Phys 2310 Mon. Nov. 6, 2017
Today's Topics

- **Begin Chapter 3: Interference**

  Y&F Chapter 35

- **Reading for Next Time**

Reading this Week

By Wed.:

- **Begin Ch. 3 (3.1 – 3.3) General Considerations, Conditions for Interference, Wavefront-splitting Interferometers**
- **Read Y&F Ch. 35**

Homework this Week

By Monday Nov. 13:

Y&F Chapter 33: 33.33, 33.34, 33.36

Y&F Chapter 35: 35.32, 35.38, 35.41, 35.43, 35.45, 35.59

Chapter 3: Interference

- **Preliminaries:**
  - **Interference occurs when two or more EM wave overlap in space such that the result is a superposition.**
  - **The result will depend on both amplitude and phase differences.**
  - **Examples include**
    - **Colored bands when light reflects off oil sheen over water**
    - **Colored bands when light reflects off soap bubbles**
    - **Interferometers**

Chapter 3: Interference

- **General Considerations**
  - **Principle of Superposition**
    - **Recall the addition of two plane waves (also applicable to spherical):**

$\vec{E}_1(r,t) = \vec{E}_{01}\cos(\vec{k}_1 \cdot \vec{r} - \omega t + \varepsilon_1)$ and $\vec{E}_2(r,t) = \vec{E}_{02}\cos(\vec{k}_2 \cdot \vec{r} - \omega t + \varepsilon_2)$

We saw that the irradience at any one point is the square of the amplitude:

$I = \left\langle \vec{E}^2 \right\rangle_T$ (neglecting some numerical constants). Thus since:

$\vec{E}^2 = \vec{E} \cdot \vec{E} = (\vec{E}_1 + \vec{E}_2) \cdot (\vec{E}_1 + \vec{E}_2)$ and we have:

$\vec{E}^2 = \vec{E}^2{}_1 + \vec{E}^2{}_2 + 2\vec{E}_1 + \vec{E}_2$ where the second term is the interference term.

Thus we might rewrite this as:

$I = I_1 + I_2 + I_{12}$ with $I_1 = \left\langle \vec{E}^2{}_1 \right\rangle_T$, $I_2 = \left\langle \vec{E}^2{}_2 \right\rangle_T$, and $I_{12} = 2\left\langle \vec{E}^2{}_1 \cdot \vec{E}^2{}_2 \right\rangle_T$

It can be shown (cosine product + averaging, see text) that $I_{12} = \vec{E}_{01} \cdot \vec{E}_{02}\cos\delta$, where:

$\delta = (\vec{k}_1 \cdot \vec{r} - \vec{k}_2 \cdot \vec{r} + \varepsilon_1 + \varepsilon_2)$ is the phase difference from both the initial phases and the path length vs. position at any point $\vec{r}$. Note though that there remains a polarization issue. We will assume $\vec{E}_1$ and $\vec{E}_2$ are parallel. In this case:

$I_{12} = E_{01}E_{02}\cos\delta$ but since $I_1 = E_{01}{}^2/2$ and $I_2 = E_{02}{}^2/2$ we have:

$I_{12} = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\delta$ Two special cases are:

Total constructive interference ($\cos\delta = 1$):

$I_{max} = I_1 + I_2 + 2\sqrt{I_1 I_2}$ when $\delta = 0, \pm 2\pi, \pm 4\pi...$ and

Total destructive interference ($\cos\delta = -1$):

$I_{min} = I_1 + I_2 - 2\sqrt{I_1 I_2}$ when $\delta = 0, \pm \pi, \pm 3\pi...$

Chapter 3: Interference

- **General Considerations**
  - **Condition for maxim and minima**

Total constructive interference ($\cos\delta = 1$):

$I_{max} = I_1 + I_2 + 2\sqrt{I_1 I_2}$ when $\delta = 0, \pm 2\pi, \pm 4\pi...$ and

Total destructive interference ($\cos\delta = -1$):

$I_{min} = I_1 + I_2 - 2\sqrt{I_1 I_2}$ when $\delta = 0, \pm \pi, \pm 3\pi...$

In the special case where both amplitudes are the same:

$I = 2I_0(1+\cos\delta) = 4I_0\cos^2\frac{\delta}{2}$ and so we have:

$I_{max} = 4I_0$ and $I_{min} = 0$

If we now consider δ:

$\delta = k(r_1 - r_2) + (\varepsilon_1 - \varepsilon_2)$ and so:

$I_{max} : (r_1 - r_2) = [2\pi m + (\varepsilon_1 - \varepsilon_2)]/k$

$I_{min} : (r_1 - r_2) = [\pi m + (\varepsilon_1 - \varepsilon_2)]/k$

The two equations above simplify further if $(\varepsilon_1 - \varepsilon_2) = 0$,

and it is apparent that they are the equations of a hyperbola.

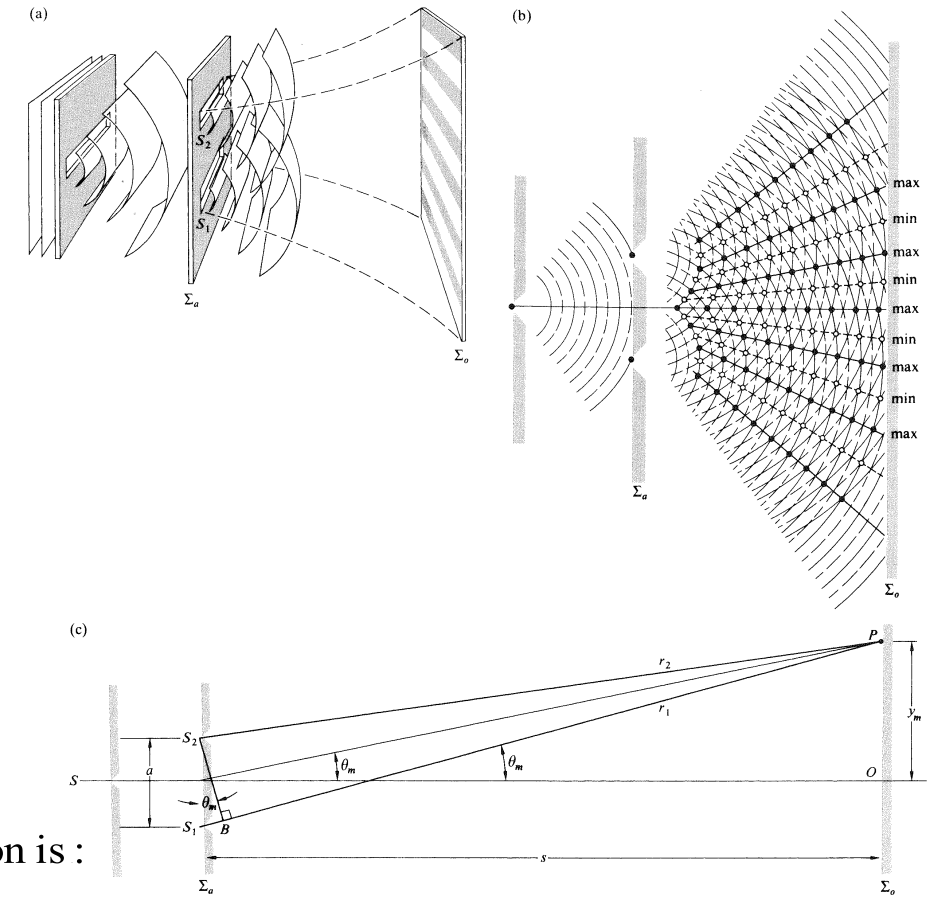
So the maxima and minima are distributed along hyperbolic surfaces (see Fig. 9.3)

Chapter 3: Interference

- **Conditions for Interference**
  - **Temporal and Spatial Coherence**
    - **Recall that atoms emit wave packet and not continuous wave trains**
      - **Coherence time is only about 10 ns or so depending on how monochromatic the source is. There is a corresponding coherence length (speed of light). See fig. 9.4**
      - **White-light fringes can be seen if the path-length differences are small.**
      - **Lasers make it easy, otherwise we must use a single source**
  - **Fresnel-Arago Laws**
    - **Two orthogonal, coherent polarizations cannot interfere since $I_{12} = 0$.**
    - **Two parallel, coherent polarizations will interfere**
    - **Two orthogonal polarizations cannot be made to interfere via rotation since they are incoherent.**

Chapter 3: Interference

- **Wavefront-splitting Interferometers**
  - **Secondary wavelet sources produce interference**
  - **Young's 2-slit Experiment**
    - **Young showed light could interfere by passing light from a semi-coherent source through two slits (see lab)**
    - **Difference in OPL from each slit to given point on screen results in interference**



$$\Delta \mathrm{OPL} = a\sin\theta = a\frac{y}{s} \ \text{(small angle approx.)}$$

and interference requires:

$$\Delta \mathrm{OPL} = a\sin\theta = a\frac{y}{s} = m\lambda \ \text{ and so the separation is:}$$

$$\Delta y \approx \frac{s}{a}\lambda \ \text{ (measure s, a, and } \Delta y \text{ to measure } \lambda\text{)}$$

Chapter 3: Interference

- **Wavefront-splitting Interferometers**
  - **Young's 2-slit Experiment**
    - **Effects of Finite Coherence Length**
      - **Finite coherence means that as the path difference increases the fringes loose contrast as due to a lack of temporal coherence (light travel time to the point of interference)**
      - **Similarly, white light fringes can be seen only at small OPL since they begin to overlap due to the λ dependence of the OPL.**
  - **Fresnal's Double Mirror and Double Prism**
    - **Interference seen from the differing OPL from two slightly tilted mirrors or very thin prisms**

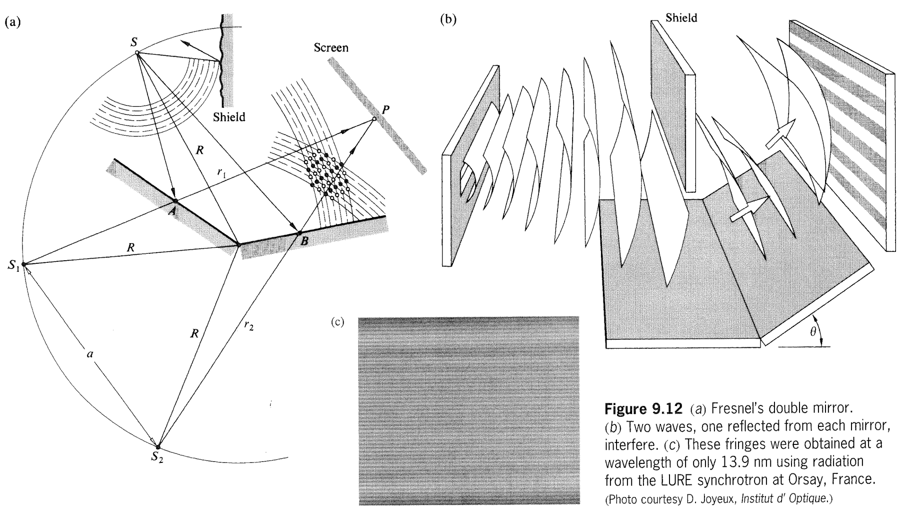
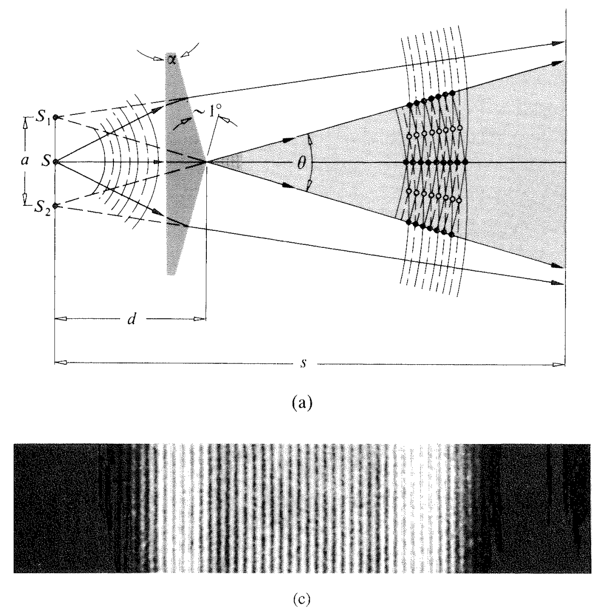


Figure 9.12 (*a*) Fresnel's double mirror. (*b*) Two waves, one reflected from each mirror, interfere. (*c*) These fringes were obtained at a wavelength of only 13.9 nm using radiation from the LURE synchrotron at Orsay, France. (Photo courtesy D. Joyeux, *Institut d' Optique*.)



Chapter 3: Interference

- **Amplitude-splitting Interferometers**
  - **Interference produced from an original source by splitting path**
  - **Fringes from Thin Films**
    - **Fringes from a film of uniform thickness (equal inclination fringes)**
    - **Circular fringes due to symmetry**

$OPL = \frac{2n_f d}{\cos\theta_t} - n_1(\bar{A}\bar{D})$ but:

$(\bar{A}\bar{D}) = (\bar{A}\bar{C})\sin\theta_i$ but substuting using Snell's Law:

$(\bar{A}\bar{D}) = (\bar{A}\bar{C})\frac{n_f}{n_i}\sin\theta_t$ and since:

$(\bar{A}\bar{C}) = 2d\tan\theta_t$ we have:

$(\bar{A}\bar{D}) = (2d\tan\theta_t)\frac{n_f}{n_i}\sin\theta_t$

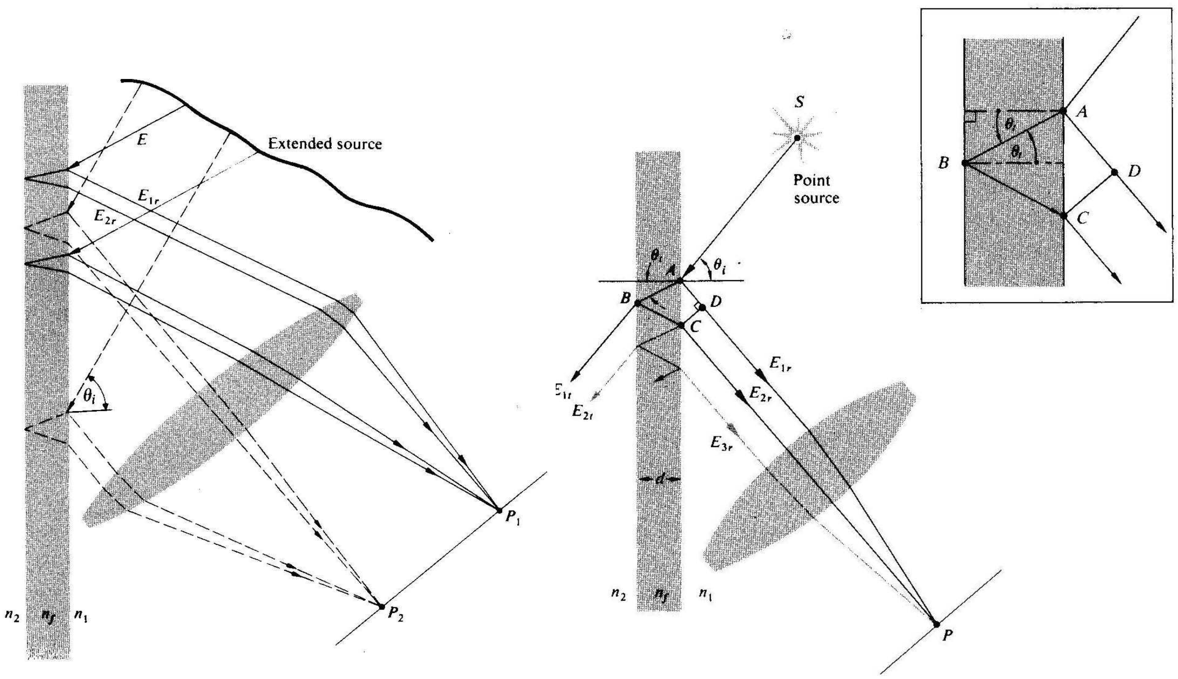
$OPL = \frac{2n_f d}{\cos\theta_t}(1-\sin^2\theta_t)$ or:

$OPL = 2n_f d\cos\theta_t$ and since the corresponding phase shift will be:

$\phi = \frac{4\pi n_f}{\lambda_0} d\cos\theta_t \pm \pi$ (note refl. phase shift)

$d\cos\theta_t = (2m+1)\frac{\lambda_0}{4n_f}$ (for maxima)

$d\cos\theta_t = (2m)\frac{\lambda_0}{4n_f}$ (for minima)



Chapter 3: Interference

- **Amplitude-splitting Interferometers**
  - **Fringes from Thin Films**
    - **Fringes from a film of changing thickness (equal thickness fringes)**
      - **Examples of Fizeau fringes include soap bubble films (and wedge shown in class), wedge produces parallel bands**

$$d = x\tan\alpha \cong x\alpha \quad \text{(small angle approx.)}$$

The condition for interference maxima:

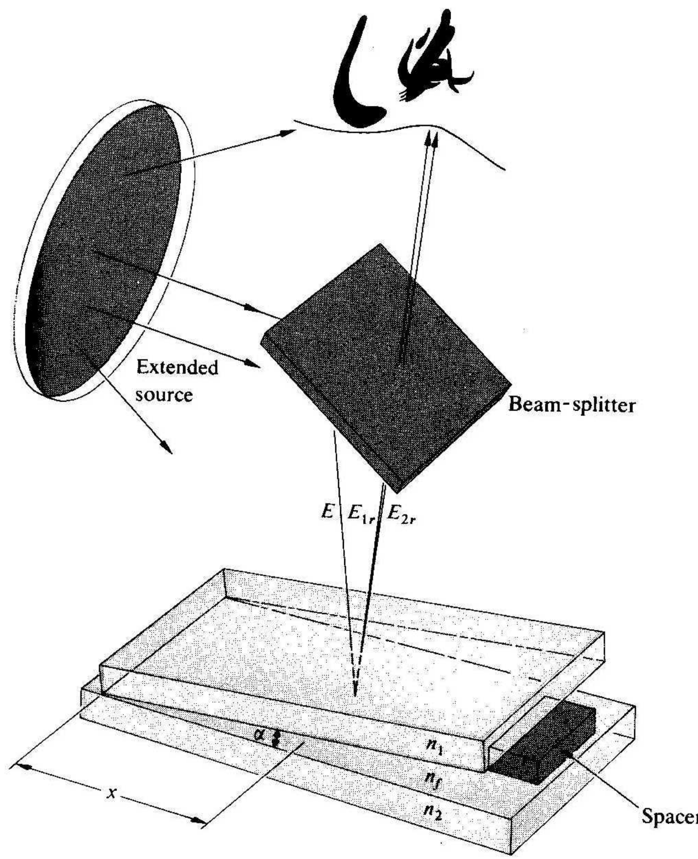
$$(m+1/2)\lambda_0 = 2n_f d_{max} = 2\alpha n_f x_{max}$$

and so the location of the maxima are:

$$x_{max} = \left(\frac{m+1/2}{2\alpha}\right)\lambda_f \quad \text{where } \lambda_f = \lambda_0 / n_f$$

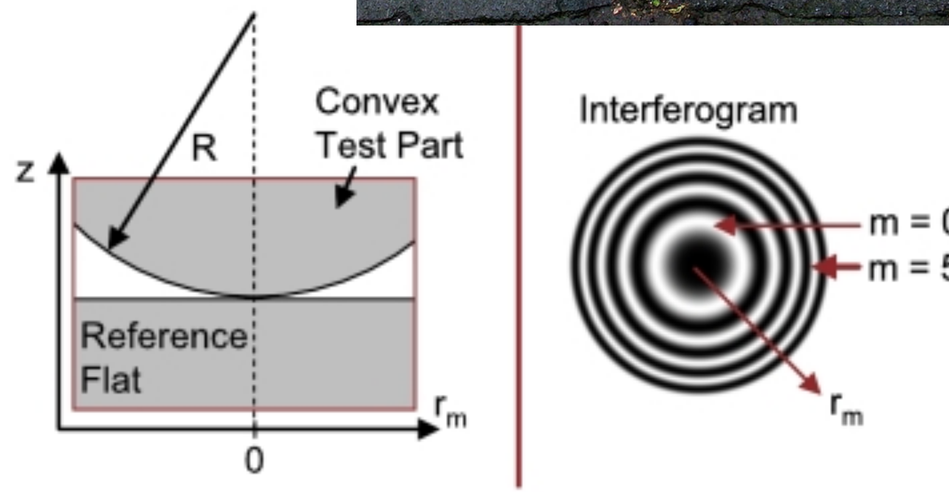
Note that the film thickness at each maxima is:

$$d_{max} = (m+1/2)\frac{\lambda_f}{2} \quad \text{(odd multiples of } \lambda_f/4\text{)}$$



Chapter 3: Interference

- **Newton's Rings are a special case where circular symmetry is involved.**
  - **The wedge thickness varies according to the radius of curvature of the lens (fig. 9.23)**
  - **Used in optical testing to determine radius of curvature and surface accuracy.**
  - **See the derivation in the text**



$$R = \frac{r_m^2}{\lambda\left(m + \frac{1}{2}\right)}$$

Chapter 3: Interference

- **Amplitude-splitting Interferometers**
  - **Michelson Interferometer**

One of the simplest and best-known interferometers.

Beam-splitter at (O) diverts light down each of two beams.

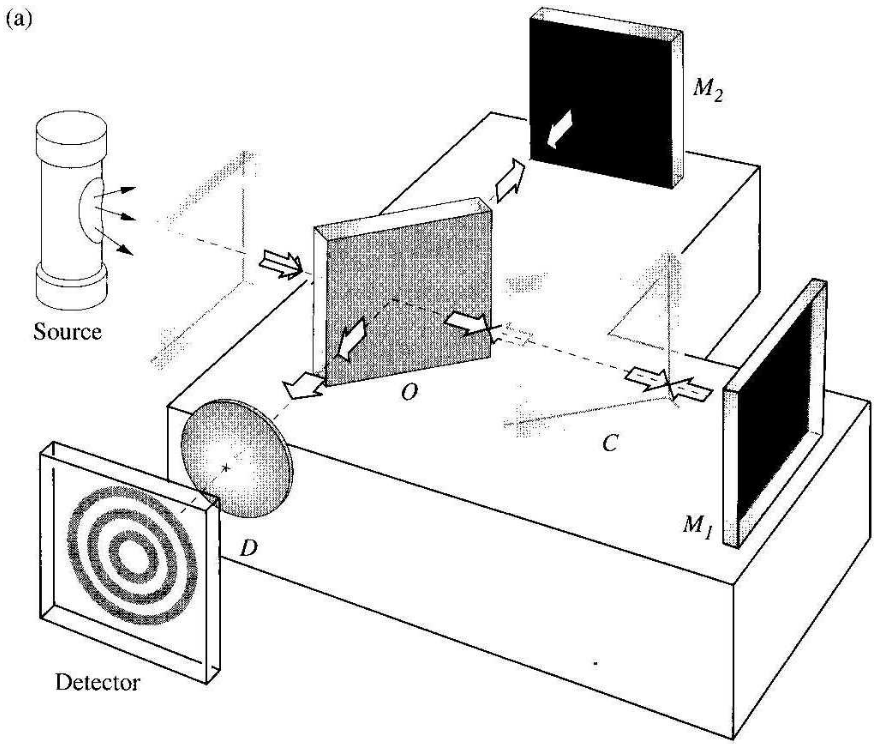
These in turn reflect off mirrors.

The beams recombine at (O) before emerging from the interferometer.

The compensator plate (C) equalizes the phase difference between the two beams. It also reduces dispersion.

Maxima occur when
$2d\cos\theta_m = m\lambda_0$ (circular fringes)

Useful for precise measurements of small distances (small changes in path length).



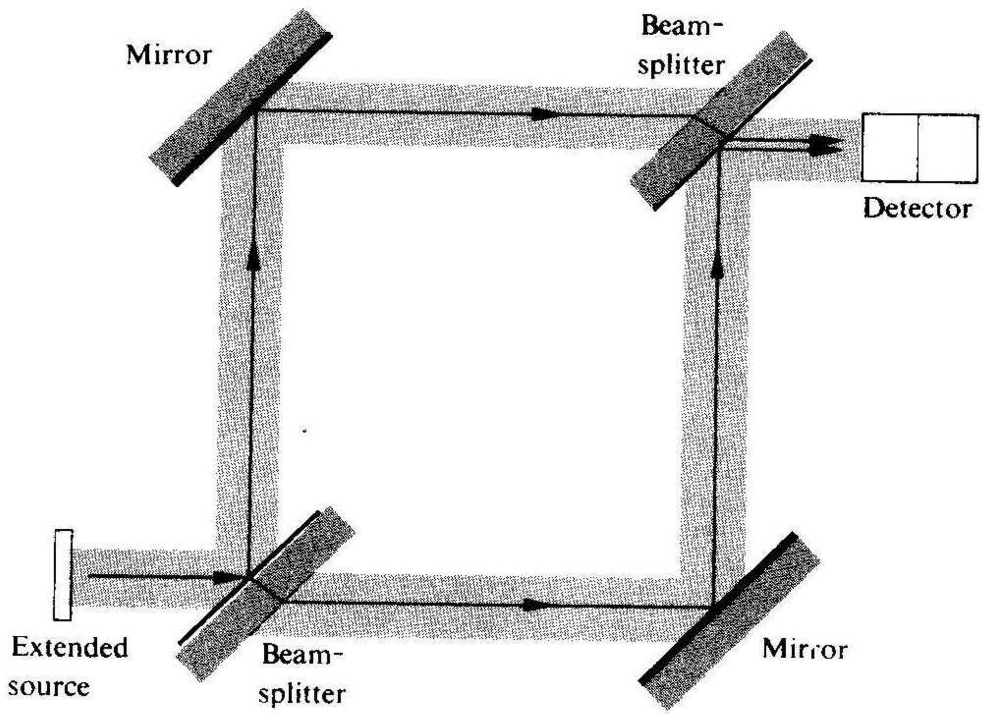
Chapter 3: Interference

- **Amplitude-splitting Interferometers**
  - **Mach-Zehnder Interferometer**

Used to measure the OPL of different materials inserted into one of the beams.

Extremely versatile laboratory instrument.

Examples include wind tunnels and shock tubes. Since index of air depends on density.



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