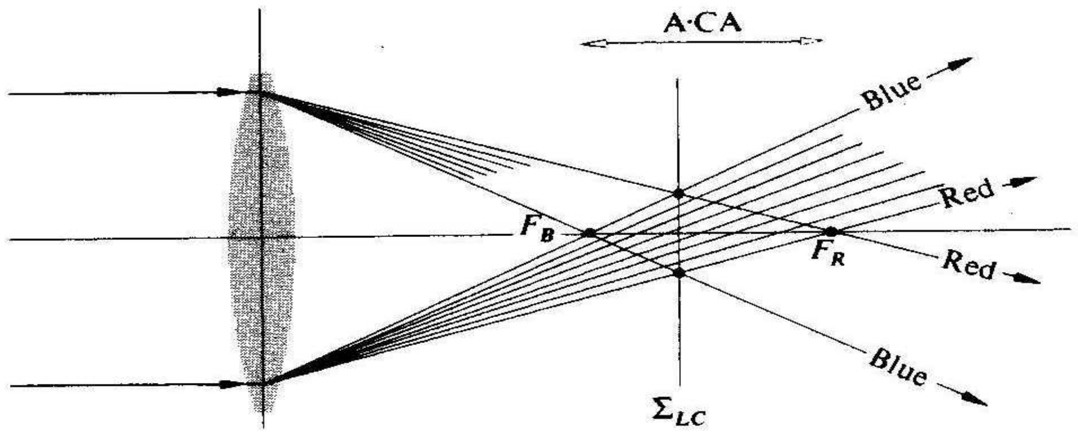
Phys 2310 Wed.. Oct. 30, 2017
Today's Topics

- Finish Geometric Optics
- Reading for Next Time

Supplemental: Optical Aberrations

- **Chromatic Aberration**
  - Dispersion of optical material (change in index with λ) results in lenses with chromatic aberration
  - Positive lenses have positive chromatic aberration
  - Negative lenses have negative chromatic aberration
  - A combination of positive and negative lenses with differing index and dispersions can be used to minimize chromatic aberration (achromatic doublet)



Supplemental: Designing an Achromat

- **Consider an Achromatic Doublet**
  - **Choose positive and negative lens focal lengths so combo has a non-zero focal length.**
  - **Choose dispersions of glasses to cancel at two wavelengths, e.g. blue and red.**
  - **Positive Crown: low-index, high-dispersion glass.**
  - **Negative Flint: high-index, low dispersion glass.**

$$\frac{1}{f}=\frac{1}{f_1}+\frac{1}{f_2}-\frac{d}{f_1 f_2} \quad \text{and since:}$$

$$\frac{1}{f_1}=(n-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)=(n-1)\rho_1 \quad \text{we have:}$$

$$\frac{1}{f}=(n_1-1)\rho_1+(n_2-1)\rho_2 \quad \text{(let d} \sim 0\text{, thin lens)}$$

If we require equal $\frac{1}{f}$ for Blue and Red light:

$$(n_{1B}-1)\rho_1+(n_{2B}-1)\rho_2=(n_{1R}-1)\rho_1+(n_{2R}-1)\rho_2$$

grouping radii (left) and refractive indices (right):

$$\frac{\rho_1}{\rho_2}=-\left(\frac{n_{2B}-n_{2R}}{n_{1B}-n_{1R}}\right)$$

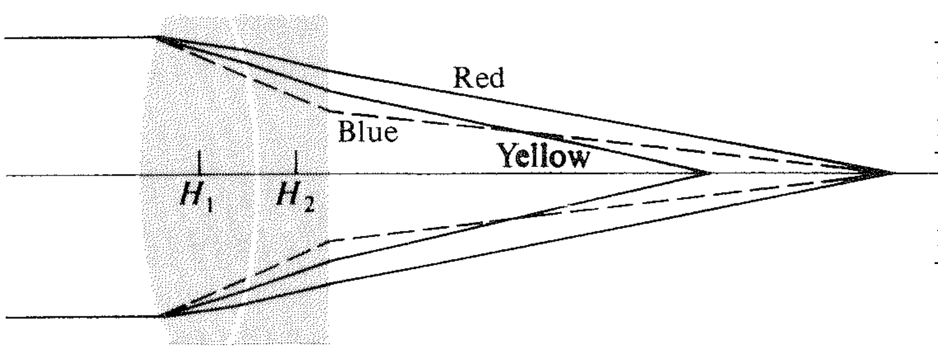
For an intermediate wavelength (e.g., yellow):

$$\frac{1}{\mathrm{f}_{1y}}=(n_{1y}-1)\rho_1 \quad \text{and} \quad \frac{1}{\mathrm{f}_{2y}}=(n_{2y}-1)\rho_2 \quad \text{so:}$$

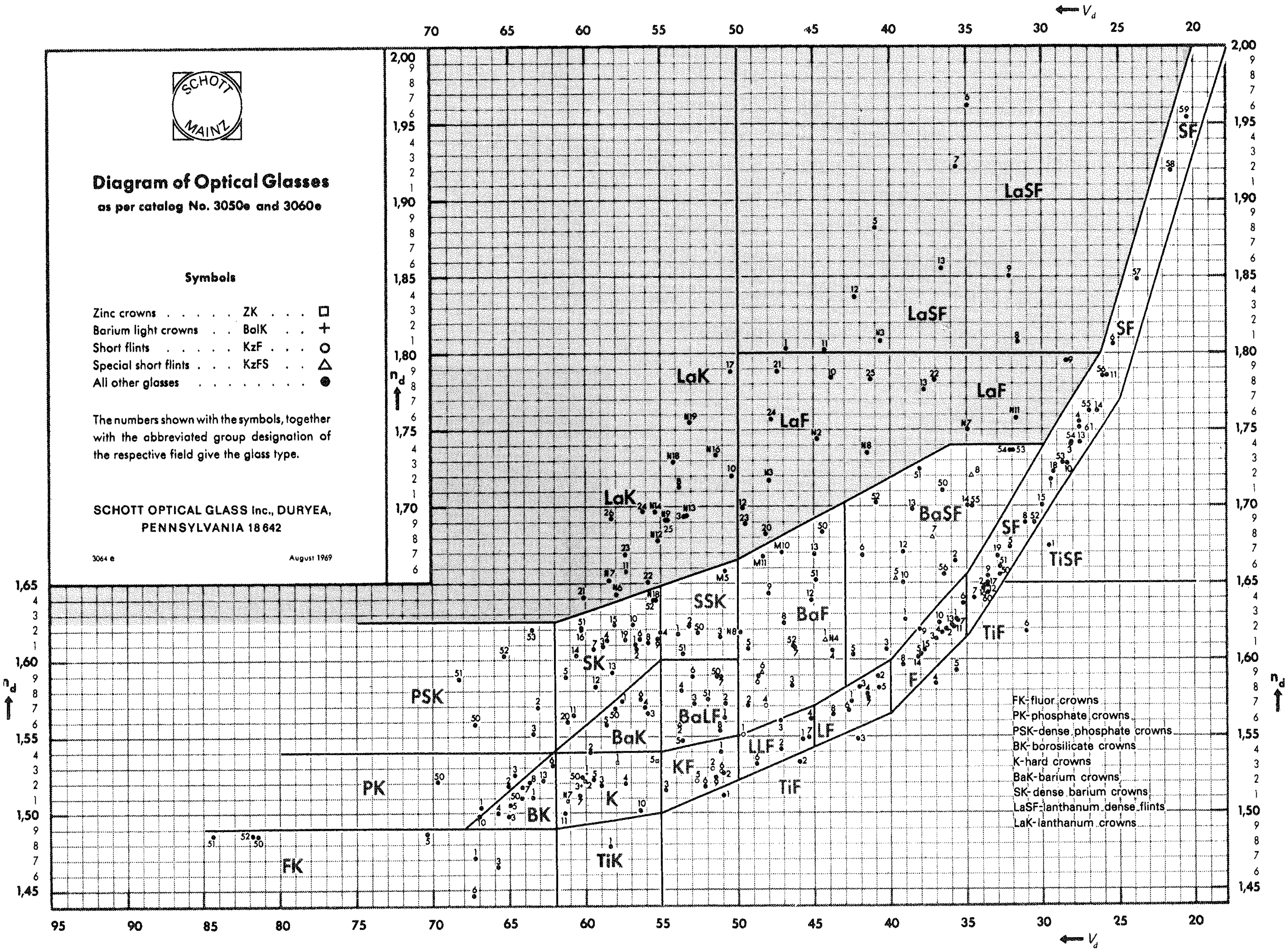
$$\frac{\mathrm{f}_{2y}}{\mathrm{f}_{1y}}=\frac{(n_{1y}-1)\rho_1}{(n_{2y}-1)\rho_2}=-\frac{(n_{2B}-n_{2R})/(n_{2y}-1)}{(n_{1B}-n_{1R})/(n_{1y}-1)} \quad \text{or:}$$

$$\frac{\mathrm{f}_{2y}}{\mathrm{f}_{1y}}=-\frac{V_1}{V_2} \quad \text{(V is dispersive power of the glass)}$$

$$V=\frac{(n_y-1)}{(n_B-n_R)} \quad \text{for BK7 (crown)} \quad V_{BK7}=\frac{(1.51673-1)}{(1.52238-1.51432)}=64.1$$



Supplemental: Dispersive Properties of Optical Glass



Supplemental: Designing an Achromat cont.

Now if we choose the Sodium d-line for yellow light:

$$\frac{f_{2d}}{f_{1d}} = -\frac{V_1}{V_2} \quad \text{or} \quad f_{1d}V_{1d} + f_{2d}V_{2d} = 0 \quad \text{(1) and since:}$$

$$\frac{1}{f_{1d}} + \frac{1}{f_{2d}} = \frac{1}{f_d} \qquad \text{(2) eliminating } f_{2d}\text{:}$$

$$\frac{1}{f_{1d}} - \frac{V_{2d}}{f_{1d}V_{1d}} = \frac{1}{f_d} = \frac{\left(V_{1d} - V_{2d}\right)}{f_{1d}V_{1d}} \quad \text{and so:}$$

$$\frac{1}{f_{1d}} = \frac{V_{1d}}{f_d(V_{1d} - V_{2d})} \qquad \text{(3) the focal length of the 1-st lens and so:}$$

$$\frac{1}{f_{2d}} = \frac{V_{2d}}{f_d(V_{2d} - V_{1d})} \qquad \text{(4) the focal length of the 2-nd lens}$$

Note that $(V_{1d} - V_{2d})$ is chosen to be large (~ 20) so radii are large.

Thus the procedure is to:

1) select the focal length of the two lens combination,

2) select the glass types (maximizing $V_{1d} - V_{2d}$),

3) compute the focal lengths of each lens (f_{1d}, f_{2d}),

4) select a design type (Fraunhofer: $R_1 = -R_2 = -R_3$)

and then compute radii $(R_{11}, R_{12}, R_{21}, R_{22})$

5) Now enter preliminary design into optical design software for optimization.

Example: Designing an Achromat

- **Let's design an achromat with OD = 128mm and FL = 1900mm**
  - **Pos. Crown: N-BK7 (1.51680, 64.17, Neg. Flint: N-SF5 (1.67271, 32.25)**

We've done steps (1) and (2) so using equations 3 & 4 derived in previous slide:

$$\frac{1}{f_{1d}} = \frac{V_{1d}}{f_d(V_{1d} - V_{2d})} = \frac{64.17}{1900(64.17 - 32.25)} = 1.0581 \text{ x } 10^{-3}$$

$$\frac{1}{f_{2d}} = \frac{V_{2d}}{f_d(V_{2d} - V_{1d})} = \frac{32.25}{1900(32.25 - 64.17)} = -5.3176 \text{ x } 10^{-4}$$

Now we have the individual focal lengths. Let's choose the Fraunhofer design (easiest):

$$\frac{1}{f_{1d}} = (n_d - 1)\left(\frac{2}{R}\right)$$ since 1-st lens is bi-convex. So:

$R_1 = -R_2 = 2(n_d - 1) f_{1d} = 1271.5mm$ and for the second lens:

$$\frac{1}{f_{2d}} = (n_d - 1)\left(\frac{1}{R_{21}} - \frac{1}{R_{22}}\right)$$ and so since $R_{21} = R_{12}$ (both negative):

$$\frac{1}{R_{22}} = \frac{1}{R_{21}} - \frac{1}{(n_d - 1) f_{2d}} = -7.8645 \text{ x } 10^{-4} - (-5.3176 \text{ x } 10^{-4})/0.67271$$

$R_{22} = 2.485 \text{ x } 10^5 mm \approx \infty$ (plano, as expected for Fraunhofer design)

So we have all four radii and we could either have this made as is or

enter the preliminary design into optical design software to optimize it further.

Supplemental: Analytical Ray Tracing: Exact

- **Geometry and Formulae**
  - **First derive the exact formulae and then do an example**

Law of sines to MTC gives:

$$\frac{\sin(\pi-\phi)}{r+s}=\frac{\sin\theta}{r}$$

Since sine of suppl. angle = sine of angle:

$$\frac{\sin\phi}{r+s}=\frac{\sin\theta}{r} \quad \text{and so:}$$

$$\sin\phi=\frac{r+s}{r}\sin\theta \qquad (1)$$

$$\sin\phi'=\frac{n}{n'}\sin\phi \quad \text{(Snell's law)} \quad (2)$$

Sum of angles in MTM' must equal π:

$$\theta+(\pi-\phi)+\phi'+(-\theta')=\pi \quad (\text{note } \theta' \text{ is negative) and so:}$$

$$\theta'=\phi'+\theta-\phi \quad \text{(angle where ray crosses axis)} \quad (3)$$

To find where the ray crosses the axis:

$$\frac{-\sin\theta'}{\mathrm{r}}=\frac{\sin\phi'}{s'-r} \quad \text{(law of sines) and so we have:}$$

$$s'=r-r\left(\frac{\sin\phi'}{\sin\theta'}\right) \qquad (4)$$

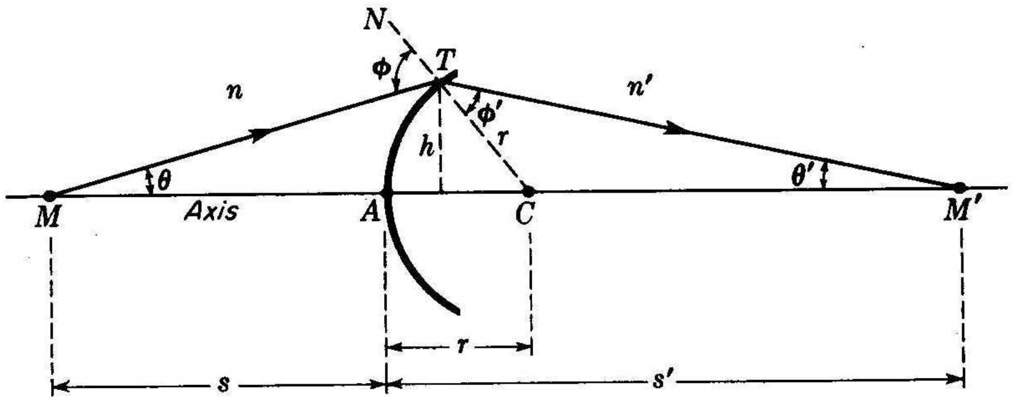
But for the special case of rays parallel to axis:

$$\sin\phi=\frac{h}{r} \qquad (5)$$

For TCM' sum of interior angles = exterior angle:

$$\theta'=\phi'-\phi \qquad (6)$$

The 6 numbered equations are used for raytracing.



Supplemental: Analytical Ray Tracing: Example #1

- **Ray-tracing of parallel rays refracting at a single spherical surface**
  - **r = +5cm, n = 1.00, n' = 1.672**

Eqn.	Unknown	H = 1.0	H = 2.0	H = 3.0
5	sin ϕ			
2	sin ϕ'			
Trig.	ϕ ϕ'			
6	θ'			
Trig.	sin θ'			
4	r - s'			
	s'			

Supplemental: Analytical Ray Tracing: Example #1

- **Ray-tracing of parallel rays refracting at a single spherical surface**
  - **R = +5cm, n = 1.00, n' = 1.672**

Eqn.	**Unknown**	**H = 1.0**	**H = 2.0**	**H = 3.0**
5	sin ϕ	0.200000	0.400000	0.600000
2	sin ϕ'	0.1196172	0.2392344	0.3588517
Trig.	ϕ ϕ'	11.536959 6.8700110	23.578178 13.841356	36.869898 21.029692
6	θ'	-4.6669480	-9.7368220	-15.840206
Trig.	sin θ'	-0.0813636	-0.1691228	-0.2729554
4	r - s'	-7.3507809	-7.0728015	-6.5734494
	s'	12.35081	12.072802	11.573449

Supplemental: Analytical Ray Tracing: Approximate

- **Geometry and Formulae**

$n_{i1}\theta_{i1} = n_{t1}\theta_{t1}$ Snell's law (for small angles) so:

$n_{i1}(\alpha_{i1} + \alpha_1) = n_{t1}(\alpha_{t1} + \alpha_1)$ and since $\alpha_1 = \dfrac{y_1}{R_1}$ (small angles) then

$n_{i1}(\alpha_{i1} + \dfrac{y_1}{R_1}) = n_{t1}(\alpha_{t1} + \dfrac{y_1}{R_1})$ or:

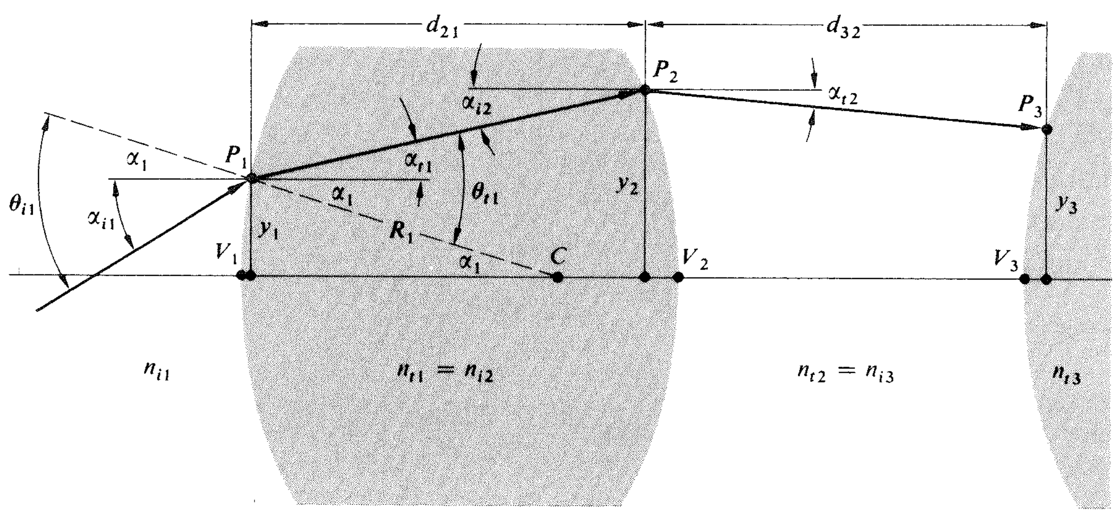
$n_{t1}\alpha_{t1} = n_{i1}\alpha_{i1} - \left(\dfrac{n_{t1} - n_{i1}}{R_1}\right)y_1$ (refraction equation)

$n_{t1}\alpha_{t1} = n_{i1}\alpha_{i1} - D_1 y_1$ (refraction equation)

Next consider the propagation between surfaces:

$y_2 = y_1 + d_{21}\alpha_{t1}$ (transfer equation (small angles): tan $\alpha_{t1} \approx \alpha_{t1}$)

Successive application will trace rays through a system.



Supplemental: Analytical Ray Tracing cont.

- **Matrix Methods**
  - **Successive application of refraction equation and transfer equation**
  - **Repeat for the second surface and apply the transfer equation**
  - **Ray matrix can be thought of as an "operator" that transforms incident ray into the refracted ray. Same for the transfer matrix.**

$n_{t1}\alpha_{t1} = n_{i1}\alpha_{i1} - D_1 y_{i1}$ (refraction equation)

$\mathrm{y}_{\mathrm{t1}} = 0 + y_{i1}$ (transfer equation for 1-st surface)

Rewriting in matrix form:

$$\begin{bmatrix} n_{t1}\alpha_{t1} \\ y_{t1} \end{bmatrix} = \begin{bmatrix} 1 & -D_1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} n_{i1}\alpha_{i1} \\ y_{i1} \end{bmatrix} \quad \text{or :}$$

$r_{t1} = R_1 r_{i1}$ where r is a "ray matrix" and R is the refraction matrix

Recall the transfer equation for the 2-nd surface:

$\mathrm{y}_{\mathrm{t2}} = d_{21}\alpha_{t1} + y_{i1}$

The corresponding matrix equation is:

$$\begin{bmatrix} n_{i2}\alpha_{i2} \\ y_{i2} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ d_{21}/n_{t1} & 1 \end{bmatrix} \begin{bmatrix} n_{t1}\alpha_{t1} \\ y_{t1} \end{bmatrix} \quad \text{or :}$$

$r_{i2} = T_{21} r_{t1}$

Note that the symmetry of the matrix equation formulation. Thus:

$r_{i2} = T_{21} R_1 r_{i1}$

Supplemental: Analytical Ray Tracing cont.

- **Define a <u>system matrix</u>**
  - **Product of refraction, transfer, refraction matrices can be defined as the <u>system matrix</u>**
  - **Some elements of system matrix have meaning:**

    $\mathbf{a_{12} = -1/f}$
  - **A multiple lens system will have a more complicated system matrix but it is composed of just successive application of ray-transfer-ray-transfer operations (see Tessar example in textbook).**
  - **Mirrors can be treated similarly (see text).**

The product of the refraction and transfer matrices:

$$A = R_2 T_{21} R_1$$

is known as the system matrix of the lens:

$$A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \quad \text{Thus:}$$

$$\mathrm{A} = \begin{bmatrix} 1 & -D_2 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ d_{21}/n_{t1} & 1 \end{bmatrix} \begin{bmatrix} 1 & -D_1 \\ 0 & 1 \end{bmatrix}$$

Expanding D and letting $\mathrm{d_{21} = d_l}$ and $\mathrm{n_{t1} = n_l}$ gives:

$$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} = \begin{bmatrix} 1 - \dfrac{D_2 d_l}{n_l} & -D_1 - D_2 + \dfrac{D_1 D_2 d_l}{n_l} \\ d_l / n_l & 1 - \dfrac{D_l d_l}{n_l} \end{bmatrix}$$

Note that the elements of the matrix correspond to properties of the lens.

$$-a_{12} = D_1 + D_2 - D_1 D_2 d_l / n_l = 1/f$$

To follow the rays from the object plane to the lens:

$$r_{I1} = T_{1O} r_O$$

To follow the rays from the lens to the image plane:

$$r_I = T_{I2} r_{t2} \quad \text{and so we now have:}$$

$$r_I = T_{I2} A_{21} T_{1O} r_O$$

See the example in the text of how a Tessar lens can be modeled.