Phys 2310 Mon. Sept. 20, 2017
Today's Topics

- Supplemental: Forced Oscillations
  - Harmonic Forcing with No Damping
    - Resonance
  - Harmonic Forcing with Damping
- Supplemental: Normal Modes
  - Normal Modes in 1d
  - Normal Modes in 2d

- Reading for Next Time

Homework this Week

French Chapter 3: 3-2, 3-13, 3-14, 4-3, 4-4, 4-5

SZ Chapter 32: #8, 9, 10, 11, 20, 29, 46

Due Mon. Sept. 26

Supplementary: Normal Modes in 1d

- **Let’s Return to Standing Waves (Normal Modes) on a String**

Recall the Wave Equation in 1d:

$$\frac{\partial^2 y}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 y}{\partial t^2} \quad \text{with the speed } v = \left(\frac{T}{\mu}\right)^{1/2}$$

If we assume a solution for a stationary vibration of the form:

$y(x,t) = f(x)\sin(\omega t)$ then we have:

$$\frac{\partial^2 y}{\partial x^2} = -\frac{d^2 f}{dx^2}\sin(\omega t) \quad \text{and} \quad \frac{\partial^2 y}{\partial t^2} = -\omega^2 f(x)\sin(\omega t) \text{ and substituting:}$$

$$\frac{d^2 f}{dx^2} = -\frac{\omega^2}{v^2} f \quad \text{and so the solution must be of the form:}$$

$f(x) = A\sin\left(\frac{\omega x}{v}\right)$ but our boundary condition requires $f = 0$ at $x = L$ so:

$A\sin\left(\frac{\omega L}{v}\right) = 0$ and so $\frac{\omega L}{v} = n\pi$ where n is any positive integer.

In terms of frequency $\nu = \omega / 2\pi$:

$\nu_n = \frac{nv}{2L} = \frac{n}{2L}\left(\frac{T}{\mu}\right)^{1/2}$ and since the length must be an integer number of half-wavelengths:

$\lambda_n = \frac{2\pi}{n}$ and so $\frac{\omega}{v} = \frac{n\pi}{L} = \frac{2\pi}{\lambda_n}$ and so we can rewrite $f(x)$:

$f_n(x) = A_n \sin\left(\frac{2\pi x}{\lambda_n}\right) = A_n \sin(nkx) = A_n \sin\left(\frac{n\pi x}{L}\right)$ and so:

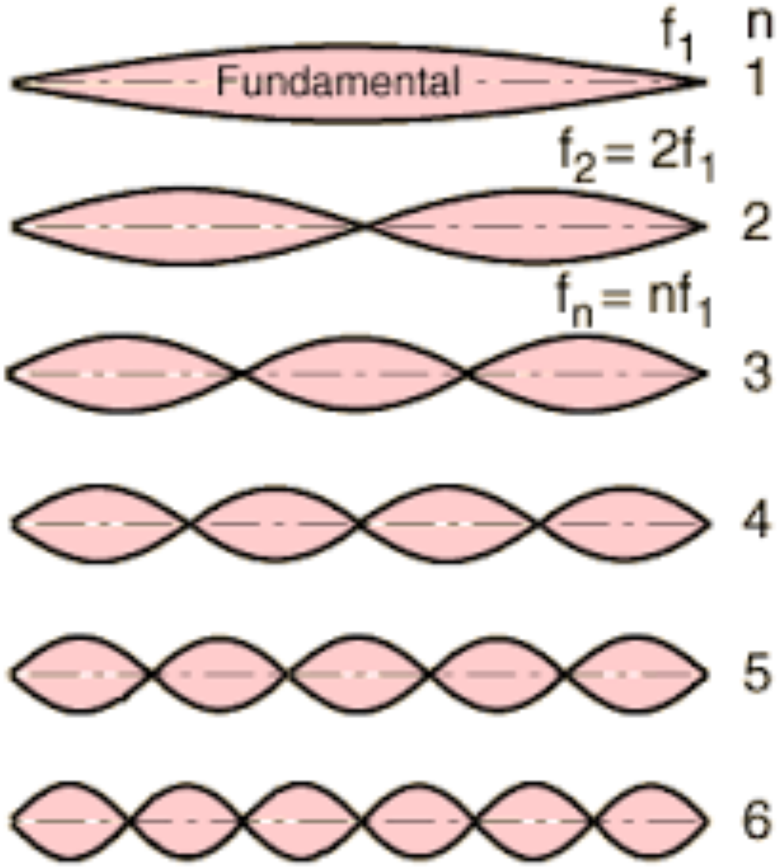
$$y(x,t) = A_n \sin\left(\frac{n\pi x}{L}\right)\cos\omega_n t$$

Example: The E string of a violin is to be tuned to a frequency of 640 Hz. Its length and mass are 33 cm and 0.125 g, respectively. What is the tension?

$\nu_n = \frac{nv}{2L} = \frac{n}{2L}\left(\frac{T}{\mu}\right)^{1/2}$ and setting $\mu = m / L$ and n = 1, we have:

$$T = 4mL\nu_1^2 = 4(1.25\text{x}10^{-4})(0.33)(6.4\text{x}10^2)^2 = 68N$$

Note: the superposition of nodes on a string occurs if we "pluck" the string. In that case the string supports several modes: fundamental + harmonics. This occurs because of the superposition of waves: the sum of solutions of the wave equation is also a solution of the wave equation. Touching a plucked string at the node for one harmonic will supress all other modes and only one survives.



Supplementary: Normal Modes in 2d

- **Now Consider Standing Waves (Normal Modes) on a Surface**

Principle of superposition means waves in x (1) and waves in y (2) are independent. Thus we have a 2d wave equation:

$$\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} = \frac{1}{v^2}\frac{\partial^2 z}{\partial t^2} \quad \text{with the speed } v = \left(\frac{S}{\sigma}\right)^{1/2}$$

where S is the force/length (surface tension), and σ is the mass/unit area.

By analogy, if the retangular membrane has a fixed outer boundary we assume a solution for a stationary vibration of the form:

$z(x,y,t) = f(x,y)\cos(\omega_{12}t)$ with separate standing waves in x and in y. In that case we have:

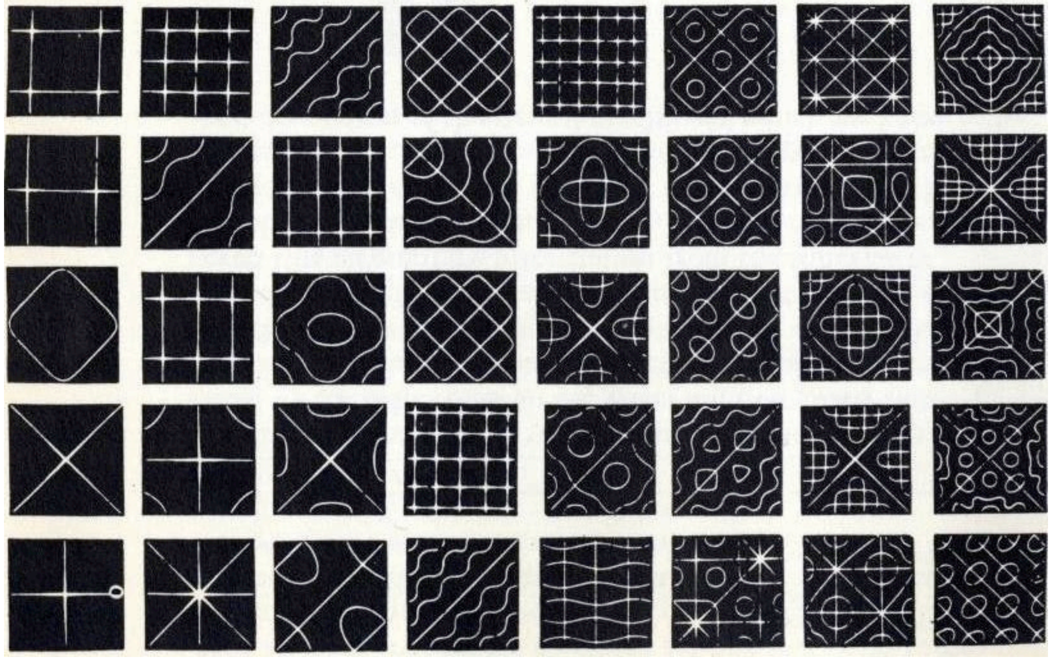
$$z(x,y,t) = C_{n1}C_{n2}\sin\left(\frac{n_1\pi x}{L_x}\right)\sin\left(\frac{n_2\pi y}{L_y}\right)\cos\omega_{12}t$$

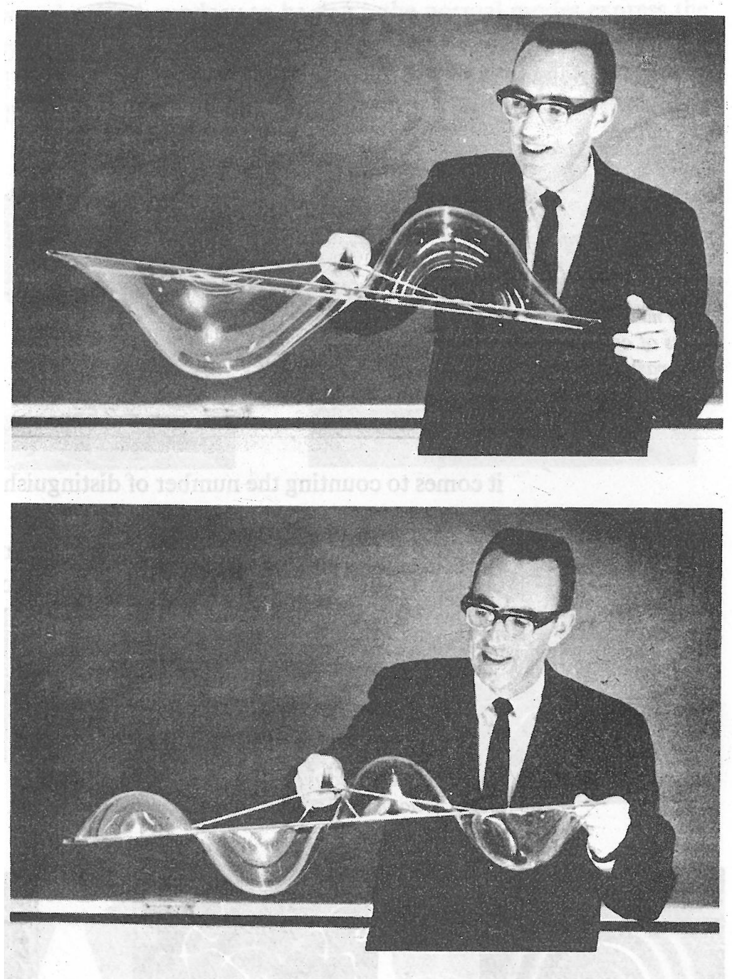
The normal mode frequencies (for standing waves) are then:

$$\omega_{12} = \left(\frac{S}{\sigma}\right)^{1/2}\left[\left(\frac{n_1\pi}{L_x}\right)^2 + \left(\frac{n_2\pi}{L_y}\right)^2\right]^{1/2}$$

where n is any positive integer and L is the length of the membrane (right).

If the boundary conditions are not such that z = 0 at x = 0, L_x and y = 0, L_y then the motion is more complcated (below).





Next Time: Begin Geometric Optics

- **Read SZ Ch. 32 (Electromagnetic Waves)**