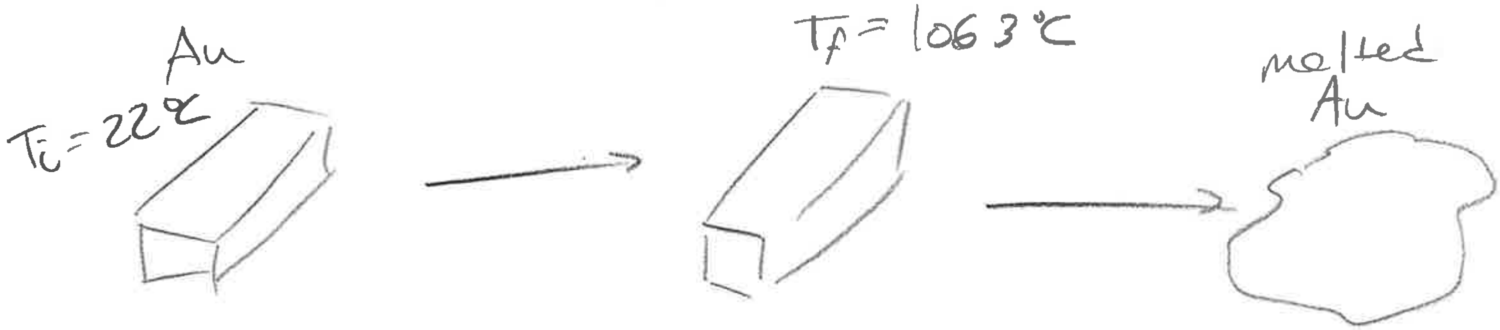
Warm up Problem



$c = 129 \ J \ kg^{-1} \ °C^{-1}$

$m = 1 kg$

$\Delta T = 1063°C - 22°C = 1041°C$

$L_f = 64500 \ J/kg$

Energy budget

39.785 kcal

$4186 \ J = 1 \ kcal$

convert to J

$$39.785 \ kcal \times \frac{4186 \ J}{1 \ kcal} = 166540 \ J$$

Determine heat required to bring gold to its melting point.

$Q = mc\Delta T$

$Q = (1)(129)(1041)$

$Q = 134289 \ J$ ✓

Energy budget

166540 J − 134289 J = 32251 J remaining

Determine maximum mass of gold remaining energy can melt.

$Q = mL_f$

$m = \frac{Q}{L_f}$

$m = \frac{32251}{64500}$

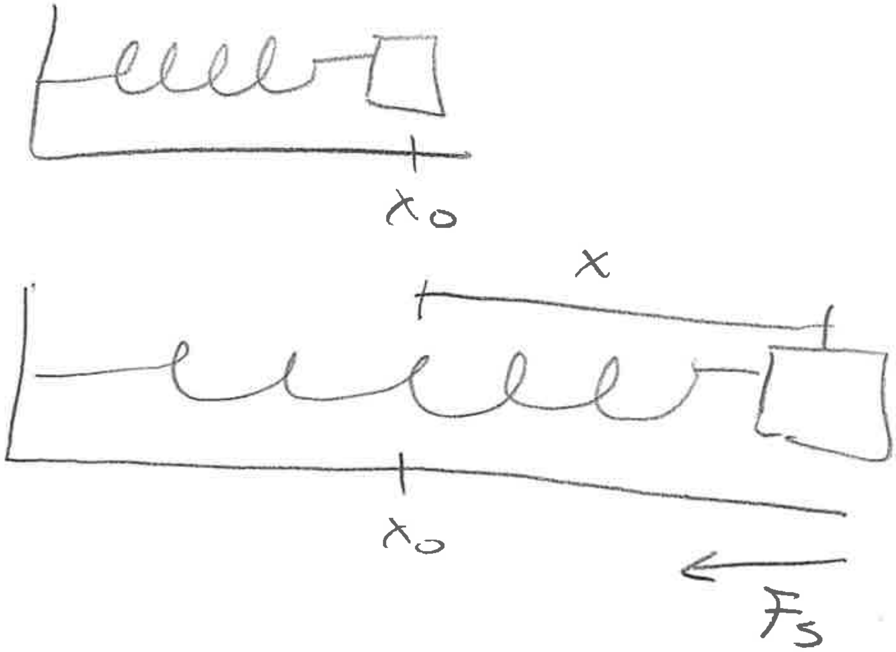
$m = 0.500$ kg

Remaining energy can only melt 500g of Au. 500g remains solid.

Determine how many rings we can make with melted gold.

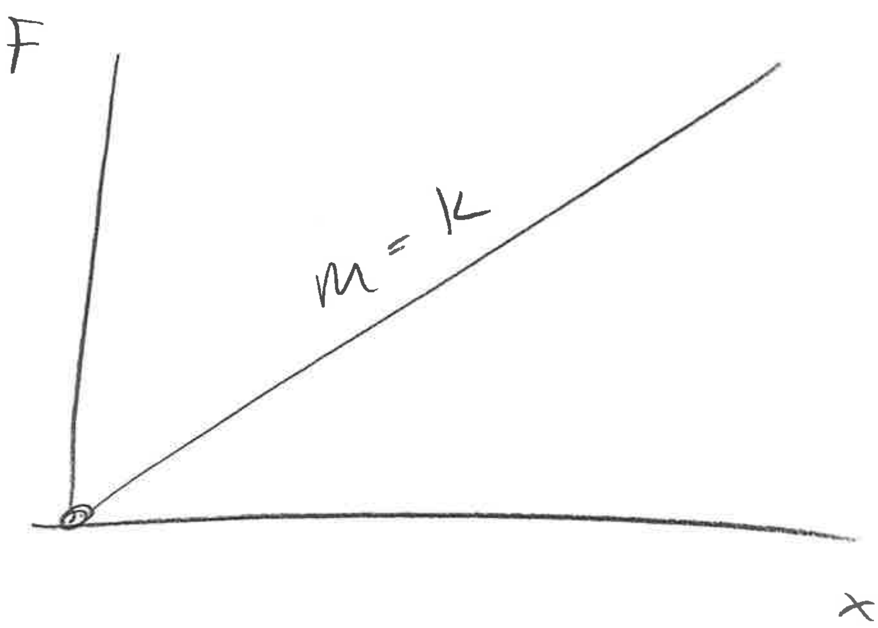
$500\,g\,Au \times \frac{1\,ring}{5\,g\,Au} = \boxed{100\ rings}$

Hooke's Law



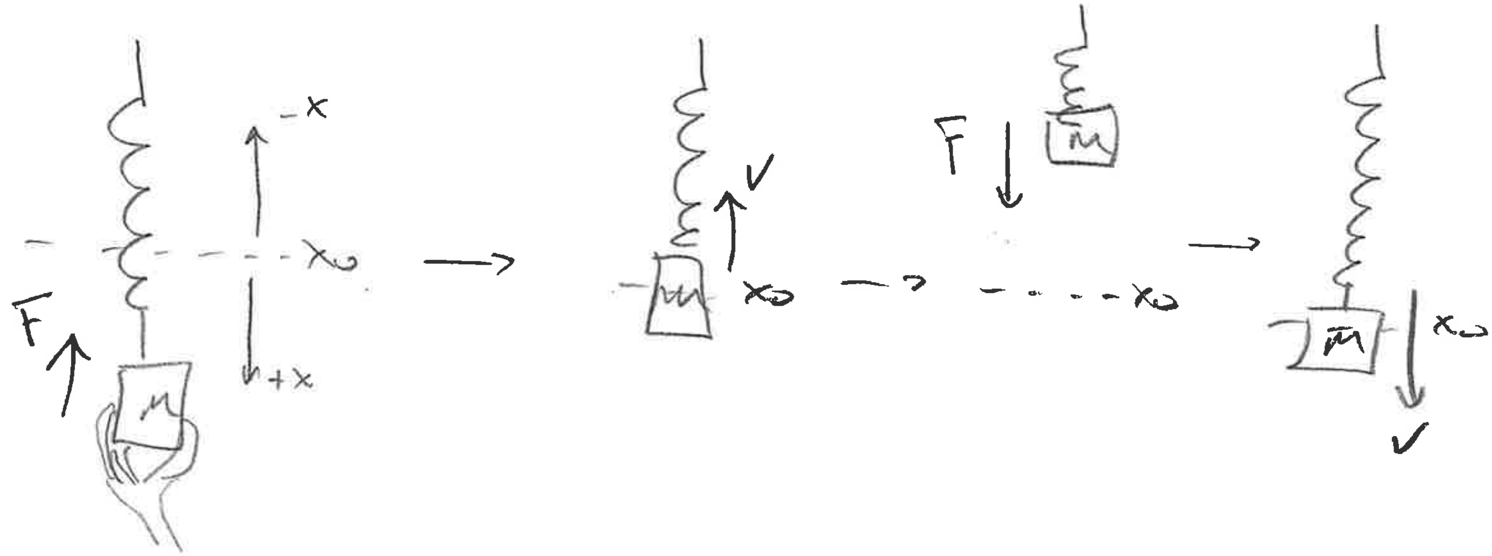
$$F_s = -kx$$

- Restoring force is proportional to negative of displacement.



$$U = \frac{1}{2} k x^2$$

Simple Harmonic Motion



$$F = -kx$$

$$ma = -kx$$

$$m\ddot{x} = -kx$$

$$x = -\frac{m}{k}\ddot{x}$$

Solution is well known

$$x = A\cos\left(\sqrt{\frac{k}{m}}\,t + \varphi\right)$$

or

$$x = A\sin\left(\sqrt{\frac{k}{m}}\,t + \varphi\right)$$

Both work, choose whichever is easiest (sin if it starts at x_0, cos if it starts at A)

A - Amplitude (m). Maximum displacement.

φ - phase angle (rad). Not important.

$\sqrt{\frac{k}{m}}$ - Angular frequency of oscillations

$$\omega = \sqrt{\frac{k}{m}} \quad (rad/s)$$

$$\omega = 2\pi f$$

f - linear frequency of oscillations

(cycles/s or Hz).

$$T = \frac{1}{f}$$

T - period of oscillations

(seconds per cycle)

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}} \quad \left(\frac{1}{s} \text{ or Hz}\right)$$

$$T = 2\pi\sqrt{\frac{m}{k}} \quad (s)$$

For <u>simple harmonic motion</u>

$$x = A\cos\omega t$$

$$v = -A\omega\sin\omega t$$

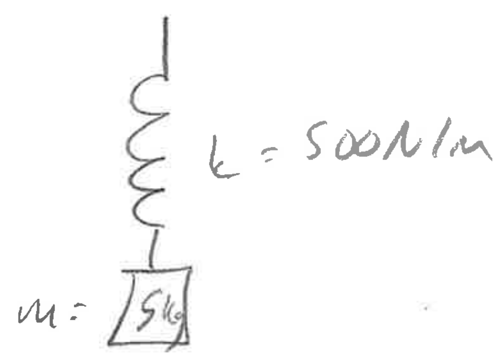
$$a = -A\omega^2\cos\omega t$$

maximum v: $A\omega$

maximum a: $A\omega^2$

<u>Spring Example</u>

A spring with $k = 500$ N/m is made to oscillate with a 5 kg mass. The block is displaced 1 m. Calculate the angular frequency and maximum velocity.



$$\omega = \sqrt{\frac{k}{m}}$$

$$\boxed{\omega = 10 \text{ rad/s}}$$

$V_0 = A\omega$

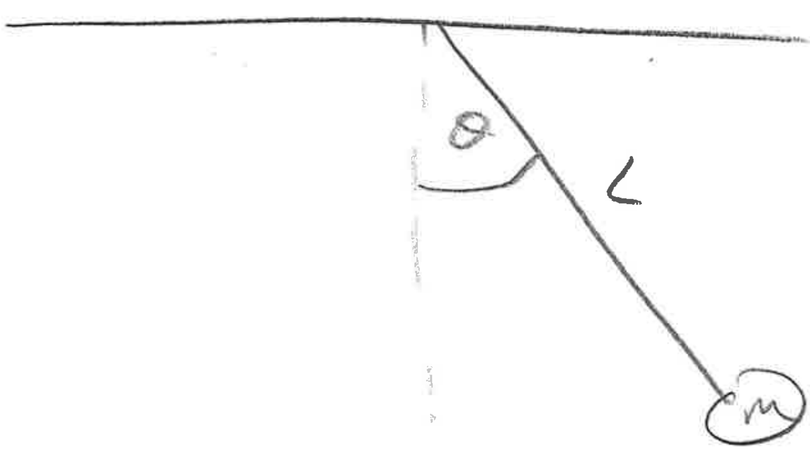
$$\boxed{V_0 = 10 \text{ m/s}}$$

Conservation of Energy

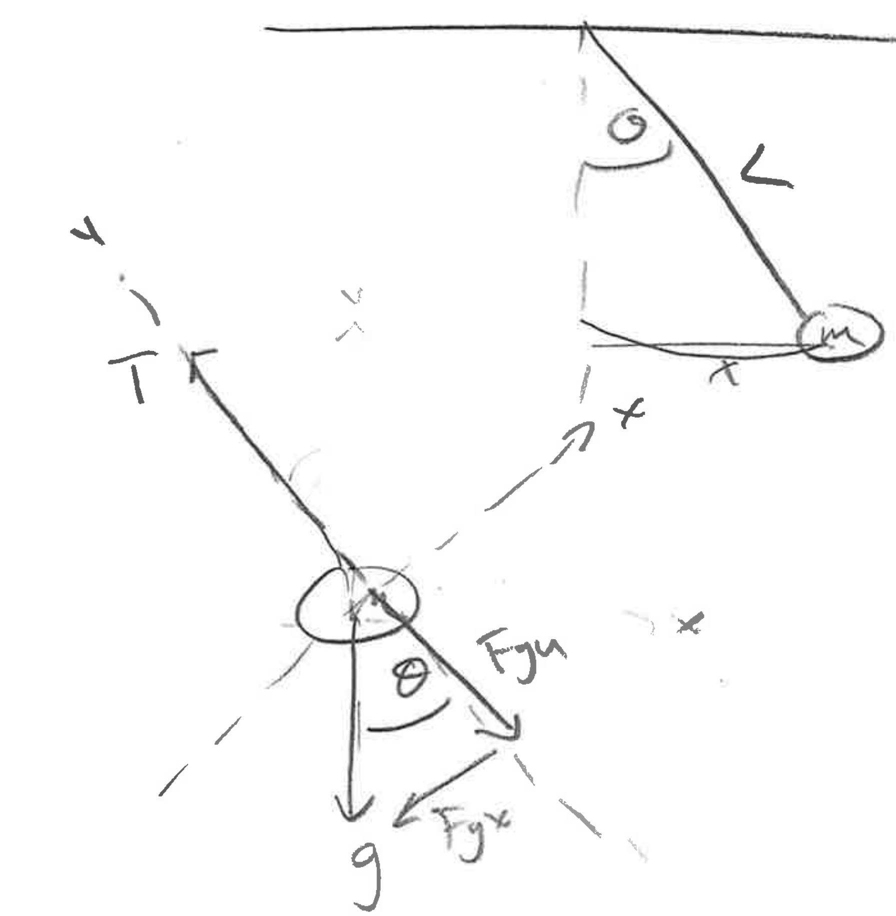
- Energy is transformed between kinetic and potential forms.

$$\underbrace{\frac{1}{2}kA^2}_{\text{Maximally displaced}} = \underbrace{\frac{1}{2}mV_0^2}_{\text{Maximum kinetic energy}} = \underbrace{\frac{1}{2}kx^2 + \frac{1}{2}mv^2}_{\text{Somewhere in between}}$$

Simple Pendulum



- what factors dictate the period of the pendulum?



$$\sum F_y = T - F_{gy} = 0$$

Nothing interesting

$$\sum F_x = F_{gx} = ma_x$$

$$-mg\sin\theta = ma_x$$

for small θ

$$\sin\theta \sim \theta$$

$$\theta = \frac{x}{L}$$

$$-\cancel{m}g\frac{x}{L} = \cancel{m}\ddot{x}$$

$$\ddot{x} = -\frac{g}{L}x$$

Solution is well known, simple harmonic motion!

$$x = A\sin\left(\sqrt{\frac{g}{L}}\,t + \varphi\right) \quad (\text{or } \cos(\ldots))$$

Angular frequency

$$\omega = \sqrt{\frac{g}{L}}$$

Linear frequency

$$\omega = 2\pi f$$

$$f = \frac{1}{2\pi}\sqrt{\frac{g}{L}}$$

Period

$$T = \frac{1}{f}$$

$$T = 2\pi\sqrt{L/g}$$

Period of oscillations only depends on L and g!

Pendulum Problem

A pendulum of length L oscillates with a period of 6 s. What is the period for a pendulum of length $4L$?

$$T = 2\pi\sqrt{\frac{L}{g}}$$

$$T \propto \sqrt{L}$$

$$T_1 = 6\ s$$

$$L_2 = 4L_1$$

$$\frac{T_2}{T_1} = \frac{\sqrt{L_2}}{\sqrt{L_1}}$$

$$T_2 = T_1\sqrt{\frac{L_2}{L_1}}$$

$$T_2 = T_1\sqrt{\frac{4L_1}{L_1}}$$

$$T_2 = T_1\sqrt{4}$$

$$T_2 = 2T_1$$

$$\boxed{T_2 = 12\ s}$$