Phys 2310 Mon. Oct. 30, 2017
Today's Topics

- **Begin Modern Optics Ch. 2: The Nature of Polarized Light**
- **Reading for Next Time**

Reading this Week

By Wed.:

Begin Ch. 2 of Modern Optics (2.1 – 8.11) Nature of Polarized Light, Dichroism and Birefingence

Homework this Week

Chapter 2 Homework (due Monday Nov. 6)
#6, 7, 8, 15, 17

Chapter 2: Polarization

- **Nature of Polarized Light**
  - We’ve seen that a single light wave will oscillate in a plane.
    - In general other waves are unrelated so the oscillations will occur in all (random) planes.
    - Fresnel equations show that reflection of waves oscillating parallel to surface is enhanced.
  - Each wave is polarized by definition
    - Totality of waves from a light source is in general unpolarized
      - Physical interaction between light and matter can polarize light

Chapter 2: Polarization

- Linear, Circular and Elliptical Polarization

Consider two waves:

$E_x(z,t) = \hat{i} E_{0x} \cos(kz - \omega t)$ and $E_y(z,t) = \hat{j} E_{0y} \cos(kz - \omega t + \varepsilon)$

The resulting wave is the sum:

$E(z,t) = E_x(z,t) + E_y(z,t)$ [in phase if $\varepsilon = n(\pm 2\pi)$]

The plane of polarization of the resultant wave depends on the sum.

Circular polarization results when $E_{0x}(z,t) = E_{0y}(z,t)$ and $\varepsilon = -\pi/2 + m(2\pi)$

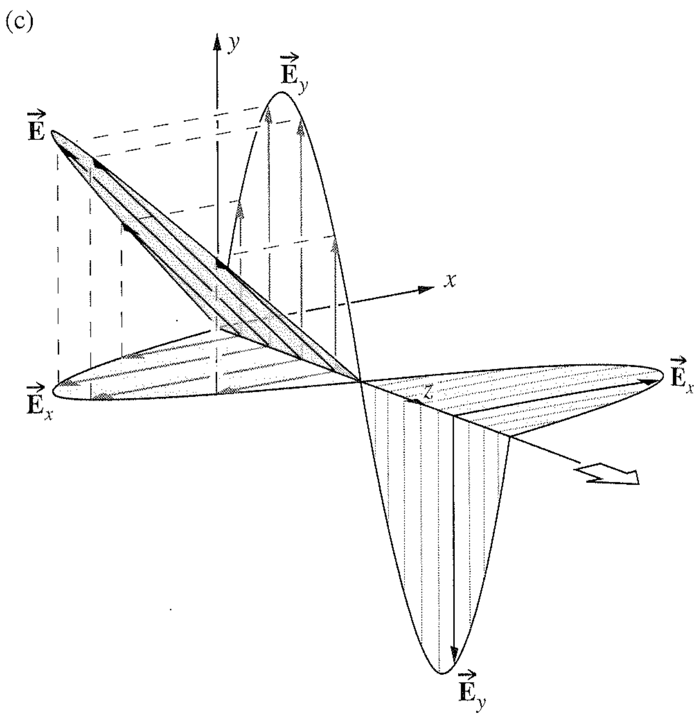
Amplitude of the result is fixed but the orientation is not: polarization rotates clockwise when seen by observer looking back at source (right circular polarization).

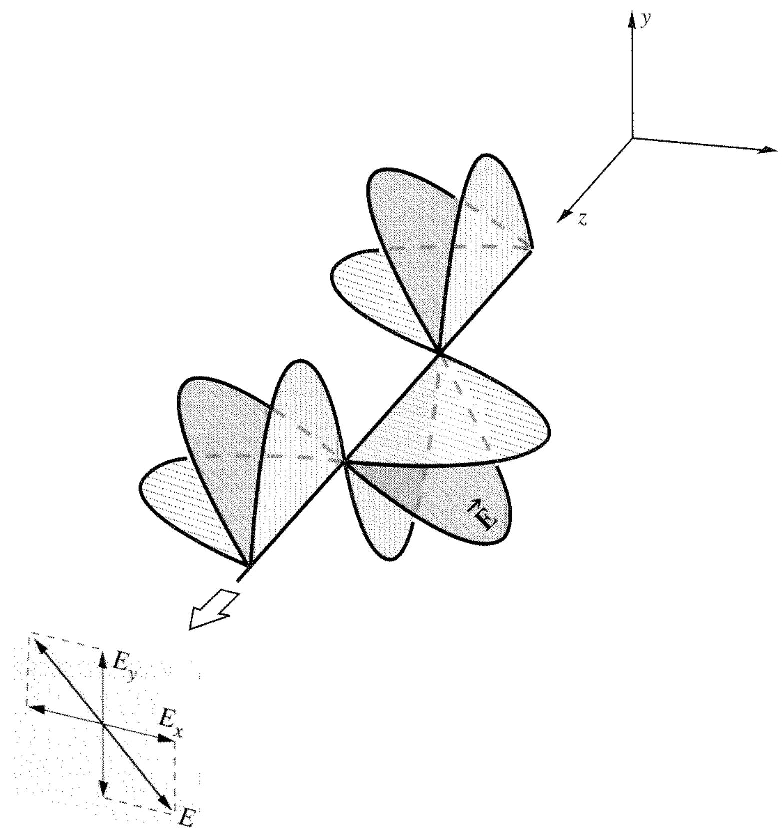
Similarly a phase difference of $\varepsilon = \pi/2 + m(2\pi)$ or left circular polarization.

The general case is one of elliptical polarization (linear and circular are special cases).

Chapter 2: Polarization

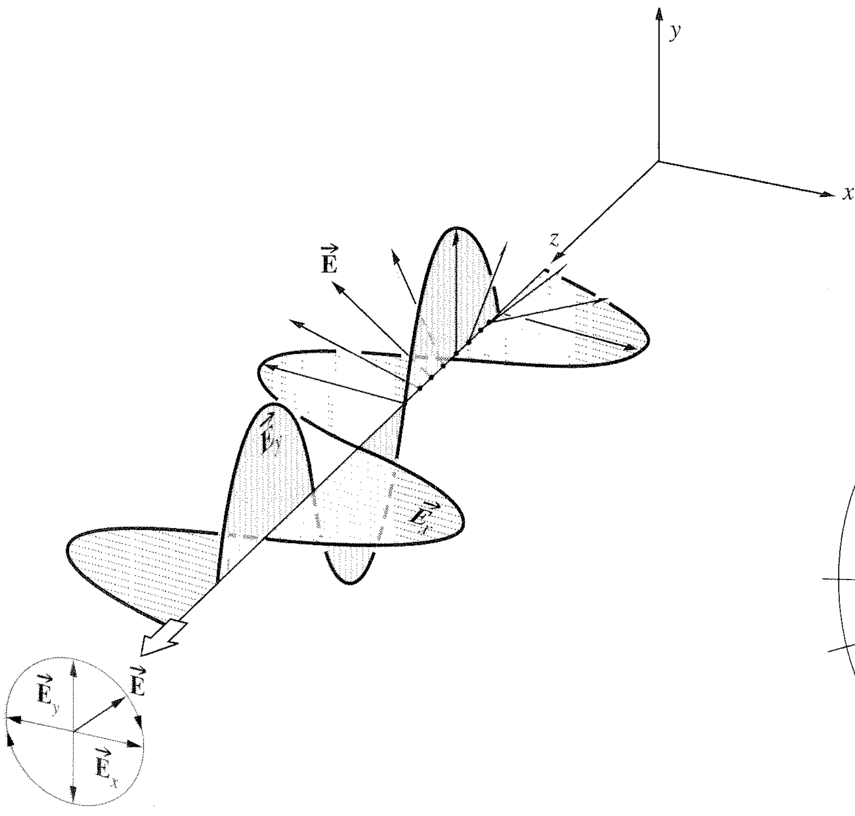
- Linear Polarization (graphical representation)

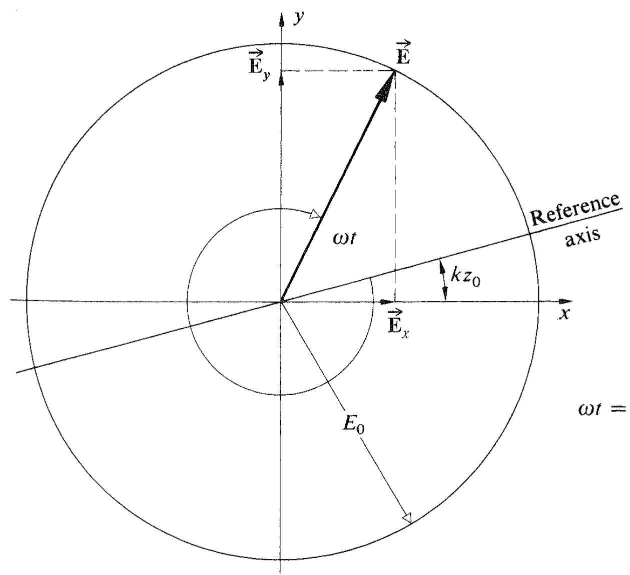


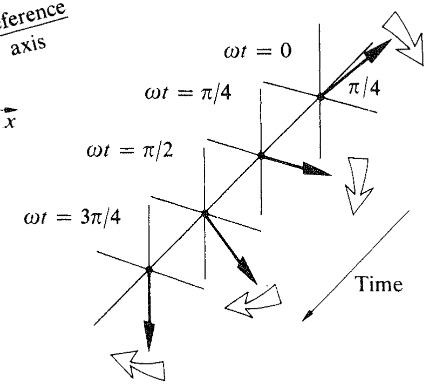


Chapter 2: Polarization

- Circular Polarization (graphical representation)

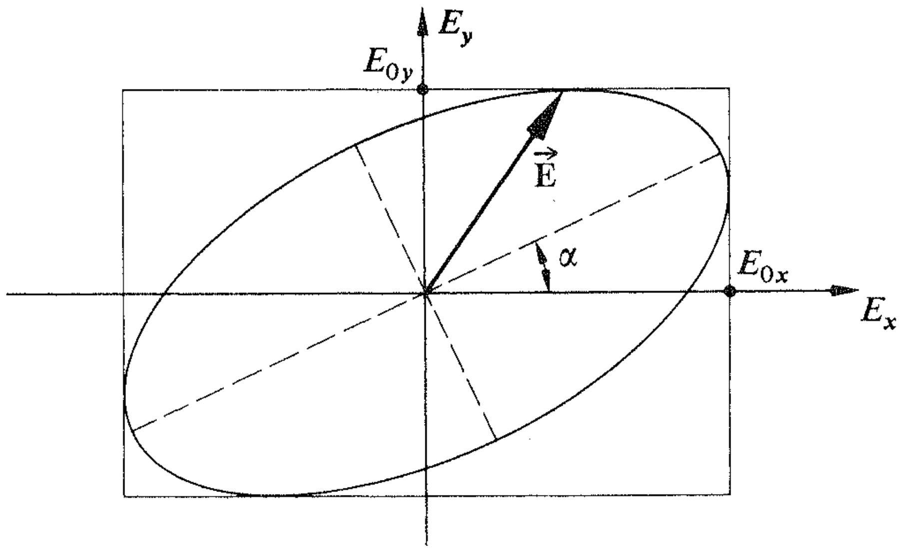


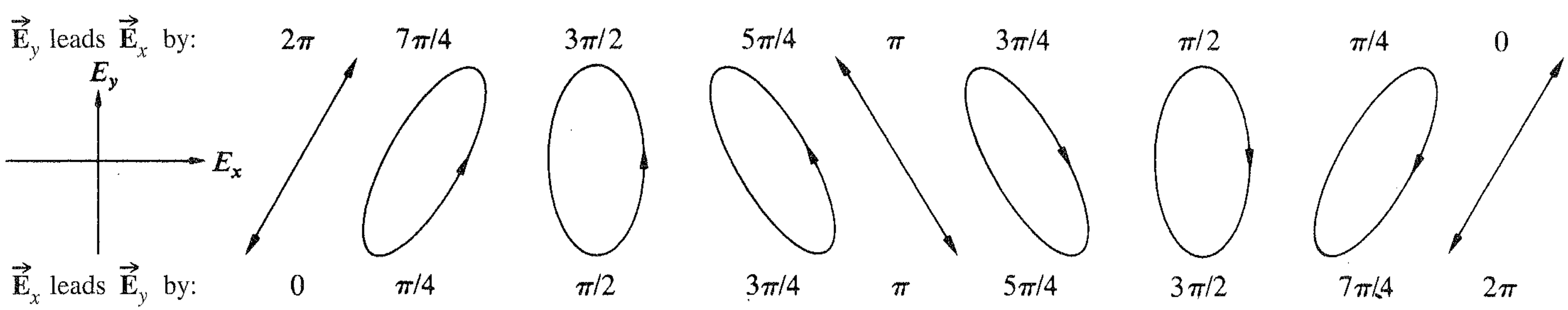




Chapter 2: Polarization

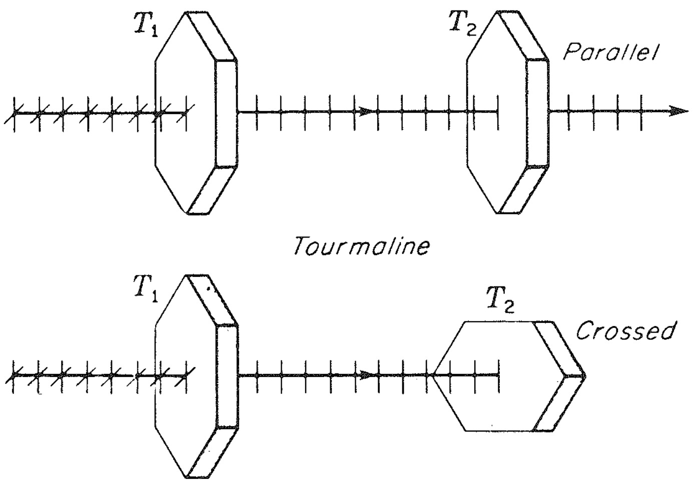
- Circular Polarization (graphical representation)





Chapter 2: Polarization

- **Polarizers**
  - **How do we generate and manipulate polarized light?**
  - **We need some sort of device.**
  - **Four mechanisms:**
    - **Dichroism: selective absorption of light according to polarization**
    - **Reflection or Scattering: makes use of polarization-dependence**
    - **Birefringence: Use crystals with differing index of refraction with polarization**
  - **Dichroism (Polaroid filter or tourmaline crystal)**

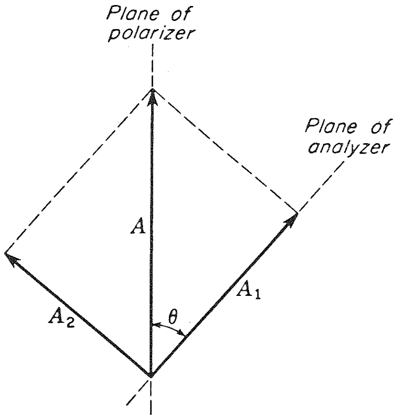


The intensity of light passed by an analyzer is (Malus' Law):

$I(\theta) = I(0)\cos^2\theta$ (empirical)

Since $I(0) = \dfrac{c\varepsilon_0}{2} E_{01}{}^2 / 2$ we conclude analyzers operate on wave amplitude:

$A_p = A\cos\theta$



Chapter 2: Polarization

- Devices for Inducing/Measuring Polarization
  - Dichroism:
    - Wire grid passes perpendicular E-field (mid-infrared and longer)
    - Dichroic crystals (e.g. tourmaline)
      - Polarized light can excite electrons in crystal to oscillate in one direction (strongly absorbed) and not the other.
    - Polaroid (stretched sheet of plastic)
      - Polarized light can excite electrons in molecule to oscillate in one direction (strongly absorbed) and not the other.
  - Reflection or Scattering:
    - Scattering of light by molecules, e.g., air can produce partially polarized light
    - Reflection can produce polarized light too (Fresnel equations)
  - Birefringence:

Chapter 2: Polarization

- Polarization by Reflection
  - Brewster's Law:
    - Reflection can produce polarized light too (Fresnel equations)
    - Recall that the amplitude of the reflected wave depends on the angle of incidence.
      - If the E-field oscillates in the plane of incidence it can be shown that the reflected intensity can be zero. Specifically this occurs when:

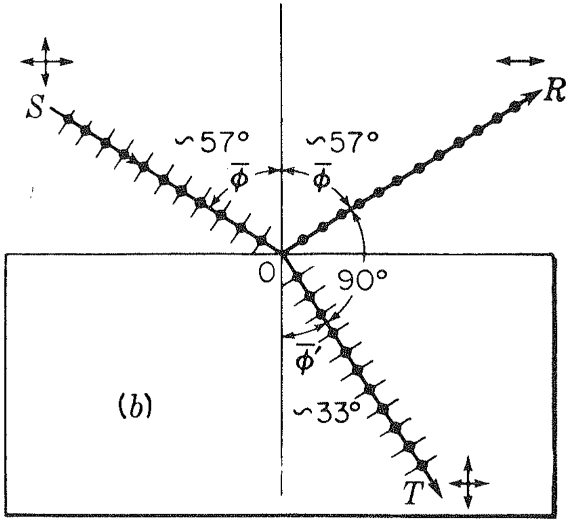
        $\theta_r + \theta_t = 90^o$ (Brewster's Law)

    Thus the component oscillating perpendicular to the plane of incidence will be maximally polarized at θ_p

$$n_i \sin\theta_p = n_t \sin\theta_t \quad \text{and since } \theta_t = 90^o - \theta_p$$

$$n_i \sin\theta_p = n_t \cos\theta_p \quad \text{and so:}$$

$$\tan\theta_p = n_t / n_i$$

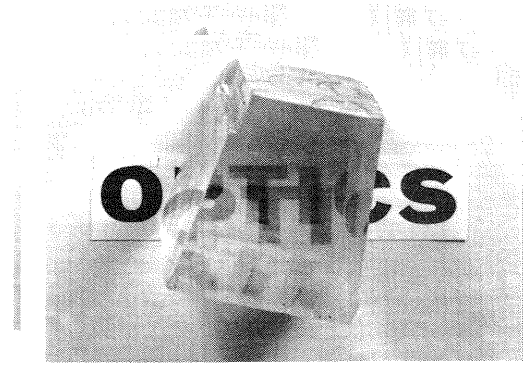


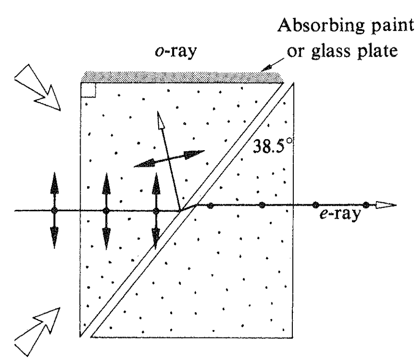
Chapter 2: Polarization

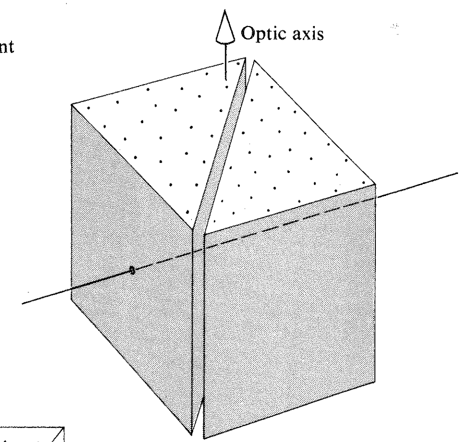
- Devices for Inducing/Measuring Polarization
  - Birefringence:
    - Crystals can have index of refractions that vary with direction if the crystalline structure is not symmetric.
    - Electric forces between atoms is asymmetric and so index of refraction has two values depending on polarization (Birefringent or Double Refraction).
      - Corresponding shift in resonance λ so crystal can refract light in one plane much more than another.
    - Examples include Calcite or Quartz
      - The plane of incidence on the crystal defines the "ordinary ray" and the other ray is called the "extraordinary ray."

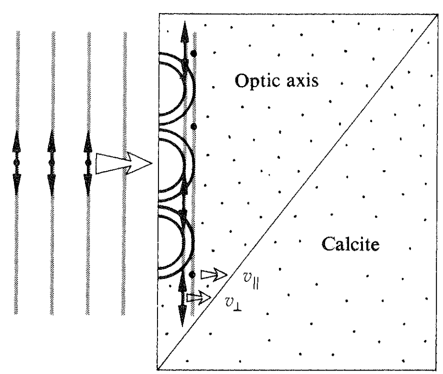
Calcite: $n_o = 1.65836$

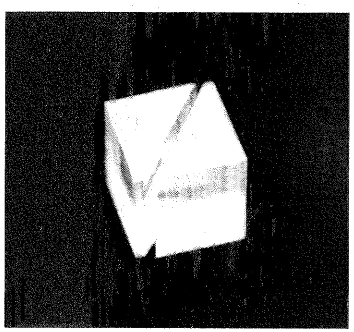
$n_e = 1.48641$





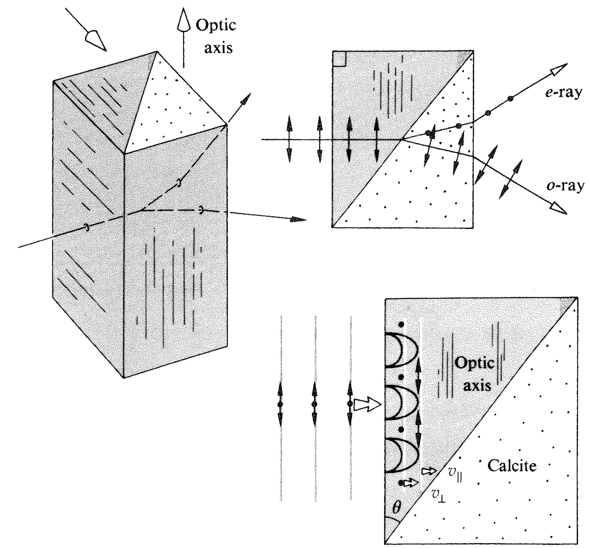


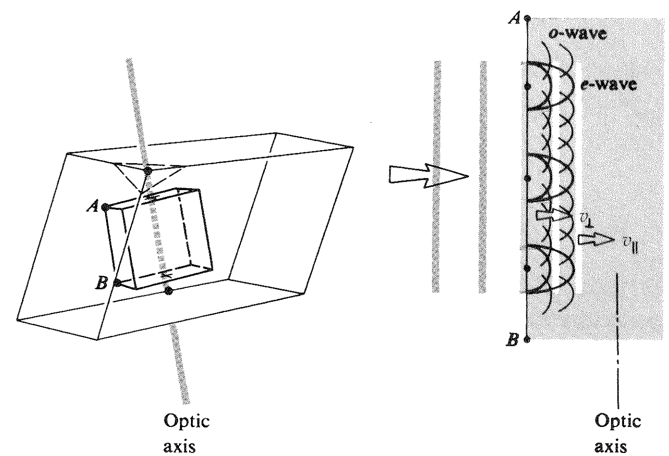




Chapter 2: Polarization

- Devices for Inducing/Measuring Polarization
  - Birefringence:
    - Double prism can separate two polarizations
    - Change orientation and we can retard (phase-shift) one with respect to another
    - Controlling the thickness can produce elliptical polarization or even circular for a given λ.



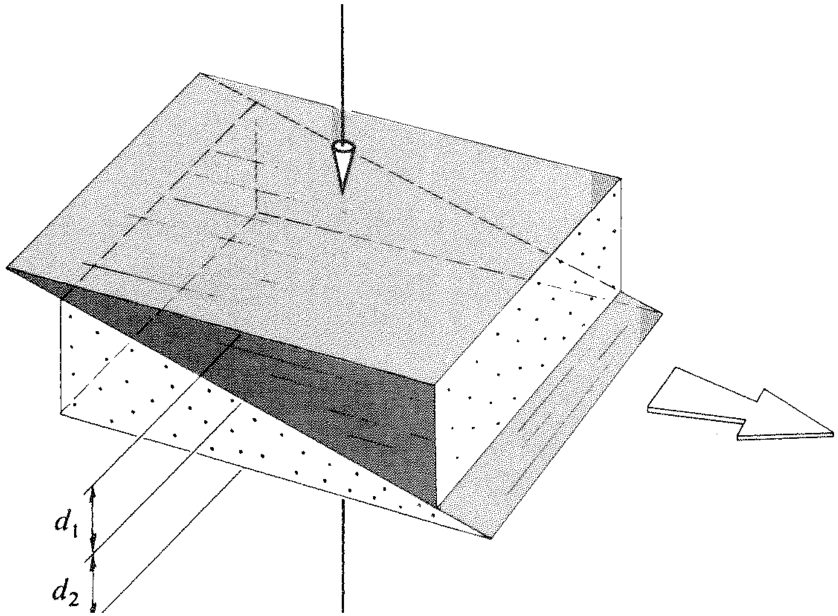


Chapter 2: Polarization

- Devices for Inducing/Measuring Polarization
  - Wave Plates:
    - Recall that by cutting a calcite plate to a specific thickness we can control the relative phases of the two polarizations.
    - Path length difference: $\Delta\phi = 2\pi d(n_o - n_e)/\lambda$
      - A full-wave plate will bring the e- and o-waves back into phase, i.e., a phase shift of 2π.
        - » Seldom used except in very specific circumstances (see text).
      - A half-wave plate produces a phase shift of π (180^o). Result is an inversion in the axes of any elliptically polarized light.
        - » Mica (muscovite) works well.
      - A quarter-wave plate produces a phase shift of $\pi/2$ (90^o).
        - » Used to convert linear polarization to elliptical and vice versa

Chapter 2: Polarization

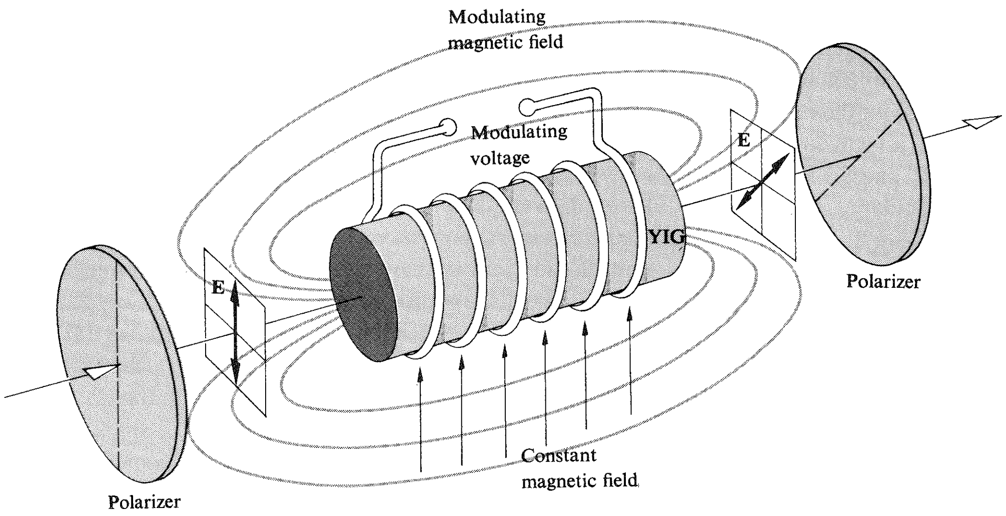
- Devices for Inducing/Measuring Polarization
  - Compensators:
    - Cutting a wedge-prism from calcite or quartz allows continuous control of phase shifts
      - Babinet compensator
      - Soleil compensator



Chapter 2: Polarization

- Devices for Inducing/Measuring Polarization
  - Faraday Effect:
    - Plane of polarization can be rotated when a strong B field is applied to some transparent substances
      - Verdet constant (ν, see table 8.2)
      - Used to modulate light via an electrical signal.
    - Seen in Interstellar medium
      - Radio waves from Pulsars interact with free electrons in B field.
        - » Amount of polarization used to estimate B field.

$$\beta = \nu B d$$

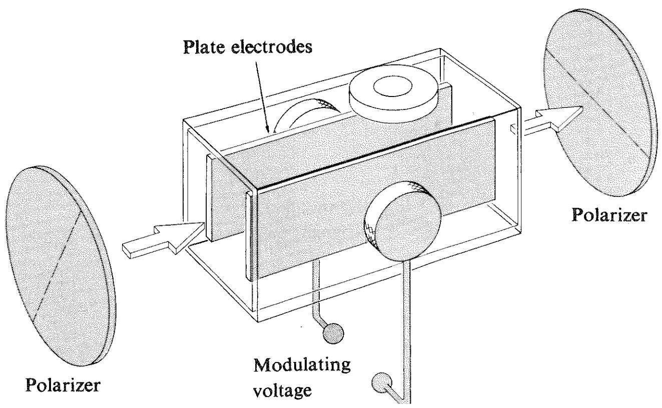


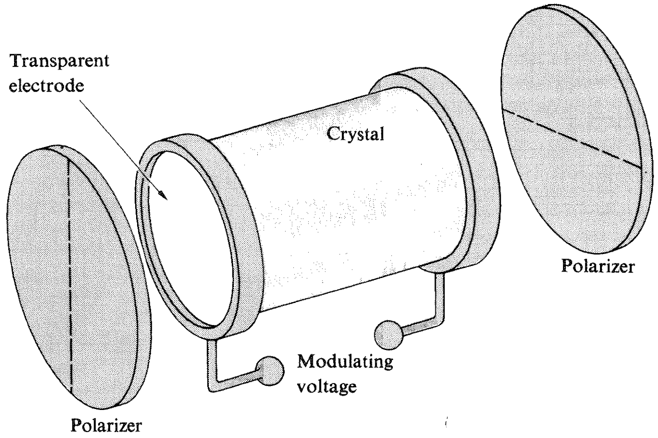
Chapter 2: Polarization

- Devices for Inducing/Measuring Polarization
  - Kerr and Pockels Effects:
    - A strong E-field can make some substances birefringent.

      $\Delta n = \lambda K E^2$ where K is the Kerr constant (table 8.3)

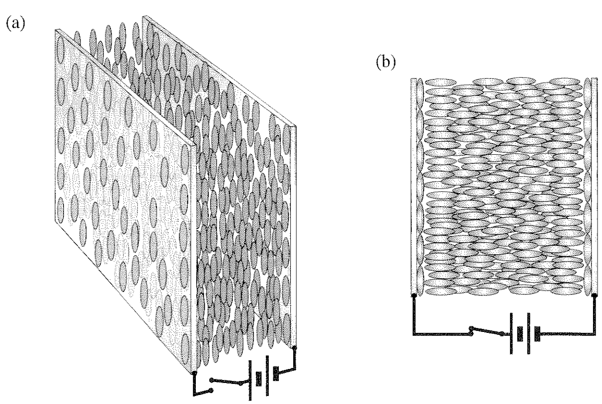
      - If placed between crossed polarizers the output intensity can be rapidly modulated (optical switch at several GHz).
      - Useful as high-speed shutter for imaging or video.
      - A Pockel Cell is similar (response time in nanoseconds)

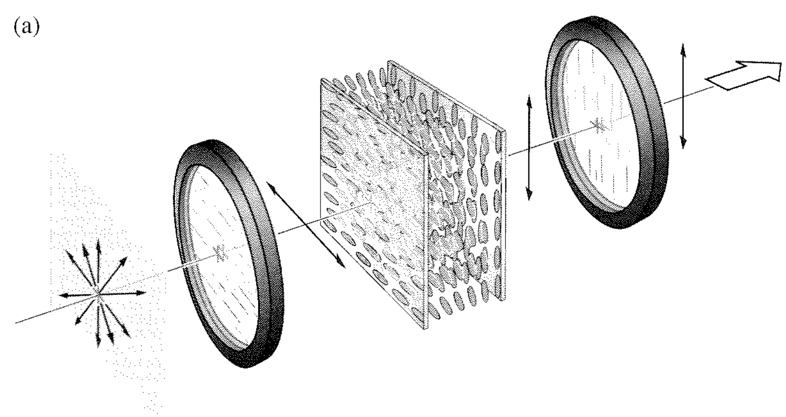




Chapter 2: Polarization

- Devices for Inducing/Measuring Polarization
  - Liquid crystals:
    - Elongated transparent crystals in a solution that can be aligned via E-field
      - Result is birefringence that is controlled when a voltage is applied.
      - Strength and wavelength of individual cells can be controlled to produced a digital display (e.g. clock) or a flat-screen display.





Mathematical Model of Polarization-I

- Stokes Parameters:
  - Used to specify the polarization state of a beam of light
  - Consider 4 filters (each transmits ½ the light)
    - 1 transmits all polarizations equally, i.e., ½ the light (I_0)
    - 2 and 3 are linear polarizers (I_1 = horizontal, $I_2 = +45^o$)
    - 4-th is a circular polarizer opaque to L (left) states (I_3)
  - We then measure the intensity passed by each <u>one at a time</u>.
  - We define Stokes Parameters as:

$$S_0 = 2I_0 \quad \text{(just the incident intensity)}$$

$$S_1 = 2I_1 - 2I_0 \quad (\text{Horizontal if } S_1 \geq 0,\ \text{Vertical if } S_1 \leq 0)$$

$$S_2 = 2I_1 + 2I_0 \quad (\text{If } S_1 = 0, \text{elliptical or circular}: +45^o \text{ if } S_2 \geq 0,\ -45^o \text{ if } S_2 \leq 0)$$

$$S_3 = 2I_3 - 2I_0 \quad (\text{Right handed}: S_3 \geq 0, \text{Left handed}: S_3 \leq 0, \text{or neither}: S_3 = 0)$$

Mathematical Model of Polarization-II

$$S_0 = 2I_0 \qquad \text{(just the incident intensity)}$$

$$S_1 = 2I_1 - 2I_0 \qquad (\text{Horizontal if } S_1 \geq 0,\ \text{Vertical if } S_1 \leq 0)$$

$$S_2 = 2I_1 + 2I_0 \qquad (\text{If } S_1 = 0, \text{elliptical or circular}: +45^{\circ} \text{ if } S_2 \geq 0,\ -45^{\circ} \text{ if } S_2 \leq 0)$$

$$S_3 = 2I_3 - 2I_0 \qquad (\text{Right handed}: S_3 \geq 0, \text{Left handed}: S_3 \leq 0, \text{or neither}: S_3 = 0)$$

- Stokes Parameters as an array (matrix) of 4 numbers:
  - If we divide each Stokes Parameter by S_o we normalize their value
    - Unpolarized light: $S_o = 1$, $S_1 = S_2 = S_3 = 0$ (1,0,0,0)
    - Horizontally polarized light: (1,1,0,0)
    - Vertically polarized light: (1,-1,0,0)
    - Polarized light at +45^o: (1,0,1,0)
    - Polarized light at -45^o: (1,0,-1,0)
    - Right-hand polarized light: (1,0,0,1)
    - Left-hand polarized light: (1,0,0,-1)
  - With the degree of polarization: $V = (S_1^2 + S_2^2 + S_3^2)^{1/2}$

Mathematical Model of Polarization - III

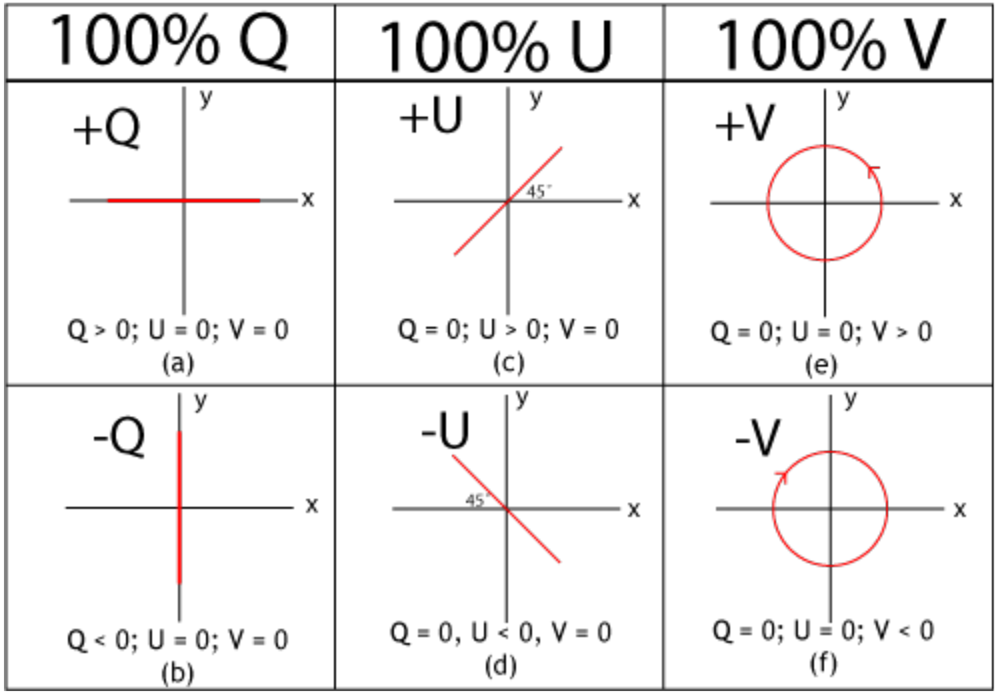
- **Figure at Right Illustrates Stokes Parameters**
  - **In the x-y plane:**

$$I = |E_x|^2 + |E_y|^2$$

$$Q = |E_x|^2 - |E_y|^2$$

$$U = 2\,\mathrm{Re}(E_x E_y^*)$$

$$V = 2\,\mathrm{Im}(E_x E_y^*)$$



Jones Vectors

- A short-hand version but only for purely polarized light was invented by Jones:

$\tilde{E} = \begin{bmatrix} E_{0x}e^{i\phi_x} \\ E_{0y}e^{i\phi_y} \end{bmatrix}$ and so if the amplitudes and phases are the same and we have polarized light we can factor out the ampltudes :

$\tilde{E} = E_{0x}\begin{bmatrix} e^{i\phi_x} \\ e^{i\phi_x} \end{bmatrix} = E_{0x}e^{i\phi_x}\begin{bmatrix} 1 \\ 0 \end{bmatrix}$ a horz. polarized wave, and so if the incident intensity is normalized the this is simplified further :

$\tilde{E}_h = 1\begin{bmatrix} 1 \\ 0 \end{bmatrix}$ where the 1 is just to explicitly show the normalization. We can now look at the matrices for other cases :

$\tilde{E}_v = 1\begin{bmatrix} 0 \\ 1 \end{bmatrix}$

$\vec{\mathrm{E}}_{45} = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ 1 \end{bmatrix}$ where $\frac{1}{\sqrt{2}}$ specifically indicates that E^2 is the intensity. A right circular polarization has $\delta\phi = \pi/2$:

$\tilde{\mathrm{E}}_{\mathrm{R}} = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ -i \end{bmatrix}$ and so left circular polarization would be : $\tilde{\mathrm{E}}_{\mathrm{L}} = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ i \end{bmatrix}$ See page 377 for other examples.

Stokes and Jones Vectors

- So now we see that a single or group of analyzers is just a matrix that operates on the incident beam:

$\tilde{E}_t = A\tilde{E}_i$ or upon expansion :

$$\begin{bmatrix} \tilde{E}_{tx} \\ \tilde{E}_{ty} \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} \tilde{E}_{ix} \\ \tilde{E}_{iy} \end{bmatrix}$$

See the book for various examples.

TABLE 8.6 Jones and Mueller matrices.

Linear optical element	Jones matrix	Mueller matrix
Horizontal linear polarizer ↔	$\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$	$\frac{1}{2}\begin{bmatrix} 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$
Vertical linear polarizer ↕	$\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$	$\frac{1}{2}\begin{bmatrix} 1 & -1 & 0 & 0 \\ -1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$
Linear polarizer at +45° ⤢	$\frac{1}{2}\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$	$\frac{1}{2}\begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$
Linear polarizer at −45° ⤡	$\frac{1}{2}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$	$\frac{1}{2}\begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$
Quarter-wave plate, fast axis vertical	$e^{i\pi/4}\begin{bmatrix} 1 & 0 \\ 0 & -i \end{bmatrix}$	$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$
Quarter-wave plate, fast axis horizontal	$e^{i\pi/4}\begin{bmatrix} 1 & 0 \\ 0 & i \end{bmatrix}$	$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \end{bmatrix}$
Homogeneous circular polarizer right ↻	$\frac{1}{2}\begin{bmatrix} 1 & i \\ -i & 1 \end{bmatrix}$	$\frac{1}{2}\begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 \end{bmatrix}$
Homogeneous circular polarizer left ↺	$\frac{1}{2}\begin{bmatrix} 1 & -i \\ i & 1 \end{bmatrix}$	$\frac{1}{2}\begin{bmatrix} 1 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 1 \end{bmatrix}$

Chapter 2 Key Concepts

Linear, Elliptical, Circular Polarization
Polarizers & Malus' Law
Birefringence of Crystals
  Waveplates
Polarization by Scattering & Reflection
Photoelasticity & Modulators
Liquid Crystals
Stokes Parameters & Matrix Models

Chapter 2 Key Equations

Mathematical Form of a Polarized Wave:

Linear:

Circular:

Elliptical:

Malus' Law:

Brewster's Law:

Degree of Polarization:

Optical Path Length from Retarders:

Faraday Effect:

Kerr Effect:

Stokes, Jones & Muller Parameters/Vectors:

Homework this Week

Chapter 2 Homework (due Monday Nov. 6)
#6, 7, 8, 15, 17