Phys 2310 Wed. Oct. 4, 2017
Today's Topics

- – Continue Chapter 33: Geometric Optics
- Reading for Next Time

Reading this Week

By Monday:

Finish Ch. 33 Lenses, Mirrors and Prisms

Homework

- Due Oct. 11, 2017
- Y&F Ch. 32: #32.1, 32.5

  Ch. 33: #33.3, 33.7, 33.9, 33.12, 33.22, 33.24

Chapter 33: Geometric Optics

- **Overview of Image Properties**
  - **No optical system produces perfect images**
    - **Always have to choose between cost and complexity (What is the minimum cost that will still do the job?)**
  - **Point source producing "rays of light"**
    - **An optical system producing a perfect image of source is <u>stigmatic</u>.**
    - **Most real systems produce a <u>blur spot</u> (region of minimum blur)**
      - **Limited by diffraction effect (later) and finite size of pixels on the detector**
    - **Image plane vs. Object Plane (conjugate points on axis)**

Chapter 33: Geometric Optics

- **Curved Surfaces (Overview)**
  - Purpose (goal) of an optic is to reshape wavefront (or deflect rays) from a source to some desired shape.
    - Lens: we might want to image an object onto a flat detector
    - Reflector: we might want to direct the light from a source toward a given direction (flashlight beam)
  - Design of an optical system depends on the goal and the requirements
  - Cost is always a factor since complexity means additional labor in manufacture, mounting of components and time to completion.

Chapter 33: Geometric Optics

- **Overview**
  - Convex surface: surface bends outward toward the object
    - Known as a converging surface or lens since light is concentrated (focused)
  - Concave surface: surface bends inward away from the object
    - Known as a diverging surface or lens since light is less concentrated (diverges)
- **Aspherical Surfaces**
  - Early lenses were spherical but it was known that aspherical lenses produce the most accurate wavefronts.
    - Greater difficulty of manufacture and expense limits their use.
      - Most common use is in parabolic reflectors and telescope mirrors
    - Computer design and manufacture has increased their use.
      - Inclusion of a single aspherical lens in a complex optical systems can often greatly reduce its complexity

Chapter 33: Geometric Optics

- **Spherical Surfaces**
  - **Aspheric optics function best for sources on their axis of rotation**
    - **Off-axis sources show increased blur (aberrations)**
  - **Geometry of spherical surfaces means that they image over a broader "field of view"**
    (i.e., the angular extent of the source)
    - **Axial aberrations are present but off-axis aberrations typically increase more slowly.**
  - **Much easier to manufacture and test**

Supplementary: Refraction at Spherical Surfaces

- **Object and Image Distances are Related**
  - **Consider a spherical convex surface between media n and n'**
  - **For small angles (α and γ) we make a small angle approximations, the so-called paraxial rays.**
  - **If C is opposite M r is positive and vice versa.**

$$\frac{\sin\phi}{\sin\phi'} = \frac{n'}{n} \quad \text{Snell's Law}$$

$$\frac{\phi}{\phi'} = \frac{n'}{n} \quad (\text{small angle approx.} \rightarrow \sin\phi \cong \phi)$$

$$\phi = \alpha + \beta \quad (\phi \text{ is exterior angle of triangle MTC})$$

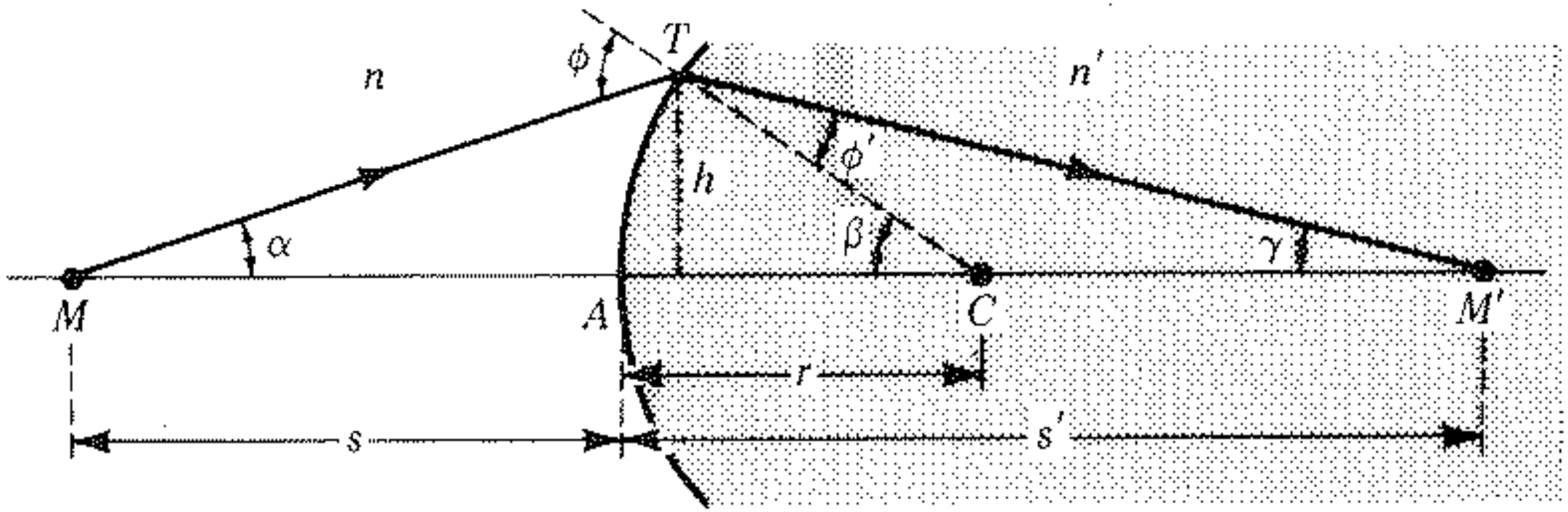
$$\beta = \phi' + \gamma \rightarrow \phi' = \beta - \gamma \quad (\text{same for } \beta \text{ and triangle TCM}')$$

$$n'\beta - n'\gamma = n\alpha + n\beta \rightarrow n\alpha + n'\gamma = (n' - n)\beta \quad (\text{substituting})$$

$$\alpha \cong \frac{h}{s}, \beta \cong \frac{h}{r} \text{ and } \gamma \cong \frac{h}{s'} (\text{small angles}: \tan\alpha \cong \alpha) \text{ and so}:$$

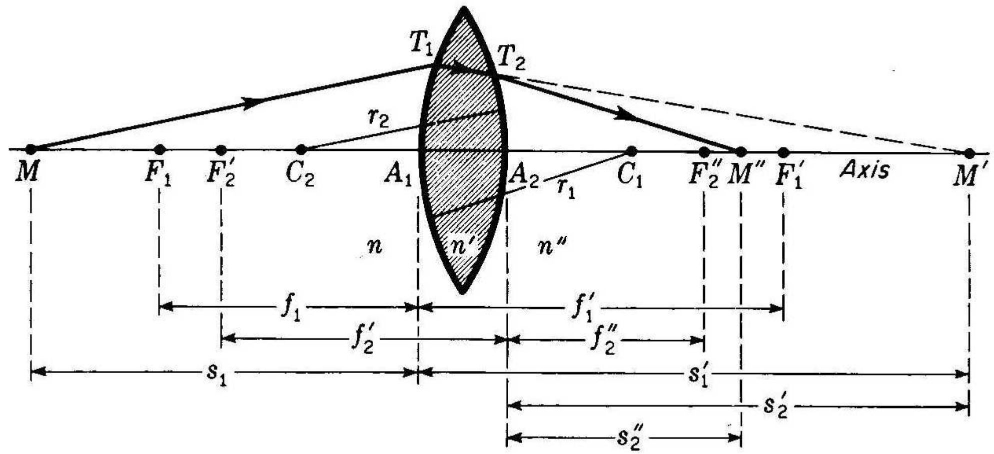
$$n\frac{h}{s} + n'\frac{h}{s'} = (n' - n)\frac{h}{r} \quad (\text{dividing through by } h):$$

$$\frac{n}{s} + \frac{n'}{s'} = \frac{n' - n}{r}$$



Chapter 33: Lens Maker's Formula

- **Convex lens**
  - **Properties can be derived by considering two surfaces. At T_1. Note that r_1 is positive and r_2 is negative.**



$$\frac{n}{s_1}+\frac{n'}{s_1'}=\frac{n'-n}{r_1}$$

and at the second surface:

$$\frac{n'}{s_2'}+\frac{n''}{s_2''}=\frac{n''-n'}{r_2}$$

Now if the thickness is negligible $s_1'=-s_2'$ so:

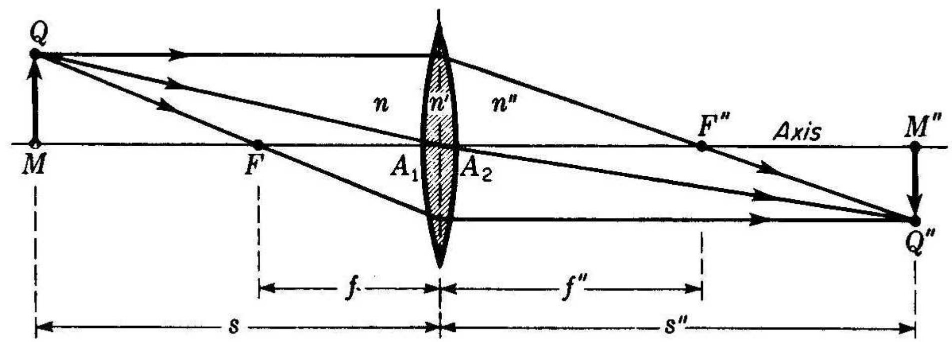
$$\frac{n'}{s_1'}=-\frac{n'}{s_2'}$$

adding the two equations:

$$\frac{n}{s_1}+\frac{n''}{s_2''}=\frac{n'-n}{r_1}+\frac{n''-n'}{r_2}$$

if the medium is the same on both sides:

$$\frac{n}{s}+\frac{n}{s''}=(n'-n)\left(\frac{1}{r_1}-\frac{1}{r_2}\right)$$



which in air (n = 1) becomes:

$$\frac{1}{s}+\frac{1}{s''}=(n'-1)\left(\frac{1}{r_1}-\frac{1}{r_2}\right)$$

The lens maker's formula (an approximation!)

Chapter 33: Gaussian Optics

- Recall that in deriving the Lens Maker's Equation (aka, the Thin Lens Equation) we made the small angle approximation:

  $\sin \phi \sim \phi$

- This is also known as first-order theory since we can see that this approximation comes from a Taylor Expansion of the Sin:

$$\sin \phi \cong \phi - \frac{\phi^3}{3!} + \frac{\phi^5}{5!} - \frac{\phi^7}{7!} + \ldots$$

$$\cos \phi \cong 1 - \frac{\phi^2}{2!} + \frac{\phi^4}{4!} - \frac{\phi^6}{6!} + \ldots$$

If we approximate $\sin \phi \sim \phi - \phi^3/3$ this is known as third-order theory (i.e., there is no second-order theory for sin!)

Chapter 33: Gaussian Optics Cont.

- If we ask where an object is located such that plane wave emerge from the lens we are stating the image will be formed at infinity.
- This is also called the first focal length of the lens.
- Incoming plane wave produces image at second focal length.
- For a thin lens these are the same.
- Note the opposite signs for r_1 and r_2. Otherwise first equation makes no sense.
- Substituting for f in the Lens Makers Formula gives the Gaussian Lens Formula
  - This gives image and object distances in terms of the focal length of the lens
- The Power of a lens is 1/f

Diopters = 1/f (meters)

$$\frac{1}{s'}+\frac{1}{s''}=(n-1)\left(\frac{1}{r_1}-\frac{1}{r_2}\right)$$

or

$$\frac{1}{s'}+\frac{1}{\infty}=(n-1)\left(\frac{1}{r_1}-\frac{1}{r_2}\right)$$

thus

$$\frac{1}{f}=(n-1)\left(\frac{1}{r_1}-\frac{1}{r_2}\right)$$

$$\frac{1}{s'}+\frac{1}{s''}=\frac{1}{f}$$

Chapter 33: Graphical Image Analysis

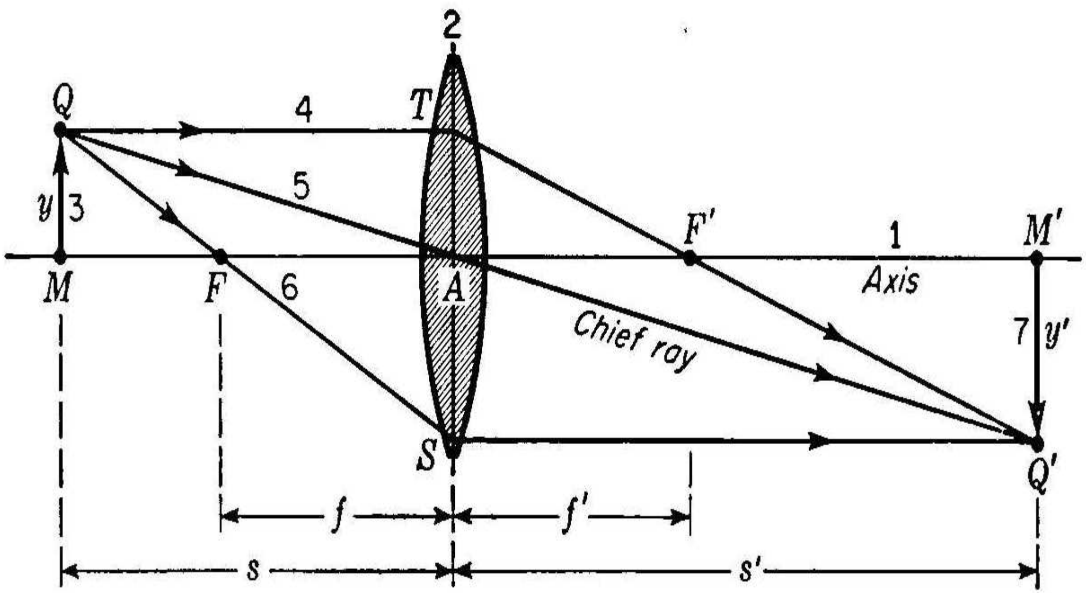
- **Rays parallel to the optical axis (QT) must pass through the focal point after going through the lens.**
- **Rays that pass through the focus and then the lens must emerge parallel to optical axis (QS).**
- **Rays that pass through the center of the lens are undeviated (QA).**
- **Triangles QMA and Q’ M’ A are similar.**

$$\frac{M'Q'}{MQ} = \frac{AM'}{AM}$$

$s' = AM'$ and $s = AM$

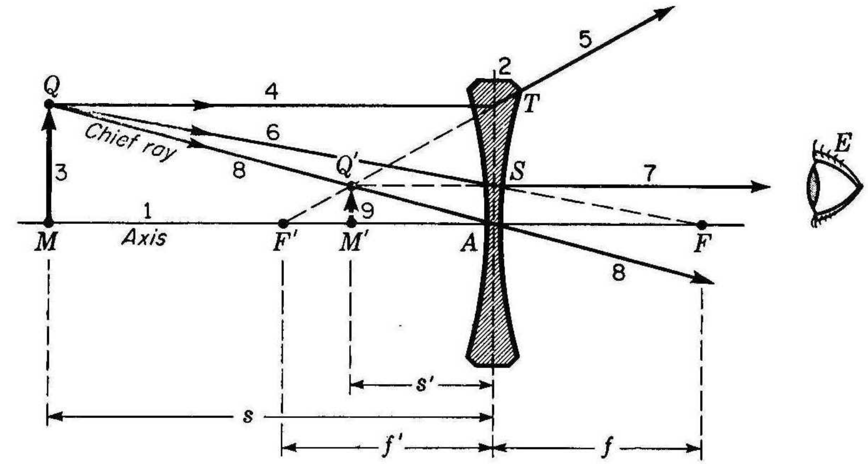
$y = MQ$ and $y' = -M'Q'$ so :

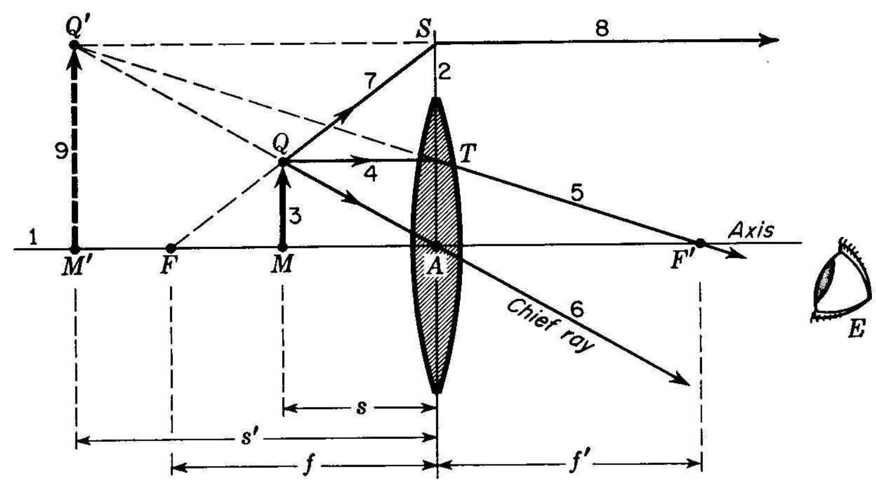
$$m = \frac{y'}{y} = -\frac{s'}{s} \quad \text{(magnification)}$$



Chapter 33: Virtual Images

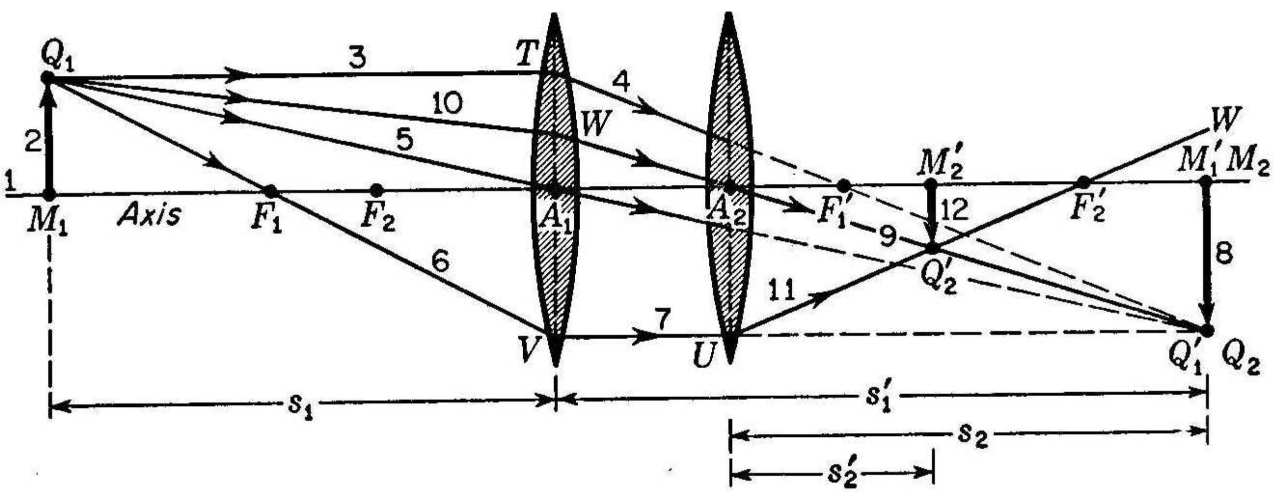
- **Concave or Negative lenses produce de-magnified virtual images.**
  - **Trace back rays 6/7 & 8**
- **Objects closer than the f.l. to a positive lens produce a virtual (but magnified) image.**
  - **Trace back rays 4/5 & 6**
  - **This is the magnifying glass!**





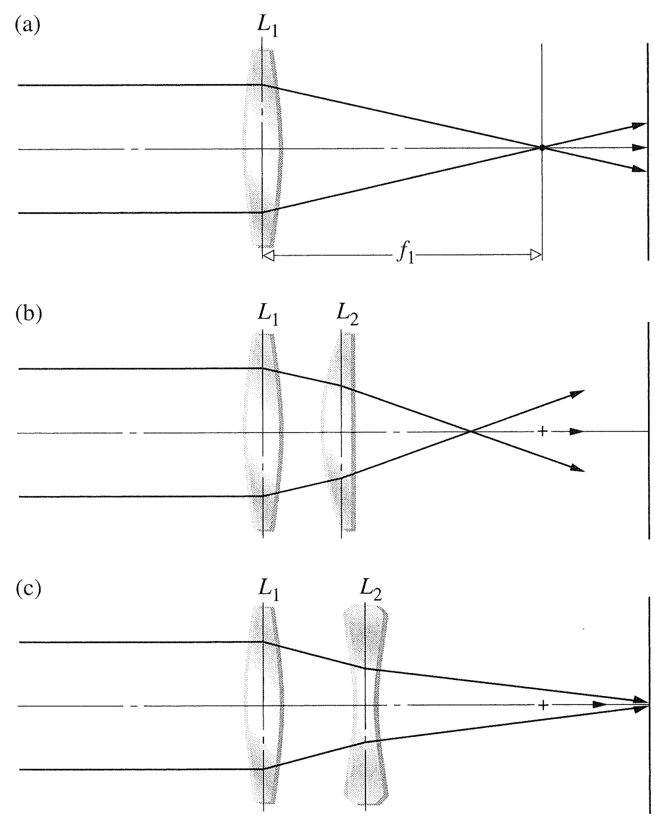
Chapter 33: Thin Lens Combinations

- **For lens combinations the object for the second lens is just the image formed by the first (subtract or add separation accordingly).**
  - **Be careful about the sign of the object distance (see table 5.2)**
  - **See pgs. 167-169 in Hecht for equations and the slide below.**
- **For the graphical method**
  - **You can solve lenses graphically by laying them out in a drawing program (or even graph paper!) and tracing the Paraxial and Chief rays**
  - **Note that the "extra" ray (#9/10) goes through center of second lens.**
  - **In addition, ray #6/7 is deviated by second lens and must go through F'_2 so together they (#6/7 & # 9/10) locates the new image.**



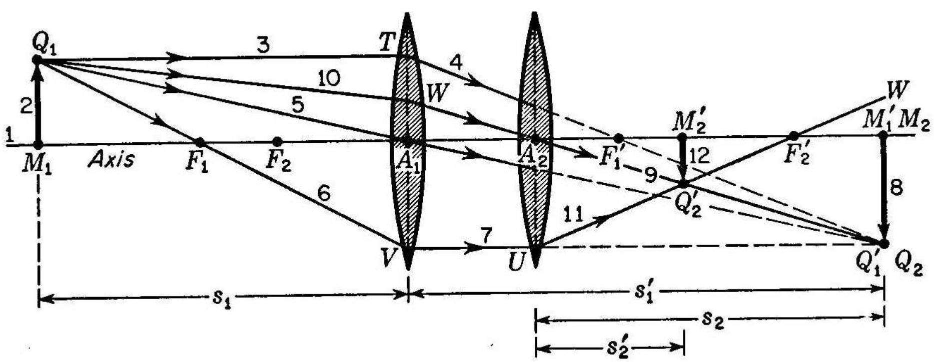
Chapter 33: Thin Len Combinations - II

- **If the second lens is inside the focus of the first:**
  - Convex lens shortens the focal length (power is higher, neg. obj. distance for 2nd)
  - Concave lens lengthens the focal length (power is increased , neg. obj. distance 2nd)



Chapter 33: Thin Len Combinations - III

- **Gaussian lens equation can be applied to a sequence of lenses: just let the image of the first lens be the object of the second and so on.**



For the First Lens:

$$\frac{1}{s_{i1}} = \frac{1}{f_1} - \frac{1}{s_{o1}} \qquad \text{or,}$$

$$s_{i1} = \frac{s_{o1} f_1}{s_{o1} - f_1}$$

Now for the second lens:

$s_{o2} = d - s_{i1}$ (Note this can be negative if the image is beyond lens #2)

and so:

$$\frac{1}{s_{i2}} = \frac{1}{f_2} - \frac{1}{s_{o2}} \qquad \text{or,}$$

$$s_{i2} = \frac{s_{o2} f_2}{s_{o2} - f_2}$$ and upon substituting for s_{o2}:

$$s_{i2} = \frac{(d - s_{i1}) f_2}{(d - s_{i1} - f_2)}$$ and substituting for s_{i1}:

$$s_{i2} = \frac{f_2 d - f_2 s_{o1} f_1 / (s_{o1} - f_1)}{d - f_2 - s_{o1} f_1 / (s_{o1} - f_1)}$$

The two focal lengths for the combination then become:

$$\text{f.f.l.} = \frac{f_1 (d - f_2)}{d - (f_1 + f_2)} \qquad \text{b.f.l.} = \frac{f_2 (d - f_1)}{d - (f_1 + f_2)}$$

Chapter 33: Thin Lens in Contact

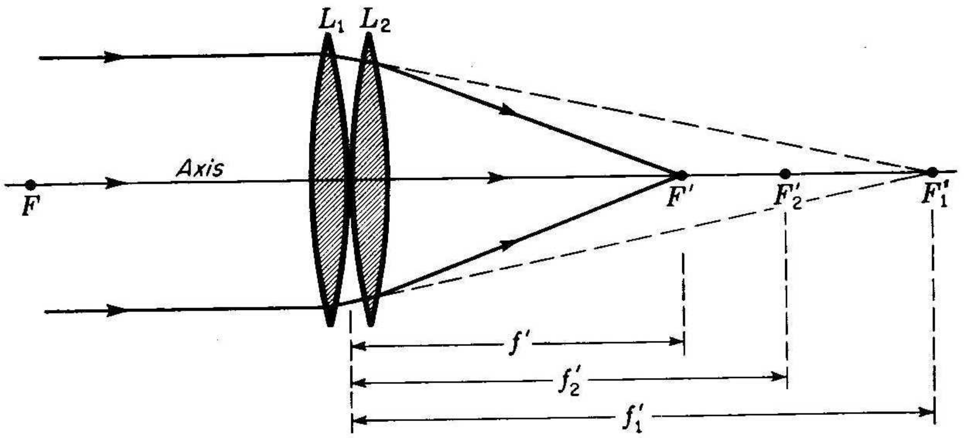
- **For lens in contact (separation is negligible)**
  - **Object distance of lens #2 = Image distance of lens #1 (let d -> 0 in b.f.l. equation above**
- **For an object at infinity:**

$$\frac{1}{-f_1} + \frac{1}{f} = \frac{1}{f_2} \quad \text{or :}$$

$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2}$$

$$P = P_1 + P_2$$

(power is sum of each power)



Chapter 33: Image Properties - I

Sign Conventions

(very important, memorize!)

TABLE 5.2 Meanings Associated with the Signs of Various Thin Lens and Spherical Interface Parameters

Quantity	Sign +	Sign −
s_o	Real object	Virtual object
s_i	Real image	Virtual image
f	Converging lens	Diverging lens
y_o	Erect object	Inverted object
y_i	Erect image	Inverted image
M_T	Erect image	Inverted image

Chapter 33: Image Properties

Convex vs. Concave Lenses

TABLE 5.3 Images of Real Objects Formed by Thin Lenses

Convex

Object	Image			
Location	Type	Location	Orientation	Relative Size
$\infty > s_o > 2f$	Real	$f < s_i < 2f$	Inverted	Minified
$s_o = 2f$	Real	$s_i = 2f$	Inverted	Same size
$f < s_o < 2f$	Real	$\infty > s_i > 2f$	Inverted	Magnified
$s_o = f$		$\pm\infty$		
$s_o < f$	Virtual	$\lvert s_i \rvert > s_o$	Erect	Magnified

Concave

Object	Image			
Location	Type	Location	Orientation	Relative Size
Anywhere	Virtual	$\lvert s_i \rvert < \lvert f \rvert$,	Erect	Minified

Example Problems

- **Consider a bi-convex lens with $R_1 = R_2 = 15cm$.**
  a) **Determine the focal length of the lens**
  b) **Find the image distance for an object located 35cm from the lens**
  c) **Make a ray diagram sketch for this configuration**
  d) **Make a sketch of the image distance vs. object distance**

Example Problems

- **Consider a concave spherical mirror with fl = 60cm.**
  a) **Is the image real or virtual?**
  b) **Find the image of an object located 10.0 m away from the mirror**
  c) **Make a ray diagram sketch for this configuration**
- **Repeat this example for a convex spherical mirror with fl = - 60cm**

Homework

- Due Oct. 11, 2017
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  Ch. 33: #33.3, 33.7, 33.9, 33.12, 33.22, 33.24

Reading this Week

By Monday:

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