Phys 2310 . Nov. 3, 2017
Today's Topics

- **Continue Chapter 2: Electromagnetic Theory, Photons, and Light**
- **Reading for Next Time**

Reading this Week

By Wednesday:

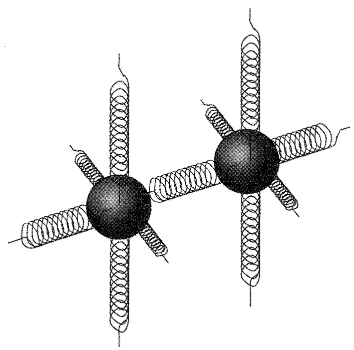
Finish Fowles Ch. 2 (2.3 – 2.11) Polarization Theory, Jones Matrices, Fresnel Equations and Brewster's Angle

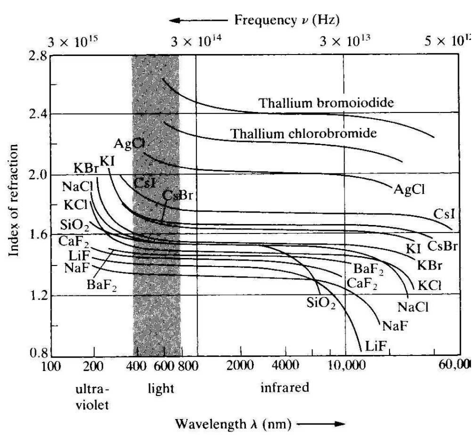
Homework this Week

Chapter 2 Homework (due Monday Nov. 6)
#6, 7, 8, 15, 17

Supplemental: Lorentz Model for Matter

- **How Does Light Interacts with Materials?**
  - **Maxwell's Equations Don't Help Us**
  - **Consider a Lorentz Model**
    - **Electrons (in atoms) Oscillate with a Restoring Force**





$$\frac{d^2x}{dt^2} = E(t)\frac{q_e}{m_e} - \kappa_x \frac{x}{m_e} - \gamma\frac{dx}{dt} \text{ (driven harmonic oscillator)}$$

Which can be solved for x if we guess a harmonic solution:

$x(t) = x_0 \cos\omega t$ and let $\omega_0^2 = \frac{\kappa_x}{m_e}$ and $\gamma \cong 0$, we can insert and solve for x_0 :

$$x(t) = \frac{q_e / m_e}{\left(\omega^2 - \omega_0^2\right)} E_0 \cos\omega t = \frac{q_e / m_e}{\left(\omega^2 - \omega_0^2\right)} E(t)$$

An alternative approach is to take the Fourier transform of the diff. equation to create an algebraic equation, solve it, then transform back (cool!):

$-\omega^2 x(\omega) + i\gamma\omega x(\omega) + \omega_0^2 x(\omega) = -\frac{q_e}{m_e} E(\omega)$ and solve for $x(\omega)$.

The moving electrons produce a polarization

$\vec{P}_r(t) = -N_v q_e \vec{x}(t)$

where N_v is the number density of electrons. So: $\vec{P}_r = \varepsilon_0 \chi \vec{E}_r \cdot \vec{x}$

Which leads to a displacement:

$\vec{D}_r = \varepsilon_0 \vec{E}_r + \vec{P}_r = \varepsilon \vec{E}_r = \varepsilon_0 (1+\chi) \vec{E}_r$ (free + bound electrons)

The polarization $\vec{P}_r = \varepsilon_0 \chi \vec{E}_r$ depends on the "spring constants."

Upon substitution for x into the diff. equation the solution is:

$$P_x = \frac{\omega_p^2}{\left(\omega_0^2 - \omega^2\right) + i\gamma\omega} \varepsilon_0 E_x \text{ where } \omega_p^2 = \frac{Nq_e^2}{\varepsilon_0 m_e}$$

Note that the amplitude get very large if driving frequency is in resonance with the "oscillators" ($\omega \cong \omega_0$). This means that:

$$\varepsilon = \frac{\omega_p^2}{\left(\omega_0^2 - \omega^2\right) + i\gamma\omega} = \frac{\omega_p^2\left(\omega_0^2 - \omega^2\right)}{\left(\omega_0^2 - \omega^2\right)^2 + \gamma\omega} - i\frac{\omega_p^2 \omega\gamma}{\left(\omega_0^2 - \omega^2\right)^2 + \gamma^2\omega^2} \text{ or:}$$

$\varepsilon = \varepsilon_r - i\varepsilon_i$ (real and imaginary parts) and if we introduce complex refractive index:

$\tilde{n} = n - i\kappa$ in which case $\tilde{\varepsilon} = \tilde{n}^2$ and so a material will have a frequency-dependent n.

Chapter 2: Interaction of Light & Matter

- **The electromagnetic Approach to Refraction and Reflection**
  - **E&M treatment is more complete and quantitative than Snell's laws.**
- **Consider EM waves at an interface**
  - **If we imagine the E field an infinitesimal distance on either side of the interface the components tangent to the surface should be equal.**

Consider a wave incident on an interface between two media (n_i and n_t):

$\vec{E}_i = \vec{E}_{oi}\cos(\vec{k}_i \cdot \vec{r} - \omega_i t)$

The reflected and transmitted waves are similarly:

$\vec{E}_r = \vec{E}_{or}\cos(\vec{k}_r \cdot \vec{r} - \omega_r t + \varepsilon_r)$ and $\vec{E}_t = \vec{E}_{ot}\cos(\vec{k}_t \cdot \vec{r} - \omega_t t + \varepsilon_t)$

Since the tangential components must be equal the cross product is tangent:

$\hat{u}_n \times \vec{E}_i + \hat{u}_n \times \vec{E}_r = \hat{u}_n \times \vec{E}_t$ or:

$\hat{u}_n \times \vec{E}_{oi}\cos(\vec{k}_i \cdot \vec{r} - \omega_i t) + \hat{u}_n \times \vec{E}_{or}\cos(\vec{k}_r \cdot \vec{r} - \omega_r t + \varepsilon_r) =$

$\hat{u}_n \times \vec{E}_{ot}\cos(\vec{k}_t \cdot \vec{r} - \omega_t t + \varepsilon_t)$

Since this condition must be true of any time and at any point wrt the origin:

$(\vec{k}_i \cdot \vec{r})_{y=b} = (\vec{k}_r \cdot \vec{r} + \varepsilon_r)_{y=b} = (\vec{k}_t \cdot \vec{r} + \varepsilon_t)_{y=b}$ (at y=b)

From the first two terms:

$\left[(\vec{k}_i - \vec{k}_r)\cdot \vec{r}\right]_{y=b} = \varepsilon_r$ which says that $(\vec{k}_i - \vec{k}_r)$ is parallel to $\hat{u}_n$

or $\hat{u}_n \times (\vec{k}_i - \vec{k}_r) = 0$ and $\vec{k}_i \sin\theta_i = \vec{k}_r \sin\theta_r$. Thus:

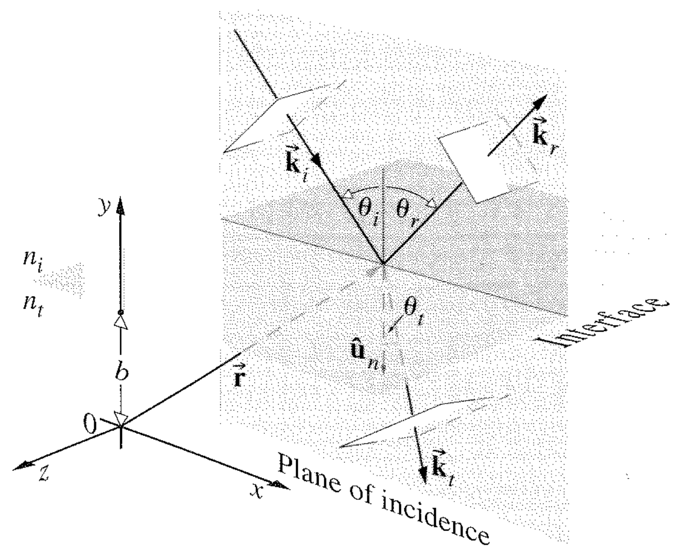
$\theta_i = \theta_i$ (law of reflection)

But if $\vec{k}_i$ and $\vec{k}_r$ and $\hat{u}_n$ are in the same plane then:

$\left[(\vec{k}_i - \vec{k}_t)\cdot \vec{r}\right]_{y=b} = \varepsilon_t$ and since the tangential components are equal:

$\vec{k}_i \sin\theta_i = \vec{k}_t \sin\theta_t$ but multiplying both sides by c/ω_i gives:

$n_i \sin\theta_i = n_t \sin\theta_t$ since n = v/c (Snell's law of refraction)



Chapter 2: Interaction of Light & Matter

- **The Fresnel Equations**
  - **Consider a plane wave incident on an interface between two media (whatever the polarization). We have seen that the incident, reflected and transmitted waves define a plane-of-incidence. Next consider two cases:**
    - **Case 1: E field is perpendicular to plane of incidence**

Since the $\vec{E}$ vectors are perpendicular to the incident plane:

$$\vec{E}_{0i} + \vec{E}_{0r} = \vec{E}_{0t}$$

The tangential component of the $\vec{B}$ field must also be cotinuous:

$$-\frac{B_i}{\mu_i}\cos\theta_i + \frac{B_r}{\mu_r}\cos\theta_r = \frac{B_t}{\mu_t}\cos\theta_t$$

and since $B_i = E_i / \mathrm{v}_i,\ B_r = E_r / \mathrm{v}_r,\ B_t = E_t / \mathrm{v}_t, \theta_i = \theta_r$ and $\mathrm{v}_r = \mathrm{v}_i$:

$$\frac{1}{\mu_i v_i}(E_i - E_r)\cos\theta_i = \frac{1}{\mu_t v_t}E_t\cos\theta_t$$ (using v allows us to introduce n)

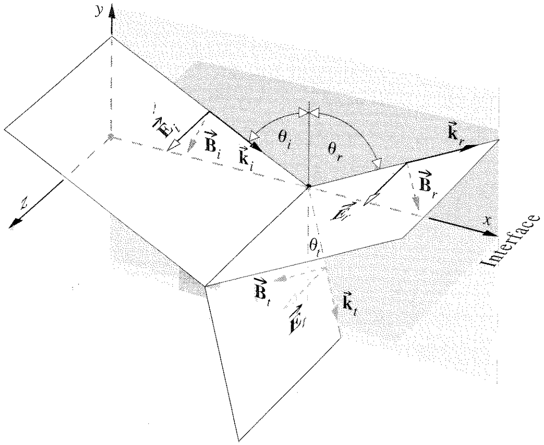
Now in terms of n the index of refraction:

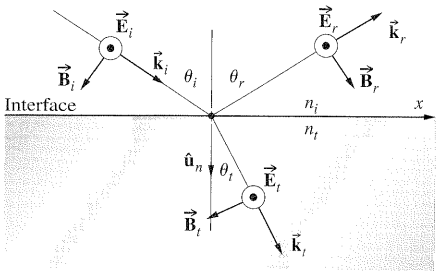
$$\frac{n_i}{\mu_i}(E_{0i} - E_{0r})\cos\theta_i = \frac{n_t}{\mu_t}E_{0t}\cos\theta_t$$ Solving for amplitudes gives:

$$r_\perp = \left(\frac{E_{0r}}{E_{0i}}\right)_\perp = \frac{\frac{n_i}{\mu_i}\cos\theta_i - \frac{n_t}{\mu_t}\cos\theta_t}{\frac{n_i}{\mu_i}\cos\theta_i + \frac{n_t}{\mu_t}\cos\theta_t} \quad \text{and} \quad t_\perp = \left(\frac{E_{0t}}{E_{0i}}\right)_\perp = \frac{2\frac{n_i}{\mu_i}\cos\theta_i}{\frac{n_i}{\mu_i}\cos\theta_i + \frac{n_t}{\mu_t}\cos\theta_t}$$

If the material is non magnetic:

$$r_\perp = \left(\frac{E_{0r}}{E_{0i}}\right)_\perp = \frac{n_i\cos\theta_i - n_t\cos\theta_t}{n_i\cos\theta_i + n_t\cos\theta_t} \quad \text{and} \quad t_\perp = \left(\frac{E_{0t}}{E_{0i}}\right)_\perp = \frac{2n_i\cos\theta_i}{n_i\cos\theta_i + n_t\cos\theta_t}$$





Chapter 2: Interaction of Light & Matter

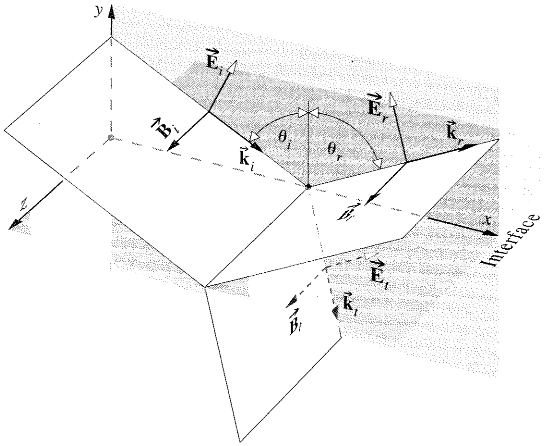
- **The Fresnel Equations**
  - **Case 2: E field is parallel to plane of incidence**

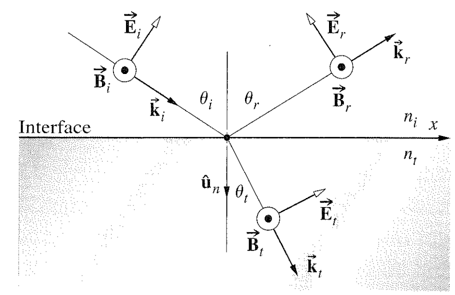
The tangential component of the $\vec{E}$ field must also be continuous:

$$E_{0i}\cos\theta_i - E_{0r}\cos\theta_r = E_{0t}\cos\theta_t$$

and since the tangential component of the $\vec{B}$ field must also be continuous:

$$\frac{1}{\mu_i v_i}E_{0i} + \frac{1}{\mu_r v_r}E_{0r} = \frac{1}{\mu_t v_t}E_t\cos\theta_t$$





Combining yields:

$$r_{\|} = \left(\frac{E_{0r}}{E_{0i}}\right)_{\|} = \frac{\dfrac{n_t}{\mu_t}\cos\theta_i - \dfrac{n_i}{\mu_i}\cos\theta_t}{\dfrac{n_i}{\mu_i}\cos\theta_i + \dfrac{n_t}{\mu_t}\cos\theta_t} \quad \text{and:}$$

$$t_{\|} = \left(\frac{E_{0t}}{E_{0i}}\right)_{\|} = \frac{2\dfrac{n_i}{\mu_i}\cos\theta_i}{\dfrac{n_i}{\mu_i}\cos\theta_t + \dfrac{n_t}{\mu_t}\cos\theta_i}$$

When both materials are non-magnetic:

$$r_{\|} = \left(\frac{E_{0r}}{E_{0i}}\right)_{\|} = \frac{n_t\cos\theta_i - n_i\cos\theta_t}{n_i\cos\theta_i + n_t\cos\theta_t} \quad \text{and:}$$

$$t_{\|} = \left(\frac{E_{0t}}{E_{0i}}\right)_{\|} = \frac{2n_i\cos\theta_i}{n_i\cos\theta_t + n_t\cos\theta_i}$$

Chapter 2: Interaction of Light & Matter

- **<u>Amplitude</u> Coefficients of the Fresnel Equations**

When combined with Snell's Law we can simplify even further:

$$r_\perp = -\frac{\sin(\theta_i - \theta_t)}{\sin(\theta_i + \theta_t)} \text{ and } r_\parallel = +\frac{\tan(\theta_i - \theta_t)}{\tan(\theta_i + \theta_t)}$$

$$t_\perp = +\frac{2\sin\theta_t \cos\theta_i}{\sin(\theta_i + \theta_t)} \text{ and } t_\parallel = \frac{2\sin\theta_t \cos\theta_i}{\sin(\theta_i + \theta_t)\cos(\theta_i - \theta_t)}$$

What does this mean (see sec. 4.6.3 in text)?

Requiring the $\vec{E}$ and $\vec{B}$ field to be continuous at the interface constrains the amplitudes of the refracted and transmitted waves. The result depends on whether the field is oscillating perpendicular or parallel to the surface and it also depends on the indices of refraction. For arbitrarily oriented waves one can consider the components.

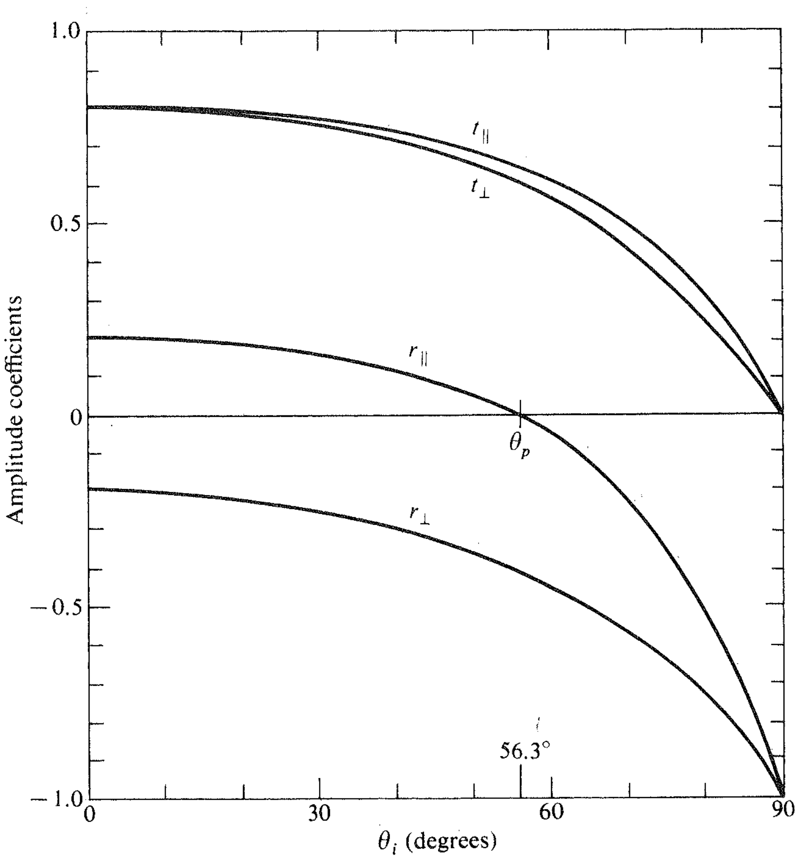
*Note that when $\theta_i + \theta_t = 90^\circ$ the value of $r_\parallel$ becomes 0 and the reflected wave is fully polarized. This is the polarization angle θ_p.

Chapter 2: Interaction of Light & Matter

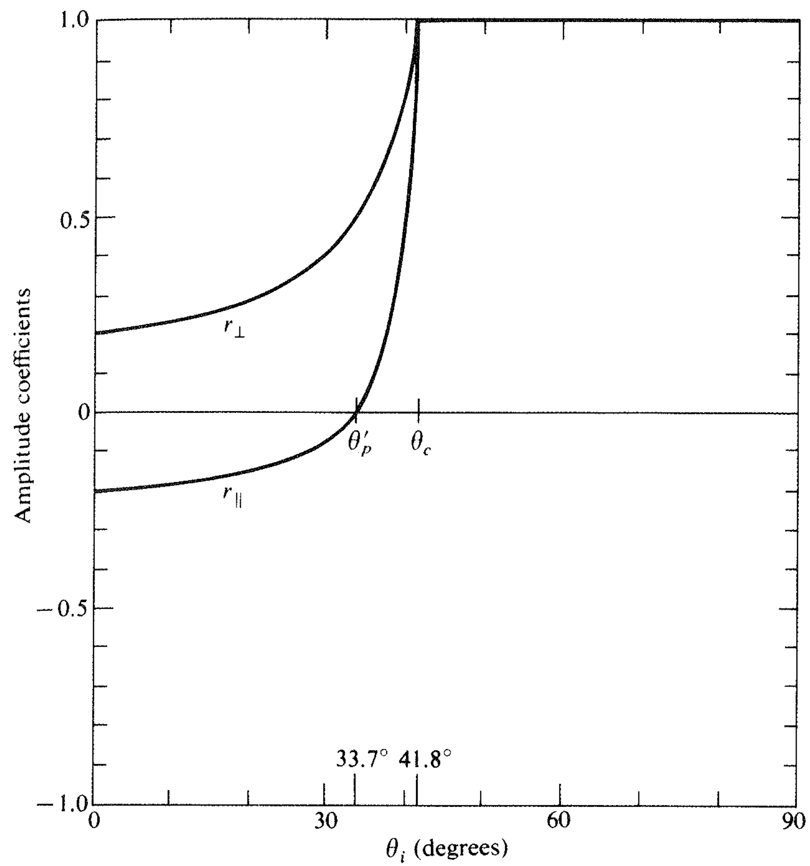
- **Meaning of the Fresnel Equations**
  - **Amplitudes of the reflected and transmitted waves can be calculated.**
  - **Normal component of E undergoes phase shift of π upon reflection if $n_i < n_t$**
    (negative sign indicates phase change: like a standing wave)

θ_p = Brewster's Angle

$n_t > n_i$ (example for $n_t = 1.5$)

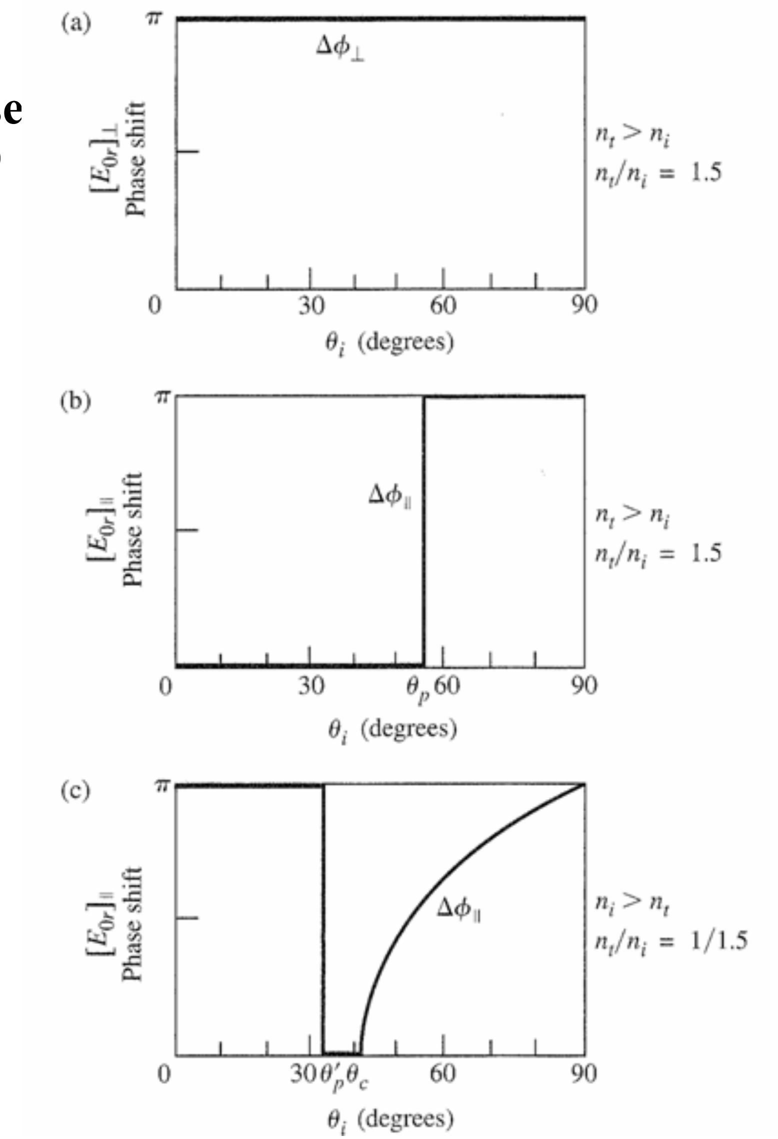


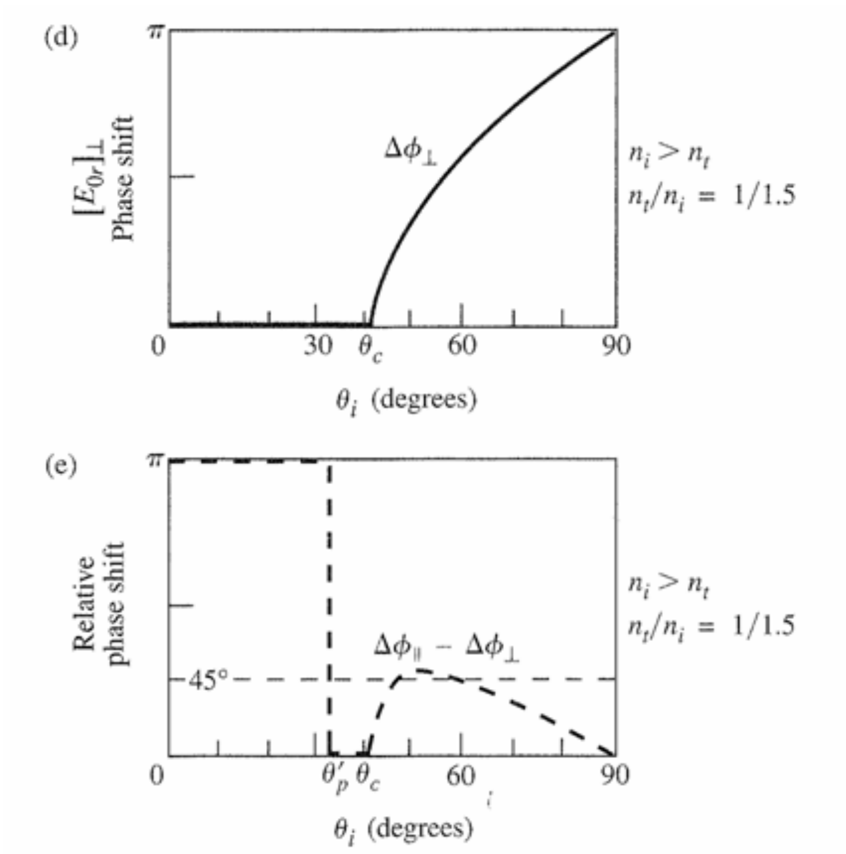
$n_t < n_i$ (example for $n_t = 1.5$)



Chapter 2: Interaction of Light & Matter

- **Fresnal equations and phase shifts**
  - **Normal component of E undergoes phase shift of π upon reflection if $n_i < n_t$ (right)**
  - **Tangential component is more complicated (figures below show phase change when $n_1 < n_2$).**





Chapter 2: Interaction of Light & Matter

- **Fresnel equations and the Reflected and Transmitted Intensity**
  - **Depends on both <u>the square of the amplitude and the cross-sectional area of the beam</u>. Left figure shows reflection intensity for two wave orientations. Note that intensity goes to 100% at high incidence and to zero for waves perpendicular to the surface. Right figure shows similar effect for internal reflection.**

We define the reflectance (R) as :

$$R \equiv \frac{I_r A\cos\theta_r}{I_i A\cos\theta_i} = \frac{I_r}{I_i} = \frac{v_r \varepsilon_r E_{0r}{}^2}{v_i \varepsilon_i E_{0i}{}^2}$$

$$R = \left(\frac{E_{0r}}{E_{0i}}\right)^2 = r^2 \text{ if } v_r = v_i \text{ and } \varepsilon_r = \varepsilon_i$$

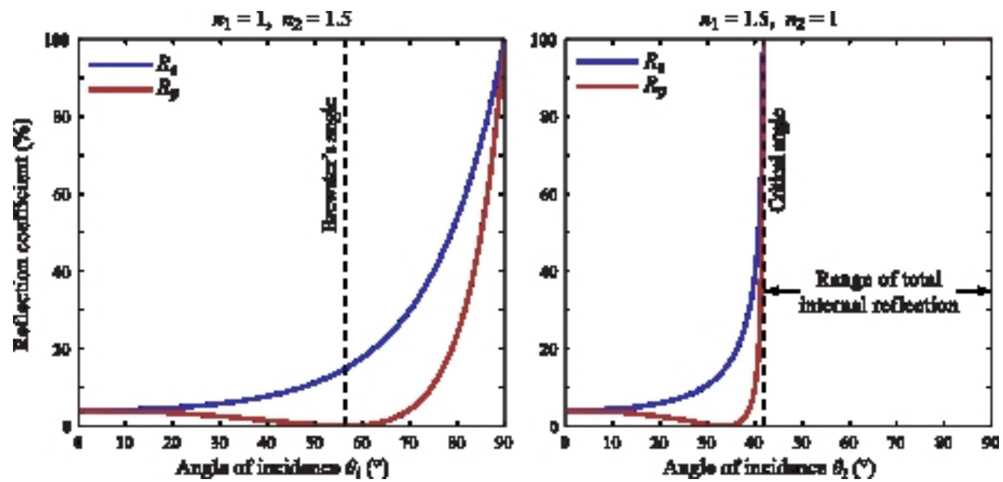
and the transmittance (T) as :

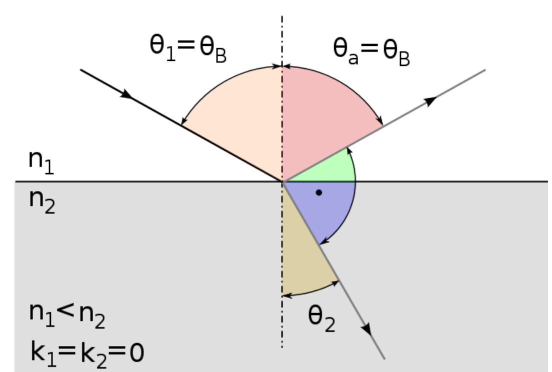
$$T \equiv \frac{I_t \cos\theta_r}{I_i \cos\theta_i} = \frac{I_t}{I_i} \text{ since } \mu_0 \varepsilon_t = 1/v_t{}^2 \text{ and}$$

$$\mu_0 v_t \varepsilon_t = n_t / c$$

$$T = \frac{n_t \cos\theta_t}{n_i \cos\theta_i}\left(\frac{E_{0t}}{E_{0i}}\right)^2 = \left(\frac{n_t \cos\theta_t}{n_i \cos\theta_i}\right) t^2$$

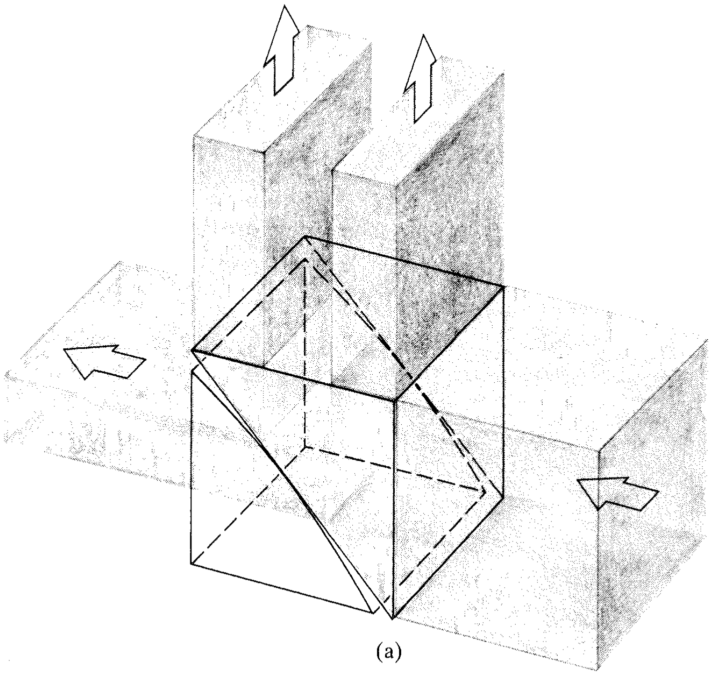
This is valid for the parallel and perpendicular components but t^2 varies.





Chapter 2: Interaction of Light & Matter

- **Total Internal Reflection and the Evanescent Wave**
  - **Frustrated total internal reflection occurs when two surfaces of a transparent substance are brought into contact to destroy (frustrate) the total internal reflection that would otherwise occur at the interface.**
  - **This occurs gradually as the two media are brought into contact as photons "tunnel" through the barrier between the two media!**
    - **Photons are not perfectly localized in space (quantum mechanics) so they can "leak" through a gap.**
  - **One application is in the construction of beamsplitters.**



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Homework this Week

Chapter 2 Homework (due Monday Nov. 6)
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