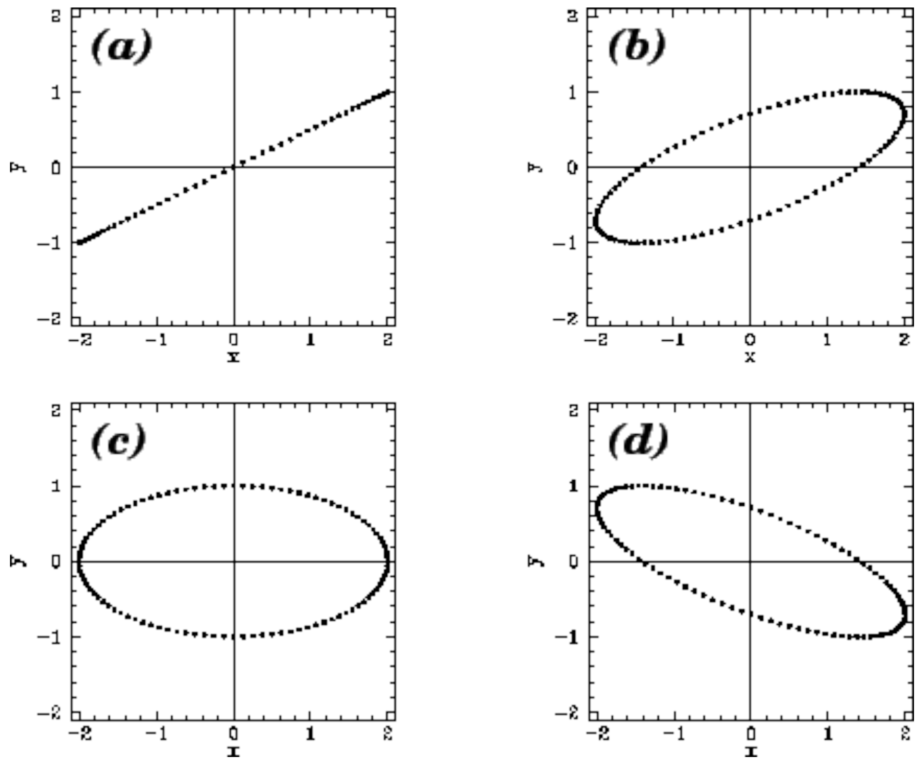
Due Monday Sept. 18

Supplementary: 2-d Harmonic Oscillator

- **Now Consider a 2-d Harmonic Oscillator**

$x = A_1\cos(\omega_1 t + \alpha_1)$ and $y = A_2\cos(\omega_2 t + \alpha_2)$
Since the cos() functions vary between +1 and -1 the x displacement is confined within $\pm A_1$ and the y displacement is confined within $\pm A_2$. Thus any point: $P(x,y,t)$ will be confined within the corresponding rectangle.

If $\omega_1 \neq \omega_2$ or $\frac{\omega_1}{\omega_2} \neq$ ratio of whole numbers then the pattern will never repeat and eventually fill in the entire region. For the special case where $\omega_1 = \omega_2$:
$x = A_1\cos(\omega t)$ and $y = A_2\cos(\omega t + \delta)$ consider a few cases:
$1: \delta = 0$. In this case: $x = A_1\cos(\omega t)$ and $y = A_2\cos(\omega t)$ and so:
$y = \frac{A_2}{A_1}x$ and so P traces out a diagonal line.
$2: \delta = \pi/2$ and so now we have:
$x = A_1\cos(\omega t)$ and $y = A_2\cos(\omega t + \delta) = -A_2\sin(\omega t)$ and since:
$\sin^2\omega t + \cos^2\omega t = 1$ we must have:
$\frac{x^2}{A_1^2} + \frac{y^2}{A_2^2} = 1$ which is the equation of an ellipse with axes along the x and y axes. Note further that as t begins to increase x decreases while y goes negative. Thus the vector between the origin and P rotates clockwise.

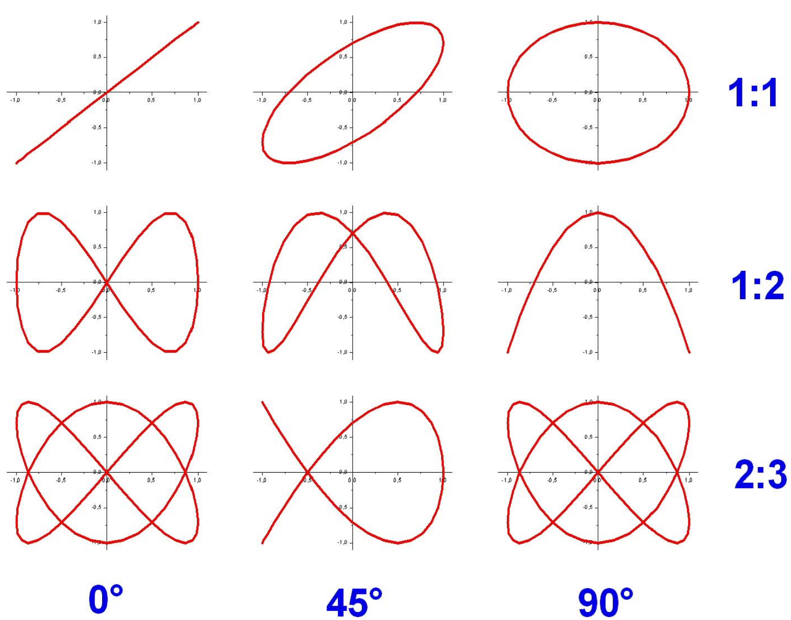


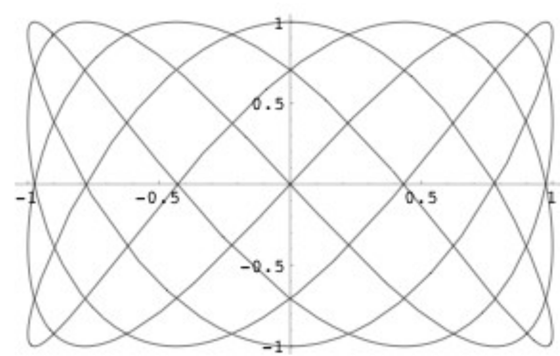
Supplementary: 2-d Harmonic Oscillator

- **2-d Harmonic Oscillator Continued**

3: If $\frac{\omega_1}{\omega_2}$ is the ratio of whole numbers we see complex but repeating pattern.

Consider the case where $\omega_2 = 2\omega_1$. During one cycle of ω_2 we go through only one half cycle of ω_1. The result is the family of curves shown below depending on the phase offset between the two wavefunctions.





Supplementary: Normal Modes in 1d

- **Let's Return to Standing Waves (Normal Modes) on a String**

Recall the Wave Equation in 1d:

$$\frac{\partial^2 y}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 y}{\partial t^2} \quad \text{with the speed } v = \left(\frac{T}{\mu}\right)^{1/2}$$

If we assume a solution for a stationary vibration of the form:

$y(x,t) = f(x)\sin(\omega t)$ then we have:

$$\frac{\partial^2 y}{\partial x^2} = -\frac{d^2 f}{dx^2}\sin(\omega t) \quad \text{and} \quad \frac{\partial^2 y}{\partial t^2} = -\omega^2 f(x)\sin(\omega t) \text{ and substituting:}$$

$$\frac{d^2 f}{dx^2} = -\frac{\omega^2}{v^2} f \quad \text{and so the solution must be of the form:}$$

$f(x) = A\sin\left(\frac{\omega x}{v}\right)$ but our boundary condition requires $f = 0$ at $x = L$ so:

$A\sin\left(\frac{\omega L}{v}\right) = 0$ and so $\frac{\omega L}{v} = n\pi$ where n is any positive integer.

In terms of frequency $\nu = \omega / 2\pi$:

$\nu_n = \frac{nv}{2L} = \frac{n}{2L}\left(\frac{T}{\mu}\right)^{1/2}$ and since the length must be an integer number of half-wavelengths:

$\lambda_n = \frac{2\pi}{n}$ and so $\frac{\omega}{v} = \frac{n\pi}{L} = \frac{2\pi}{\lambda_n}$ and so we can rewrite $f(x)$:

$f_n(x) = A_n\sin\left(\frac{2\pi x}{\lambda_n}\right) = A_n\sin(nkx) = A_n\sin\left(\frac{n\pi x}{L}\right)$ and so:

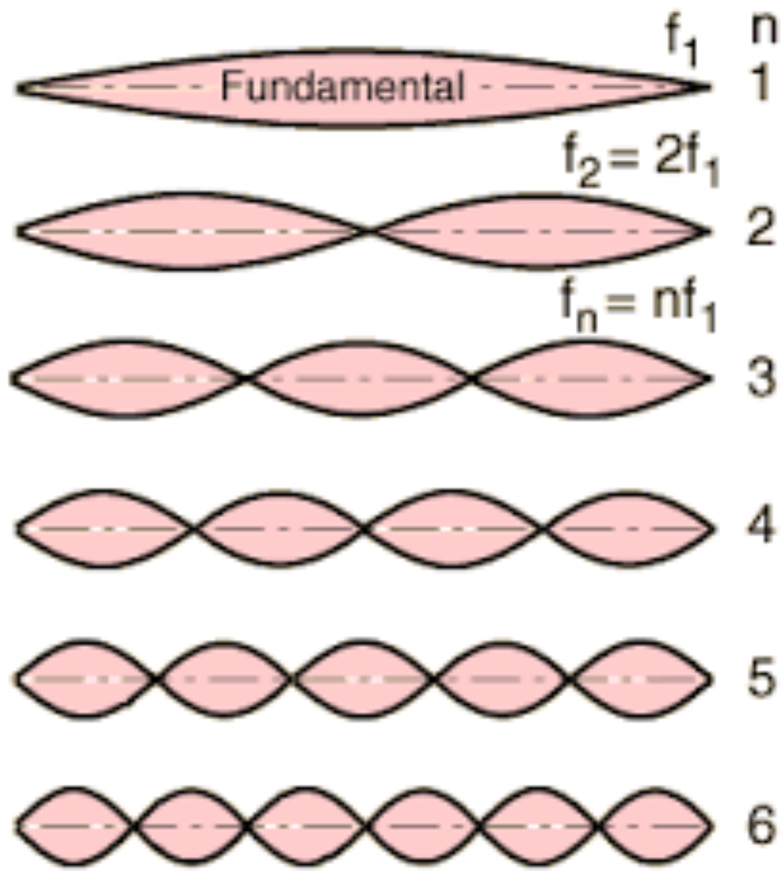
$$y(x,t) = A_n\sin\left(\frac{n\pi x}{L}\right)\cos\omega_n t$$

Example: The E string of a violin is to be tuned to a frequency of 640 Hz. Its length and mass are 33 cm and 0.125 g, respectively. What is the tension?

$\nu_n = \frac{nv}{2L} = \frac{n}{2L}\left(\frac{T}{\mu}\right)^{1/2}$ and setting $\mu = m/L$ and n = 1, we have:

$$T = 4mL\nu_1^2 = 4(1.25\text{x}10^{-4})(0.33)(6.4\text{x}10^2)^2 = 68N$$

Note: the superposition of nodes on a string occurs if we "pluck" the string. In that case the string supports several modes: fundamental + harmonics. This occurs because of the superposition of waves: the sum of solutions of the wave equation is also a solution of the wave equation. Touching a plucked string at the node for one harmonic will supress all other modes and only one survives.



Supplementary: Normal Modes in 2d

- **Now Consider Standing Waves (Normal Modes) on a Surface**

Principle of superposition means waves in x (1) and waves in y (2) are independent. Thus we have a 2d wave equation:

$$\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} = \frac{1}{v^2}\frac{\partial^2 z}{\partial t^2}$$ with the speed $v = \left(\frac{S}{\sigma}\right)^{1/2}$ where S is the force/length (surface tension), and σ is the mass/unit area.

By analogy, if the retangular membrane has a fixed outer boundary we assume a solution for a stationary vibration of the form:

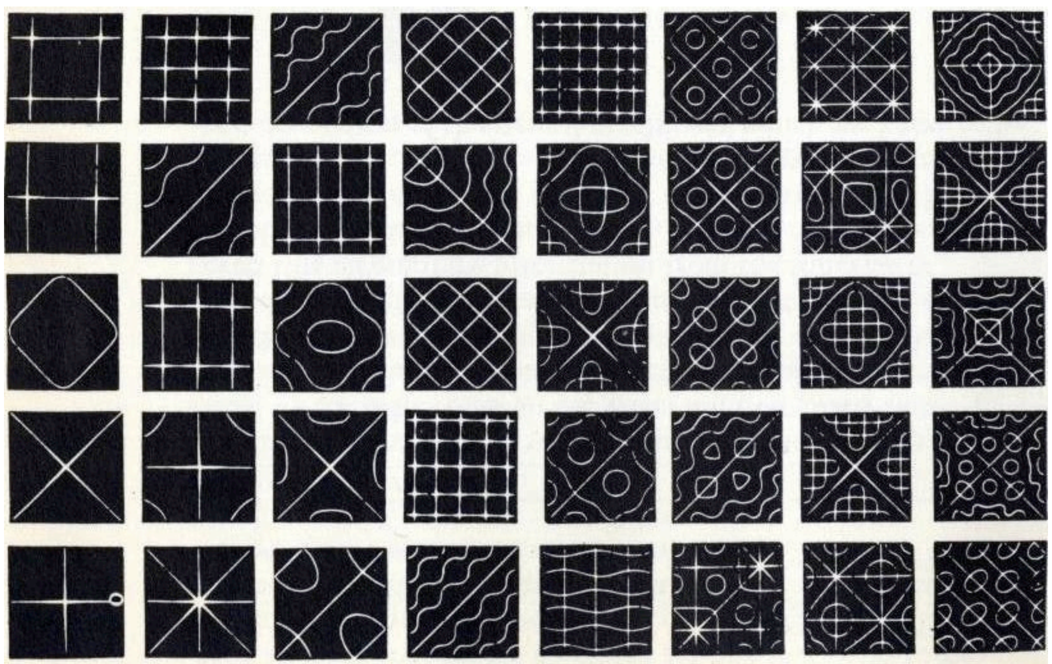
$z(x,y,t) = f(x,y)\cos(\omega_{12}t)$ with separate standing waves in x and in y. In that case we have:

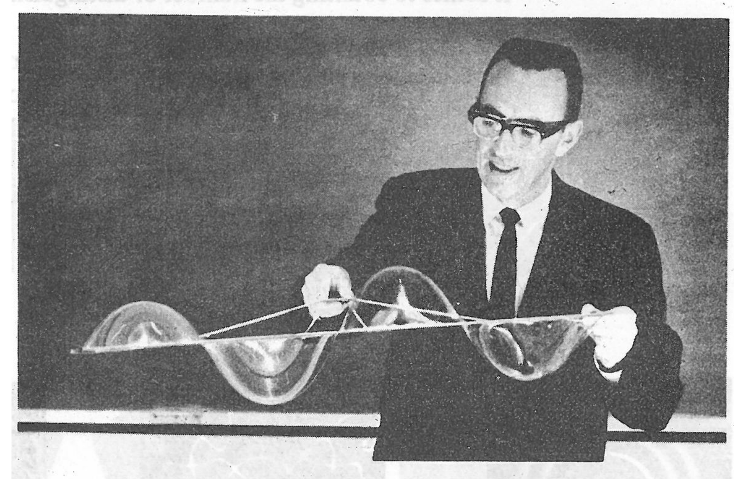
$$z(x,y,t) = C_{n1}C_{n2}\sin\left(\frac{n_1\pi x}{L_x}\right)\sin\left(\frac{n_2\pi y}{L_y}\right)\cos\omega_{12}t$$

The normal mode frequencies (for standing waves) are then:

$$\omega_{12} = \left(\frac{S}{\sigma}\right)^{1/2}\left[\left(\frac{n_1\pi}{L_x}\right)^2 + \left(\frac{n_2\pi}{L_y}\right)^2\right]^{1/2}$$ where n is any positive integer and L is the length of the membrane (right).

If the boundary conditions are not such that z = 0 at x = 0, L_x and y = 0, L_y then the motion is more complcated (below).





Homework this Week

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Next Time: Begin Geometric Optics

- **Read SZ Ch. 32 (Electromagnetic Waves)**