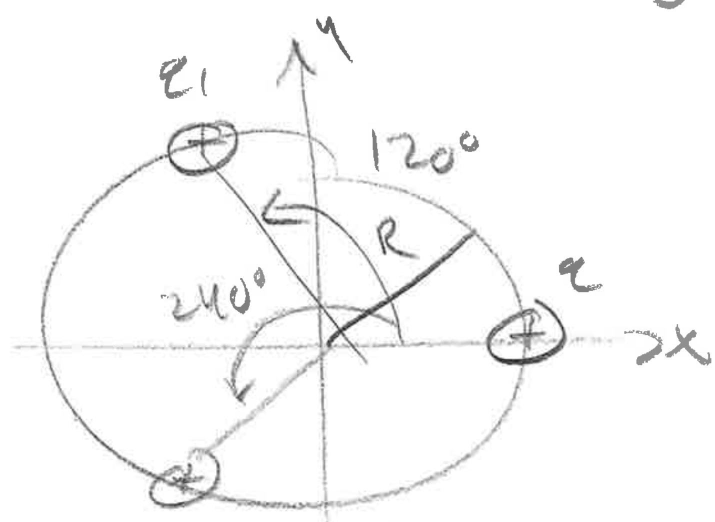
Warm Up Problem

Three charges are arranged along the circumference of a circle of radius $R = 2m$ with angles 0°, 120°, and 240°. Calculate the net $\vec{E}$ field at the center of the ring.



$$\vec{E}_{net} = \vec{E}_1 + \vec{E}_2 + \vec{E}_3$$

$$E_{net,x} = E_{1x} + E_{2x} + E_{3x}$$

$$E_{net,x} = \frac{kQ}{R^2} + \frac{kQ}{R^2}\cos(120°) + \frac{kQ}{R^2}\cos(240°)$$

$$E_{net,x} = \frac{kQ}{R^2}\left[1 + \cos(120°) + \cos(240)\right]$$

$$E_{net,x} = \frac{kQ}{R^2}\left[\, 0 \,\right]$$

$$\boxed{E_{net,x} = 0}$$

$$E_{net,y} = \frac{kQ}{R^2}\sin(120°) + \frac{kQ}{R^2}\sin(240°)$$

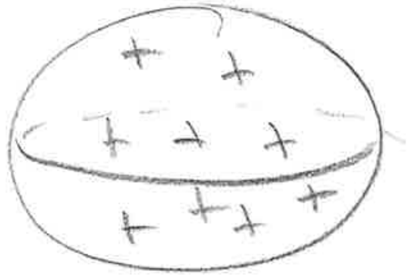
$$E_{net,y} = \frac{kQ}{R^2}\left[\sin(120°) + \sin(240°)\right]$$

$$E_{net,y} = \frac{kQ}{R^2}\left[\, 0 \,\right]$$

$$\boxed{E_{net,y} = 0}$$

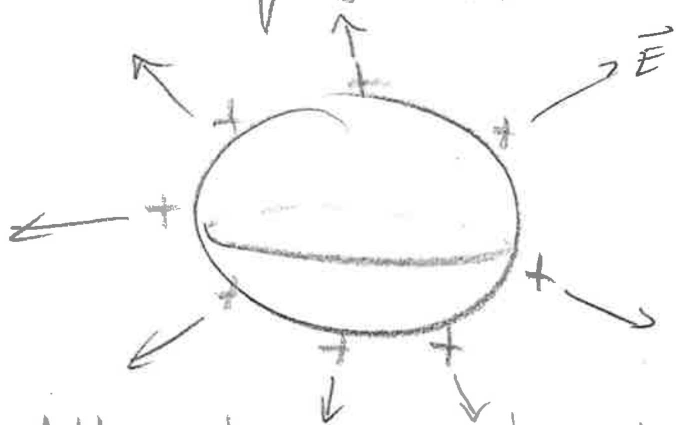
$$\boxed{\vec{E} = 0 \text{ N/C}}$$

Electric Fields and Conductors



- Imagine a conducting sphere with excess charge $+q$.

- Because it is a conductor, charges will <u>separate</u>.



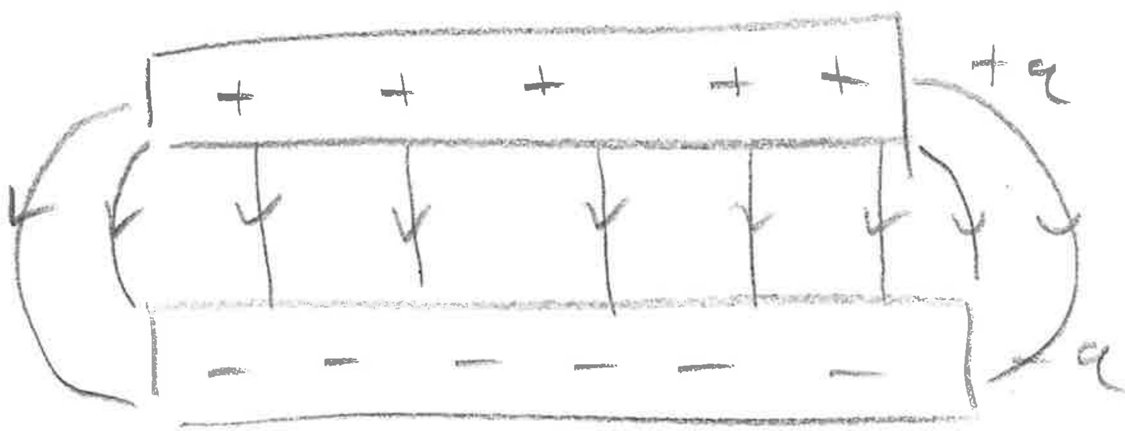
- All charges distribute on surface of conductor.

<u>3 Takeaways</u>

1. The $\vec{E}$ field inside a conductor is 0.
2. Outside of the conductor, $\vec{E}$ field lines are perpendicular to the surface.
3. All excess charge resides on the surface.

Parallel Plates

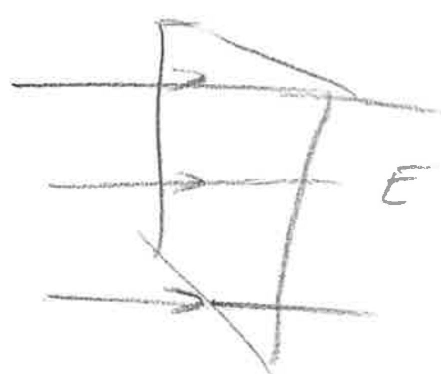
- Two conducting plates with equal but opposite charges can generate a uniform $\vec{E}$ field between the plates.



- $\vec{E}$ field bends near the edges.
- Uniform $\vec{E}$ in between plates.

Gauss' Law and $\vec{E}$ flux (Φ_E)

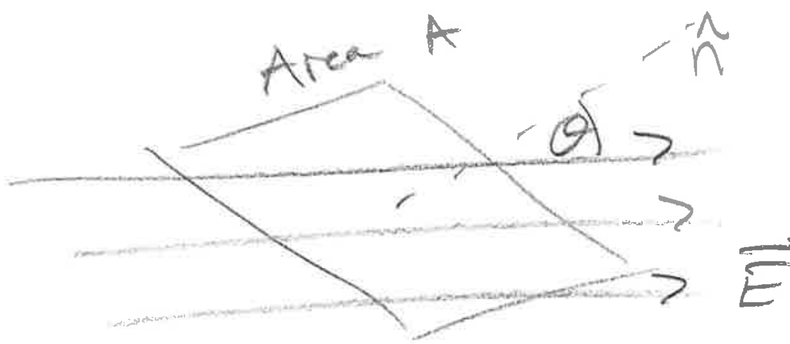
Electric flux: $\vec{E}$ field passing through area.



$$\Phi_E \equiv \vec{E} \cdot \vec{A}$$

- Electric flux is the dot product between Electric field and area vectors.
  - ↳ Area vector is <u>normal</u> (90°) to the surface

$$\Phi_E = |\vec{E}||\vec{A}| \cos\theta$$



+ if going with area (out)

− if going against area (in).

Gauss' Law

$$\Phi_E = \frac{q_{enc}}{\varepsilon_0}$$

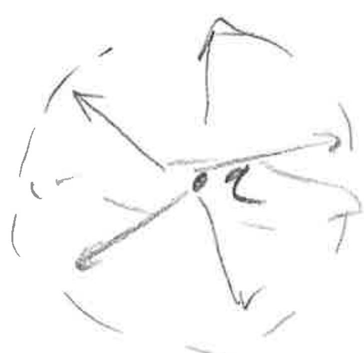
Φ_E – Electric flux (Nm^2/C)

q_{enc} – charge enclosed inside surface (C)

ε_0 – permittivity of free space ($8.85 \times 10^{-12}\ C^2/Nm^2$)

Gauss Law Example

The Electric flux through a Gaussian spherical surface is measured to be 565 Nm^2/C. Find the enclosed charge.



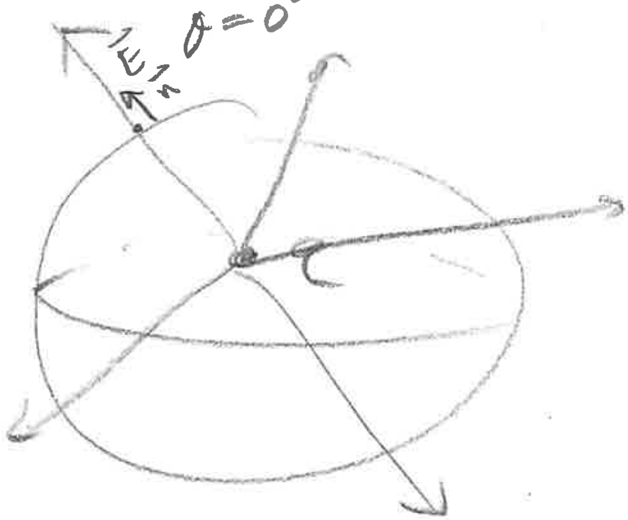
$$\Phi_E = 565 \; Nm^2/C$$

$$\Phi_E = \frac{q_{enc}}{\varepsilon_0}$$

$$q_{enc} = \varepsilon_0 \Phi_E$$

$$\boxed{q_{enc} = 5 \, nC}$$

Because $\vec{E}$ and $\vec{A}$ need to be aligned, most common to use a Gaussian surface of a sphere.



Gauss' Law Example

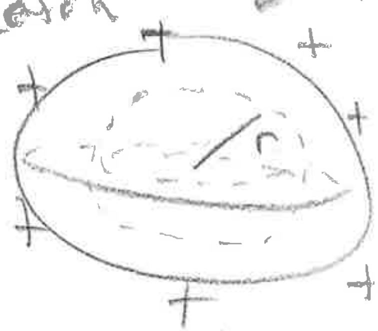
(r = 5m)

A conducting sphere has an excess charge of 10 nC. Calculate the $\vec{E}$ field:

a) Inside of the sphere.

b) Outside of the sphere

c) Sketch $\vec{E}$ vs r.



a) $\vec{E} \cdot 4\pi r^2 = \dfrac{q_{enc}}{\varepsilon_0}$ (8)

$q_{enc} = 0$

$\vec{E} \cdot 4\pi r^2 = 0$

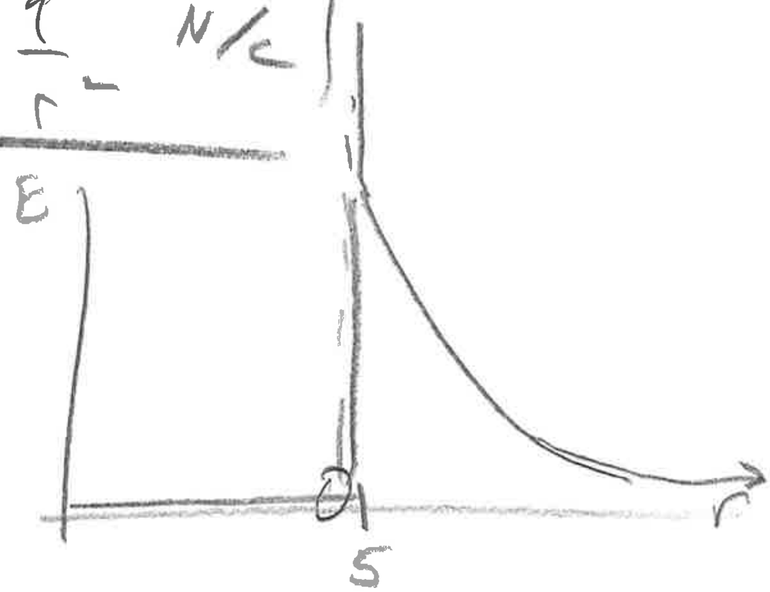
$\boxed{\vec{E} = 0}$

b) $\vec{E} \cdot 4\pi r^2 = \dfrac{q_{enc}}{\varepsilon_0}$

$q_{enc} = 10 \times 10^{-9}\, C$

$E = \dfrac{q_{enc}}{4\pi r^2 \varepsilon_0}$ $\longrightarrow$ Coulombs Law!

$\boxed{E = \dfrac{9}{r^2}\ N/c}$



Gauss' Law with an insulating sphere:

Inside the spere $(r < R)$:

$$\Phi_E = \frac{q_{enc}}{\varepsilon_0}$$

- q_{enc} is the charge density (ρ) of spere $\left(q / \tfrac{4}{3}\pi R^3 \right)$ times the volume enclosed.

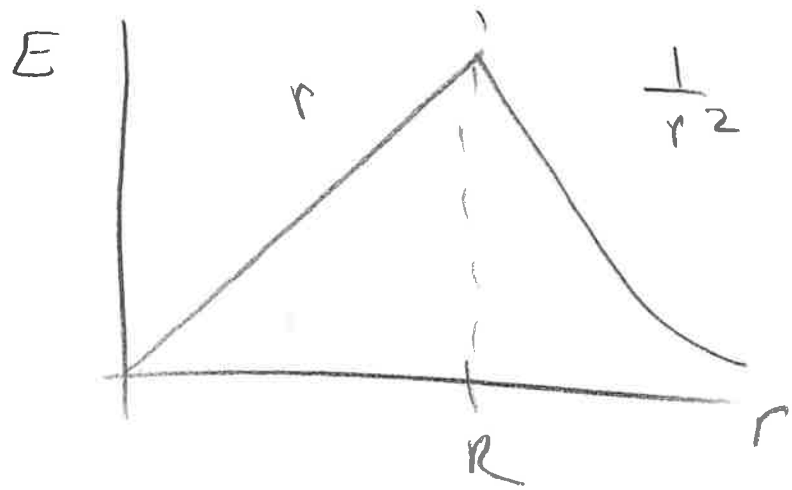
$$E\, 4\pi r^2 = \frac{1}{\varepsilon_0}\left(\rho \cdot \frac{4}{3}\pi r^3\right)$$

$$E\, \cancel{4\pi} \cancel{r^2} = \frac{1}{\varepsilon_0}\, \frac{\cancel{4}}{3}\, \cancel{\pi}\, r^{\cancel{3}} \rho$$

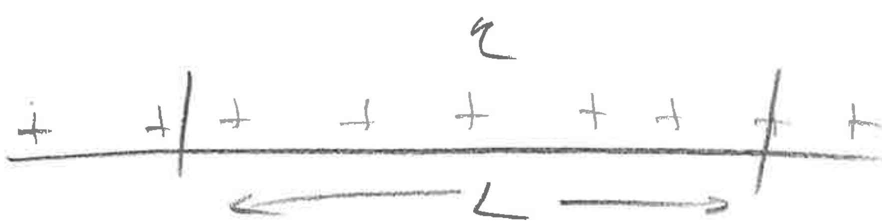
$$\boxed{E = \frac{\rho r}{3\varepsilon_0} \qquad (r < R)}$$

Outside sphere → point charge.

$$E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2} \qquad (r > R)$$

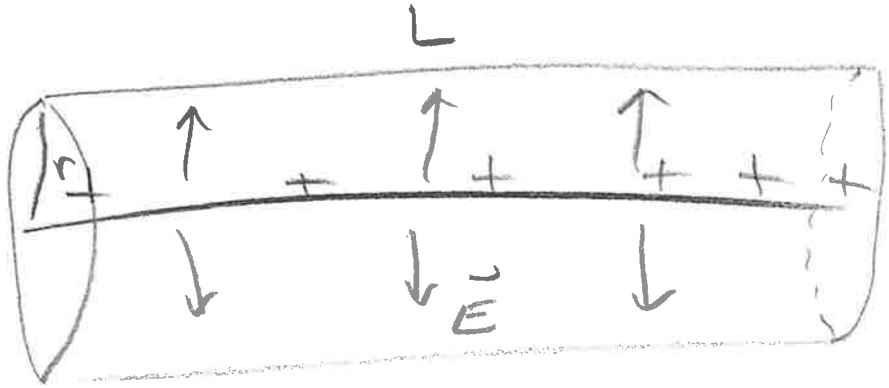


Infinite wire



Charge per unit length: $\frac{q}{L} = \lambda$

Make Gaussian cylinder



$$E \cdot A = \frac{q_{enc}}{\varepsilon_0}$$

$$E(2\pi r L) = \frac{\lambda L}{\varepsilon_0}$$ → charge enclosed in cylinder.

$$E\, 2\pi r \cancel{L} = \frac{\lambda \cancel{L}}{\varepsilon_0}$$

$$\boxed{E = \frac{\lambda}{2\pi r \varepsilon_0}}$$

$$E \propto \frac{1}{r}$$