Phys 2310 Wed. Sept. 7, 2017
Today's Topics

- Properties of Light
  - Finish Detection of Light
  - Chapter 4: Propagation of Light
- Homework Assigned

- **Reading for Next Time**

Homework #4

- Due Oct. 4
- SZ Ch. 33: #33.3, 33.7, 33.9, 33.12, 33.22, 33.24

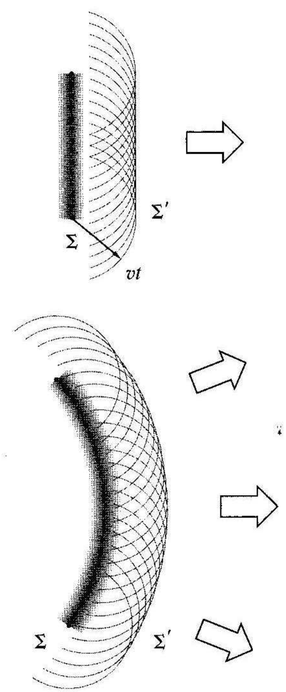
Y&F Chap. 31: Propagation of Light

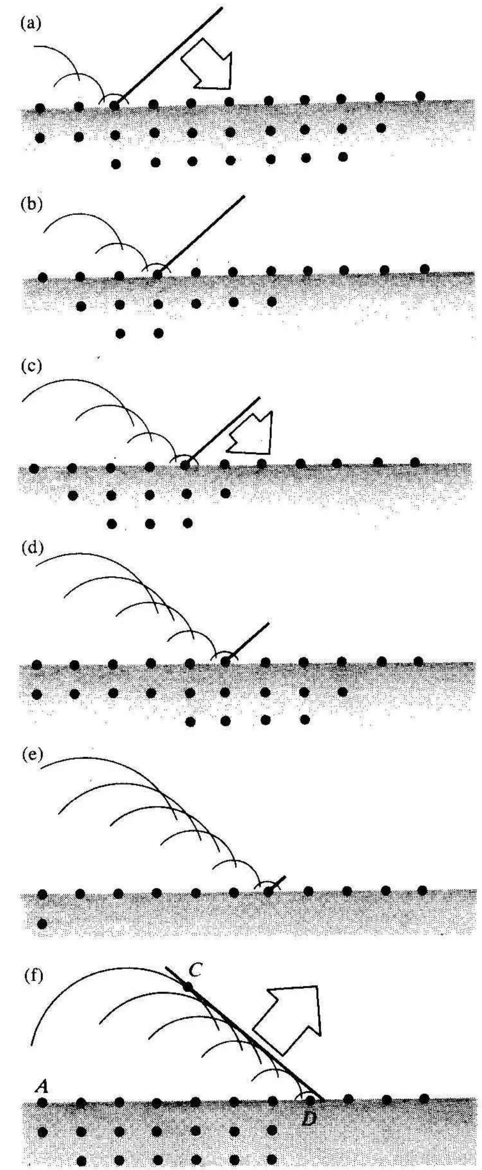
- **Photons interact with matter in a variety of ways**
  - **Photons encountering an opaque solid can be absorbed (black surface) or reflected (metal surface)**
  - **Photons encountering a transparent surface can be scattered if path length is long enough (no substance is perfectly transparent)**
  - **Enhanced scattering of bluer light in atmosphere makes sky blue**
    - **Molecules can be visualized as absorbing photons and then emitting them in a new direction (physics is complex)**
- **Huygen's Principle**
  - **Behavior of light can be understood as the scattering of "wavelets".**
  - **A surface (real or imaginary) can be thought of as a number of scattering centers**
  - **Provides an explanation for the laws of reflection and refraction**

Y&F Chap. 31: Huygen’s Principle

- **Huygen’s Principle:**
  Propagation of light can be <u>modeled</u> as if scattering off atoms in such a way that the spherical “wavelets” constructively interfere to produce a wavefront.

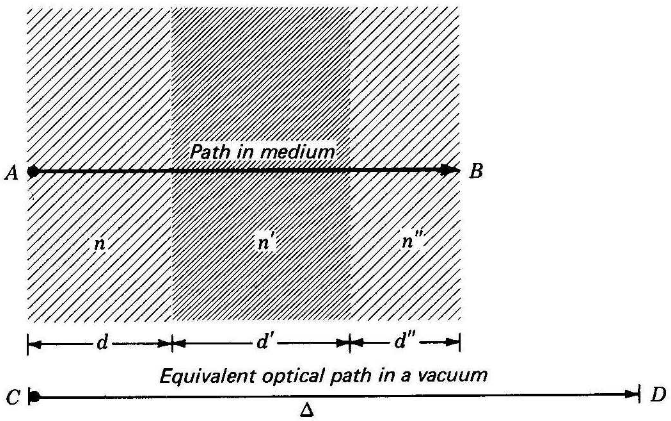
“Every point of a propagating wavefront serves as the source of spherical secondary wavelets, such that the wavefront at some later time is the envelope of these wavelets.”





Y&F Chap. 31: Properties of Optical Materials

- **We can define an equivalent optical path length by considering the index of refraction**
  - **If speed of light is slower in dense material the equivalent path in a vacuum would be longer. Hence:**
  - **O.P.L. = nd where n is the index of refraction and d is the material thickness**
- **The concepts of optical path leads to Fermat's Principle (see sec. 4.5 in text):**

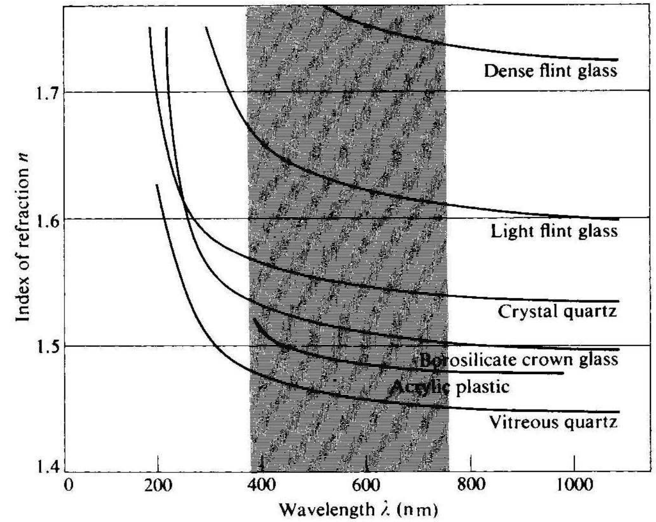


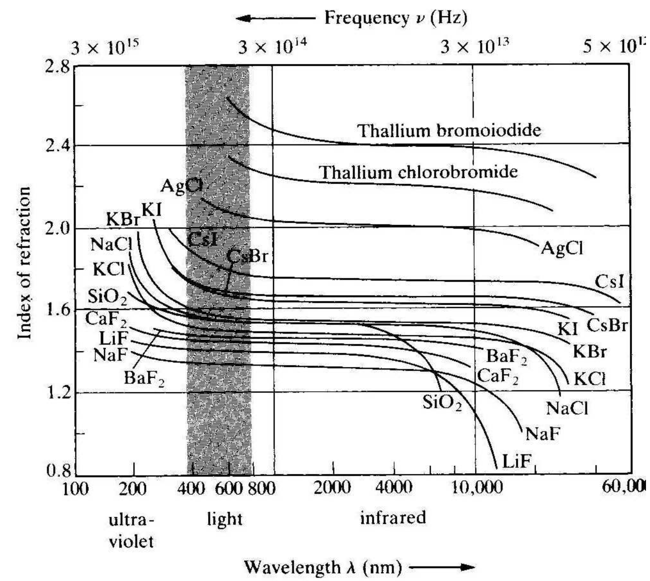
"A light ray will take the path between two points that minimizes its travel time."

This is not strictly true but it is still a useful concept for deriving Snell's Law (see below).

Y&F Chap. 31: Properties of Optical Materials

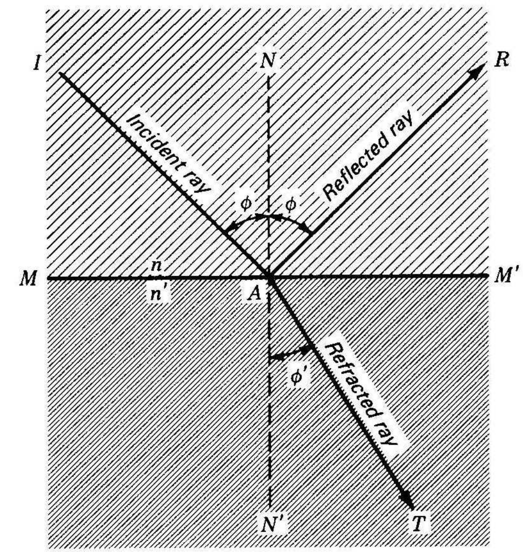
- **The optical path length is found to a function of wavelength**
  - **The index of refraction of most materials is higher at shorter wavelengths**
    - **Not strictly true over all wavelengths but applies over visible (more about this later)**
    - **Over broader wavelength this effect (*dispersion*) is readily seen**

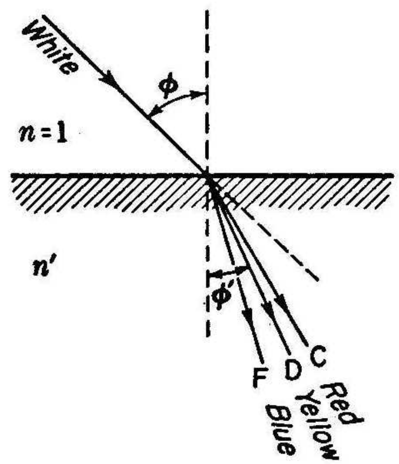




Y&F Chap. 31: Plane Surfaces

- Light slows as it enters a transparent medium
  - Light path (ray) is deflected toward normal when entering higher index medium
  - Light path (ray) is deflected away from normal when entering lower index medium
    - Note that *dispersion* is occurs such that blue light (higher index) is deflected more and vice versa





Y&F Chap. 31: Fermat's Principle and Snell's Law

Minimizing the time (optical path length) between points Q and Q' yields Snell's Law:

$$OPL = \Delta = nd + n'd$$

$$d = (h^2 + (p-x)^2)^{1/2}$$

$$d' = (h'^2 + x^2)^{1/2}$$

substituting :

$$\Delta = n[h^2 + (p-x)^2]^{1/2} + n'(h'^2 + x^2])^{1/2}$$

differentiating :

$$\frac{d\Delta}{dx} = \frac{n/2}{[h^2 + (p-x)^2]^{1/2}}(2p - 2x)(-1) + \frac{n'/2}{(h'^2 + x^2)^{1/2}} 2x = 0$$

thus,

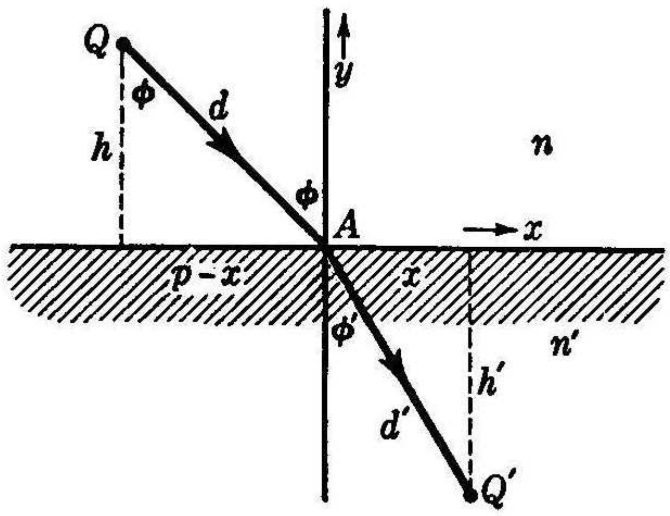
$$n\frac{p-x}{[h^2 + (p-x)^2]^{1/2}} = n'\frac{x}{(h'^2 + x^2)^{1/2}}$$

or

$$n\frac{p-x}{d} = n'\frac{x}{d'}$$

and

$$n \sin\phi = n' \sin\phi'$$



Similarly, the law of reflection can also be derived (homework)

Y&F Chap. 31: Fermat's Principle continued

- **This approach works for a variable index of refraction in which case the OPL is now an integral over the path:**

$$OPL = \int_{s_1}^{s_2} n(s)\,ds$$

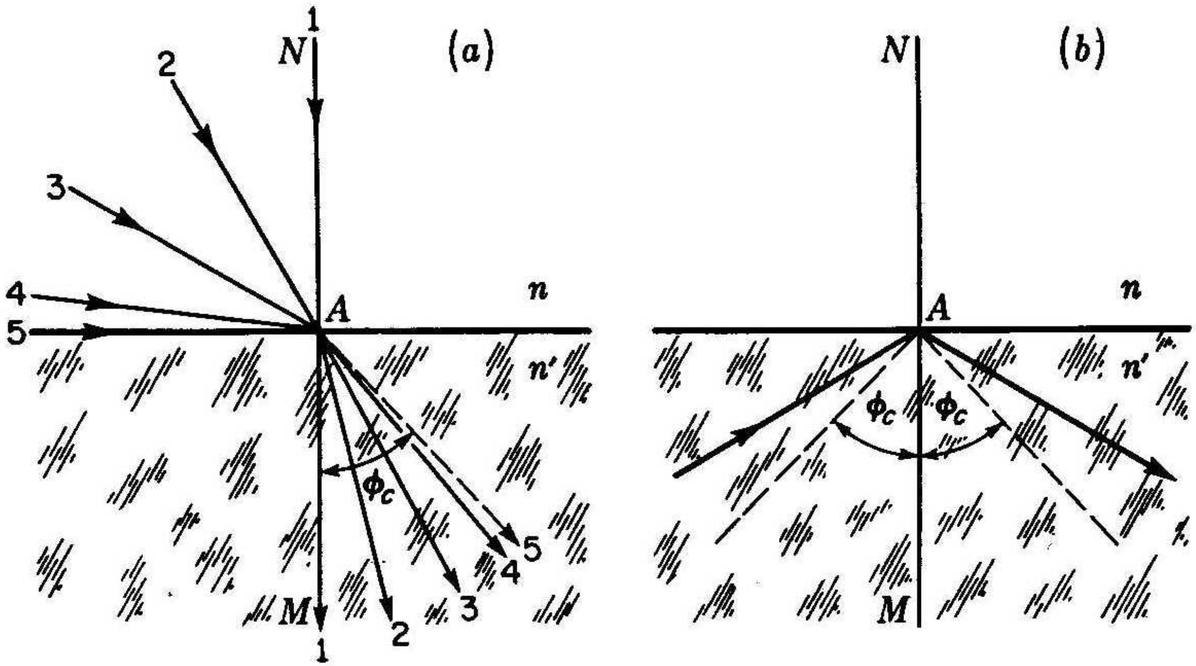
The book discusses an application of this concept to mirages.

Note, that by setting dΔ/dx = 0 the OPL doesn't strictly have to be a minimum it could also be a maximum. A modern description is that we require alternative paths to have significant differences in phase of the "wavelets". Thus we might think of the atoms as scattering the photons and the path light takes is the one where constructive interference is maximized. See the discussion on page 109-110 of text.

Y&F Chap. 31: Total Internal Reflection

The light path of rays is reversible.

- Thus we can consider total internal reflection via refraction.
- Note that for rays 1-4 all appears fine. Ray 5 is a problem.
- When we consider the reversed path any rays with $\phi > \phi_c$ will not emerge from the higher index medium. For glass (n ~ 1.52) ϕ_c ~ 42 deg. So 45-deg prisms can be used.



Substituting ϕ = 90 deg, or sin ϕ = 1 into Snell's Law gives:

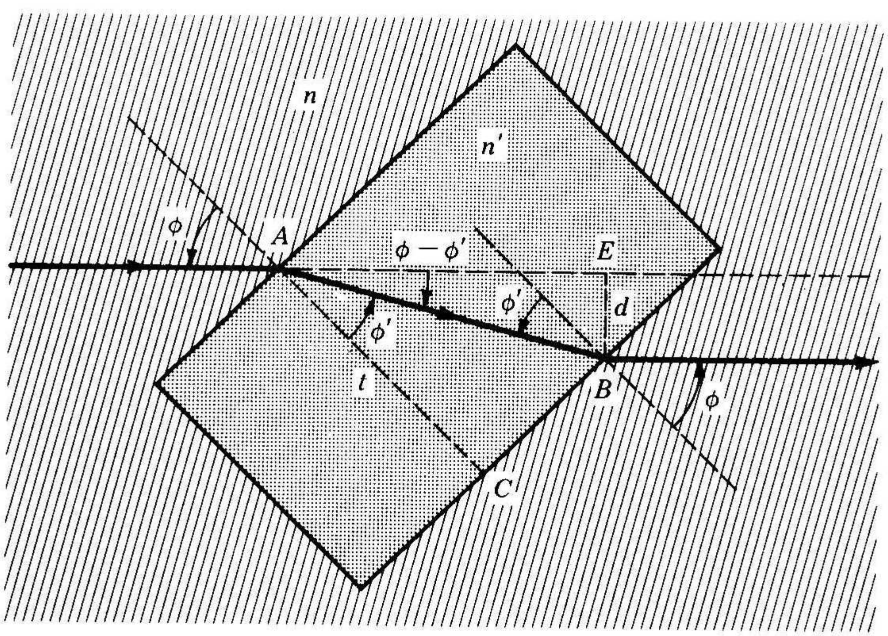
$$\sin \phi_c = \frac{n}{n'}$$

Y&F Chap. 31: Plane Surfaces continued

- **Light traversing a parallel plate is deflected**

  (e.g. a window)

- **The deflection angle can be calculated**



$$d = l\sin(\phi - \phi')$$

$$d = l(\sin\phi\cos\phi' - \sin\phi'\cos\phi)$$

$$from ABC:$$

$$l = \frac{t}{\cos\phi'}$$

$$d = t\left(\frac{\sin\phi\cos\phi'}{\cos\phi'} - \frac{\sin\phi'\cos\phi}{\cos\phi'}\right)$$

since:

$$\sin\phi' = \frac{n}{n'}\sin\phi$$

$$d = t\left(\sin\phi - \frac{\cos\phi}{\cos\phi'}\frac{n}{n'}\sin\phi\right)$$

$$or:$$

$$d = t\sin\phi\left(1 - \frac{n}{n'}\frac{\cos\phi}{\cos\phi'}\right)$$

Angle of Minimum Deviation for a Prism

- **We can trace a ray through an optical system by successive application of Snell's Law. Consider the prism at right. The deviation angle is given by:**

$\phi'_1 + \phi'_2 + (180 - \alpha) = 180$ and so $\phi'_2 = \alpha - \phi'_1$

Snell's Law at input and output :

$\sin\phi_1 = n(\lambda)\sin\phi_1'$ and $\sin\phi_2 = n(\lambda)\sin\phi_2'$ and so

since $\sin^2\theta + \cos^2\theta = 1$ and $\sin\phi_1 = n(\lambda)\sin\phi_1'$ we have :

$n\cos\phi'_1 = \sqrt{n^2 - \sin^2\phi_1}$

Substituting into Snell's law at output for ϕ'_2 gives :

$\sin\phi_2 = n(\lambda)\sin\phi_2' = n\sin(\alpha - \phi'_1) = n[\sin\alpha\cos\phi'_1 - \cos\alpha\sin\phi'_1]$

so combining gives :

$\sin\phi_2 = \sin\alpha\sqrt{n^2 - \sin^2\phi_1} - \cos\alpha\sin\phi_1'$

The total deviation angle is :

$D = \phi_1 + \phi_2 - \alpha$

Differentiaing wrt ϕ_1 and setting to 0 :

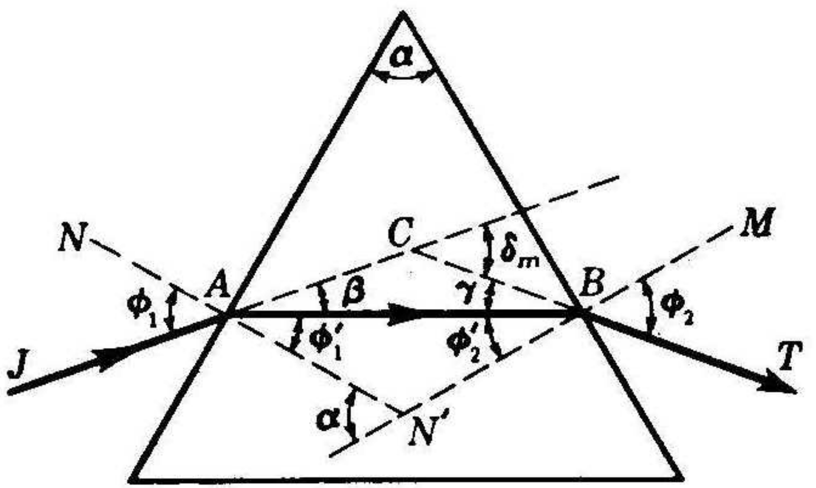
$\frac{\partial D}{\partial\phi_1} = 1 + \frac{\partial\phi_2}{\partial\phi_1} - 0 = 0$ (for minimum deviation). Diff. Snell's Law :

$\frac{\partial\phi_2}{\partial\phi_1} = -1 = \frac{\partial\phi_2\partial\phi_2'\partial\phi'_1}{\partial\phi_2'\partial\phi'_1\partial\phi_1} = \frac{n\cos\phi_2'}{\cos\phi_2}\cdot(-1)\cdot\frac{\cos\phi_1}{n\cos\phi_1'}$

So we must have :

$\cos\phi_1\cos\phi_2' = \cos\phi_1'\cos\phi_2$ and thus :

$\phi_1 = \phi_2$ and $\phi_1' = \phi_2' = \alpha/2$ (Minimum Dev. = Symmetric rays)



We can now write Snell's law as:

$$n(\lambda) = \frac{\sin\phi_1}{\sin\phi_2} = \frac{\sin\left[\frac{1}{2}[\alpha + \delta_m(\lambda)]\right]}{\sin\left(\frac{\alpha}{2}\right)}$$

Homework #4

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