Phys 2310 Mon. Oct. 30, 2017

Today’s Topics

• Begin Modern Optics Ch. 2: The Nature of Polarized Light

• Reading for Next Time
Reading this Week

By Wed.:

Begin Ch. 2 of Modern Optics (2.1 – 8.11) Nature of Polarized Light, Dichroism and Birefringence
Homework this Week

Chapter 2 Homework (due Monday Nov. 6)
#6, 7, 8, 15, 17
Chapter 2: Polarization

• **Nature of Polarized Light**
  – We’ve seen that a single light wave will oscillate in a plane.
    • In general other waves are unrelated so the oscillations will occur in all (random) planes.
    • Fresnel equations show that reflection of waves oscillating parallel to surface is enhanced.
  – Each wave is polarized by definition
    • Totality of waves from a light source is in general unpolarized
      – Physical interaction between light and matter can polarize light
Chapter 2: Polarization

• Linear, Circular and Elliptical Polarization

Consider two waves:

\[ E_x(z, t) = \hat{i} E_{0x} \cos(kz - \omega t) \quad \text{and} \quad E_y(z, t) = \hat{j} E_{0y} \cos(kz - \omega t + \varepsilon) \]

The resulting wave is the sum:

\[ E(z, t) = E_x(z, t) + E_y(z, t) \quad [\text{in phase if} \quad \varepsilon = n(\pm 2\pi)] \]

The plane of polarization of the resultant wave depends on the sum.

Circular polarization results when \( E_{0x}(z, t) = E_{0y}(z, t) \) and \( \varepsilon = -\pi / 2 + m(2\pi) \)

Amplitude of the result is fixed but the orientation is not: polarization rotates clockwise when seen by observer looking back at source (right circular polarization). Similarly a phase difference of \( \varepsilon = \pi / 2 + m(2\pi) \) or left circular polarization. The general case is one of elliptical polarization (linear and circular are special cases).
Chapter 2: Polarization

• Linear Polarization (graphical representation)
Chapter 2: Polarization

- Circular Polarization (graphical representation)
Chapter 2: Polarization

- Circular Polarization (graphical representation)
Chapter 2: Polarization

- **Polarizers**
  - How do we generate and manipulate polarized light?
  - We need some sort of device.
  - Four mechanisms:
    - **Dichroism**: selective absorption of light according to polarization
    - **Reflection or Scattering**: makes use of polarization-dependence
    - **Birefringence**: Use crystals with differing index of refraction with polarization
  - **Dichroism** (Polaroid filter or tourmaline crystal)

The intensity of light passed by an analyzer is (Malus' Law):

\[ I(\theta) = I(0) \cos^2 \theta \] (empirical)

Since \( I(0) = \frac{cE_0}{2} E_{01}^2 / 2 \) we conclude analyzers operate on wave amplitude:

\[ A_p = A \cos \theta \]
Chapter 2: Polarization

• Devices for Inducing/Measuring Polarization
  – Dichroism:
    • Wire grid passes perpendicular E-field (mid-infrared and longer)
    • Dichroic crystals (e.g. tourmaline)
      – Polarized light can excite electrons in crystal to oscillate in one direction (strongly absorbed) and not the other.
    • Polaroid (stretched sheet of plastic)
      – Polarized light can excite electrons in molecule to oscillate in one direction (strongly absorbed) and not the other.
  – Reflection or Scattering:
    • Scattering of light by molecules, e.g., air can produce partially polarized light
    • Reflection can produce polarized light too (Fresnel equations)
  – Birefringence:
Chapter 2: Polarization

• Polarization by Reflection
  – Brewster’s Law:
    • Reflection can produce polarized light too (Fresnel equations)
    • Recall that the amplitude of the reflected wave depends on the angle of incidence.
      – If the E-field oscillates in the plane of incidence it can be shown that the reflected intensity can be zero. Specifically this occurs when:
        \[ \theta_r + \theta_t = 90^\circ \] (Brewster’s Law)
    Thus the component oscillating perpendicular to the plane of incidence will be maximally polarized at \( \theta_p \)

\[ n_i \sin \theta_p = n_t \sin \theta_t \quad \text{and since} \quad \theta_t = 90^\circ - \theta_p \]
\[ n_i \sin \theta_p = n_t \cos \theta_p \quad \text{and so:} \]
\[ \tan \theta_p = n_t / n_i \]
Chapter 2: Polarization

- Devices for Inducing/Measuring Polarization
  - Birefringence:
    - Crystals can have index of refractions that vary with direction if the crystalline structure is not symmetric.
    - Electric forces between atoms is asymmetric and so index of refraction has two values depending on polarization (Birefringent or Double Refraction).
      - Corresponding shift in resonance λ so crystal can refract light in one plane much more than another.
    - Examples include Calcite or Quartz
      - The plane of incidence on the crystal defines the “ordinary ray” and the other ray is called the “extraordinary ray.”

Calcite: $n_o = 1.65836$  
$n_e = 1.48641$
Chapter 2: Polarization

- Devices for Inducing/Measuring Polarization
  - Birefringence:
    - Double prism can separate two polarizations
    - Change orientation and we can retard (phase-shift) one with respect to another
    - Controlling the thickness can produce elliptical polarization or even circular for a given $\lambda$. 
Chapter 2: Polarization

• Devices for Inducing/Measuring Polarization
  – Wave Plates:
    • Recall that by cutting a calcite plate to a specific thickness we can control the relative phases of the two polarizations.
    • Path length difference: \( \Delta \phi = 2\pi d(n_o-n_e)/\lambda \)
      – A full-wave plate will bring the e- and o-waves back into phase, i.e., a phase shift of \(2\pi\).
        » Seldom used except in very specific circumstances (see text).
      – A half-wave plate produces a phase shift of \(\pi\) (180°). Result is an inversion in the axes of any elliptically polarized light.
        » Mica (muscovite) works well.
      – A quarter-wave plate produces a phase shift of \(\pi/2\) (90°).
        » Used to convert linear polarization to elliptical and vice versa
Chapter 2: Polarization

- Devices for Inducing/Measuring Polarization
  - Compensators:
    - Cutting a wedge-prism from calcite or quartz allows continuous control of phase shifts
    - Babinet compensator
    - Soleil compensator
Chapter 2: Polarization

• Devices for Inducing/Measuring Polarization
  – Faraday Effect:
    • Plane of polarization can be rotated when a strong B field is applied to some transparent substances
      – Verdet constant ($v$, see table 8.2)
      – Used to modulate light via an electrical signal.
  • Seen in Interstellar medium
    – Radio waves from Pulsars interact with free electrons in B field.
      » Amount of polarization used to estimate B field.

\[ \beta = vBd \]
Chapter 2: Polarization

• Devices for Inducing/Measuring Polarization
  – Kerr and Pockels Effects:
    • A strong E-field can make some substances birefringent.
      \[ \Delta n = \lambda KE^2 \] where K is the Kerr constant (table 8.3)
      – If placed between crossed polarizers the output intensity can be rapidly modulated (optical switch at several GHz).
      – Useful as high-speed shutter for imaging or video.
      – A Pockel Cell is similar (response time in nanoseconds)
Chapter 2: Polarization

• Devices for Inducing/Measuring Polarization
  – Liquid crystals:
    • Elongated transparent crystals in a solution that can be aligned via E-field
      – Result is birefringence that is controlled when a voltage is applied.
      – Strength and wavelength of individual cells can be controlled to produced a digital display (e.g. clock) or a flat-screen display.
Mathematical Model of Polarization

- **Stokes Parameters:**
  - Used to specify the polarization state of a beam of light
  - Consider 4 filters (each transmits $\frac{1}{2}$ the light)
    - 1 transmits all polarizations equally, i.e., $\frac{1}{2}$ the light ($I_0$)
    - 2 and 3 are linear polarizers ($I_1 = \text{horizontal}$, $I_2 = +45^\circ$)
    - 4-th is a circular polarizer opaque to L (left) states ($I_3$)
  - We then measure the intensity passed by each one at a time.
  - We define Stokes Parameters as:

\[
S_0 = 2I_0 \quad \text{(just the incident intensity)}
\]
\[
S_1 = 2I_1 - 2I_0 \quad \text{(Horizontal if } S_1 \geq 0, \text{ Vertical if } S_1 \leq 0)\]
\[
S_2 = 2I_1 + 2I_0 \quad \text{(If } S_1 = 0, \text{ elliptical or circular: } +45^\circ \text{ if } S_2 \geq 0, -45^\circ \text{ if } S_2 \leq 0)\]
\[
S_3 = 2I_3 - 2I_0 \quad \text{(Right handed: } S_3 \geq 0, \text{ Left handed: } S_3 \leq 0, \text{ or neither: } S_3 = 0)\]
Mathematical Model of Polarization-II

\[ S_0 = 2I_0 \]  (just the incident intensity)
\[ S_1 = 2I_1 - 2I_0 \]  (Horizontal if \( S_1 \geq 0 \), Vertical if \( S_1 \leq 0 \))
\[ S_2 = 2I_1 + 2I_0 \]  (If \( S_1 = 0 \), elliptical or circular: +45° if \( S_2 \geq 0 \), -45° if \( S_2 \leq 0 \))
\[ S_3 = 2I_3 - 2I_0 \]  (Right handed: \( S_3 \geq 0 \), Left handed: \( S_3 \leq 0 \), or neither: \( S_3 = 0 \))

- **Stokes Parameters as an array (matrix) of 4 numbers:**
  - If we divide each Stokes Parameter by \( S_0 \) we normalize their value
    - Unpolarized light: \( S_0 = 1, S_1 = S_2 = S_3 = 0 \)  \((1,0,0,0)\)
    - Horizontally polarized light:  \((1,1,0,0)\)
    - Vertically polarized light:  \((1,-1,0,0)\)
    - Polarized light at +45°: \((1,0,1,0)\)
    - Polarized light at -45°: \((1,0,-1,0)\)
    - Right-hand polarized light:  \((1,0,0,1)\)
    - Left-hand polarized light:  \((1,0,0,-1)\)
  - With the degree of polarization: \( V = (S_1^2 + S_2^2 + S_3^2)^{1/2} \)
Mathematical Model of Polarization - III

• Figure at Right Illustrates Stokes Parameters
  – In the x-y plane:

\[
I = \left| E_x \right|^2 + \left| E_y \right|^2
\]

\[
Q = \left| E_x \right|^2 - \left| E_y \right|^2
\]

\[
U = 2 \text{ Re}(E_x E_y^*)
\]

\[
V = 2 \text{ Im}(E_x E_y^*)
\]
Jones Vectors

- A short-hand version but only for purely polarized light was invented by Jones:

\[
\vec{E} = \begin{bmatrix} E_{0x}e^{i\phi_x} \\ E_{0y}e^{i\phi_y} \end{bmatrix}
\]

and so if the amplitudes and phases are the same and we have polarized light we can factor out the amplitudes:

\[
\vec{E} = E_{0x} \begin{bmatrix} e^{i\phi_x} \\ e^{i\phi_y} \end{bmatrix} = E_{0x}e^{i\phi_x} \begin{bmatrix} 1 \\ 0 \end{bmatrix}
\]

a horz. polarized wave, and so if the incident intensity is normalized the this is simplified further:

\[
\vec{E}_h = \begin{bmatrix} 1 \\ 0 \end{bmatrix}
\]

where the 1 is just to explicitly show the normalization. We can now look at the matrices for other cases:

\[
\vec{E}_v = \begin{bmatrix} 0 \\ 1 \end{bmatrix}
\]

\[
\vec{E}_{45} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ 1 \end{bmatrix}
\]

where \( \frac{1}{\sqrt{2}} \) specifically indicates that \( E^2 \) is the intensity. A right circular polarization has \( \delta \phi = \pi/2 \):

\[
\vec{E}_r = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ -i \end{bmatrix}
\]

and so left circular polarization would be:

\[
\vec{E}_l = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ i \end{bmatrix}
\]

See page 377 for other examples.
Stokes and Jones Vectors

- So now we see that a single or group of analyzers is just a matrix that operates on the incident beam:

\[
\begin{align*}
\vec{E}_t &= A\vec{E}_i \quad \text{or upon expansion:} \\
\begin{bmatrix}
\vec{E}_{tx} \\
\vec{E}_{ty}
\end{bmatrix}
&= \begin{bmatrix}
a_{11} & a_{12} \\
a_{21} & a_{22}
\end{bmatrix}
\begin{bmatrix}
\vec{E}_{ix} \\
\vec{E}_{iy}
\end{bmatrix}
\end{align*}
\]

See the book for various examples.

<table>
<thead>
<tr>
<th>Linear optical element</th>
<th>Jones matrix</th>
<th>Mueller matrix</th>
</tr>
</thead>
<tbody>
<tr>
<td>Horizontal linear polarizer</td>
<td>[ \begin{bmatrix} 1 &amp; 0 \ 0 &amp; 0 \end{bmatrix} ]</td>
<td>[ \begin{bmatrix} 1 &amp; 1 &amp; 0 &amp; 0 \ 1 &amp; 1 &amp; 0 &amp; 0 \ 0 &amp; 0 &amp; 0 &amp; 0 \ 0 &amp; 0 &amp; 0 &amp; 0 \end{bmatrix} ]</td>
</tr>
<tr>
<td>Vertical linear polarizer</td>
<td>[ \begin{bmatrix} 0 &amp; 0 \ 0 &amp; 1 \end{bmatrix} \frac{1}{2} ]</td>
<td>[ \begin{bmatrix} 1 &amp; -1 &amp; 0 &amp; 0 \ -1 &amp; 1 &amp; 0 &amp; 0 \ 0 &amp; 0 &amp; 0 &amp; 0 \ 0 &amp; 0 &amp; 0 &amp; 0 \end{bmatrix} ]</td>
</tr>
<tr>
<td>Linear polarizer at +45°</td>
<td>[ \begin{bmatrix} \frac{1}{2} &amp; 1 \ 1 &amp; \frac{1}{2} \end{bmatrix} ]</td>
<td>[ \begin{bmatrix} 1 &amp; 0 &amp; 1 &amp; 0 \ 0 &amp; 1 &amp; 0 &amp; 1 \ 0 &amp; 0 &amp; 0 &amp; 0 \ 0 &amp; 0 &amp; 0 &amp; 0 \end{bmatrix} ]</td>
</tr>
<tr>
<td>Linear polarizer at -45°</td>
<td>[ \begin{bmatrix} \frac{1}{2} &amp; -1 \ -1 &amp; \frac{1}{2} \end{bmatrix} ]</td>
<td>[ \begin{bmatrix} 1 &amp; 0 &amp; -1 &amp; 0 \ 0 &amp; 0 &amp; 0 &amp; 0 \ -1 &amp; 0 &amp; 1 &amp; 0 \ 0 &amp; 0 &amp; 0 &amp; 0 \end{bmatrix} ]</td>
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<tr>
<td>Quarter-wave plate, fast axis vertical ( \exp\frac{i\pi}{4} )</td>
<td>[ \begin{bmatrix} 1 &amp; 0 \ 0 &amp; -i \end{bmatrix} ]</td>
<td>[ \begin{bmatrix} 1 &amp; 0 &amp; 0 &amp; 0 \ 0 &amp; 1 &amp; 0 &amp; 0 \ 0 &amp; 0 &amp; 0 &amp; -1 \ 0 &amp; 0 &amp; 1 &amp; 0 \end{bmatrix} ]</td>
</tr>
<tr>
<td>Quarter-wave plate, fast axis horizontal ( \exp\frac{i\pi}{4} )</td>
<td>[ \begin{bmatrix} 1 &amp; 0 \ 0 &amp; i \end{bmatrix} ]</td>
<td>[ \begin{bmatrix} 1 &amp; 0 &amp; 0 &amp; 0 \ 0 &amp; 1 &amp; 0 &amp; 0 \ 0 &amp; 0 &amp; 0 &amp; 1 \ 0 &amp; 0 &amp; -1 &amp; 0 \end{bmatrix} ]</td>
</tr>
<tr>
<td>Homogeneous circular polarizer right ( \bigcirc )</td>
<td>[ \begin{bmatrix} \frac{1}{2} &amp; 1 \ -i &amp; 1 \end{bmatrix} ]</td>
<td>[ \begin{bmatrix} 1 &amp; 0 &amp; 0 &amp; 1 \ 0 &amp; 0 &amp; 0 &amp; 0 \ 0 &amp; 0 &amp; 0 &amp; 0 \ 1 &amp; 0 &amp; 0 &amp; 1 \end{bmatrix} ]</td>
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<tr>
<td>Homogeneous circular polarizer left ( \bigcirc )</td>
<td>[ \begin{bmatrix} \frac{1}{2} &amp; 1 \ i &amp; 1 \end{bmatrix} ]</td>
<td>[ \begin{bmatrix} 1 &amp; 0 &amp; 0 &amp; -1 \ 0 &amp; 0 &amp; 0 &amp; 0 \ 0 &amp; 0 &amp; 0 &amp; 0 \ -1 &amp; 0 &amp; 0 &amp; 1 \end{bmatrix} ]</td>
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Chapter 2 Key Concepts

Linear, Elliptical, Circular Polarization
Polarizers & Malus’ Law
Birefringence of Crystals
Waveplates
Polarization by Scattering & Reflection
Photoelasticity & Modulators
Liquid Crystals
Stokes Parameters & Matrix Models
Chapter 2 Key Equations

Mathematical Form of a Polarized Wave:
Linear:
Circular:
Elliptical:
Malus' Law:
Brewster's Law:
Degree of Polarization:
Optical Path Length from Retarders:
Faraday Effect:
Kerr Effect:
Stokes, Jones & Muller Parameters/Vectors:
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