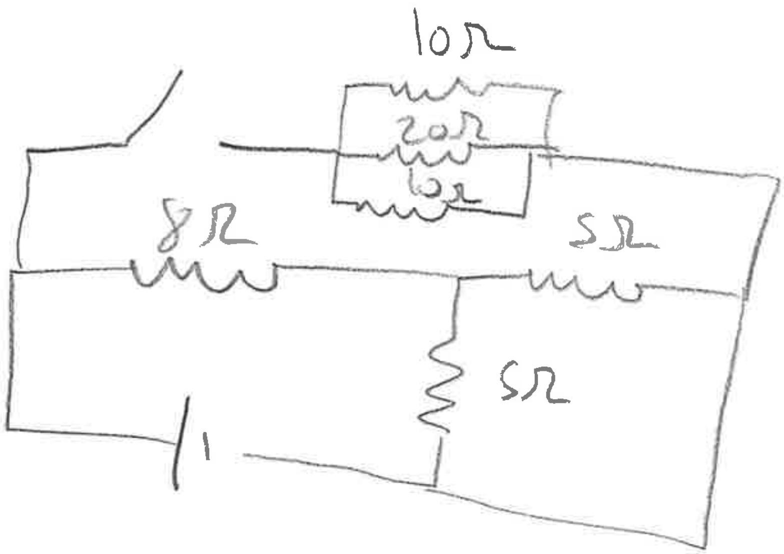
Warm Up Problem

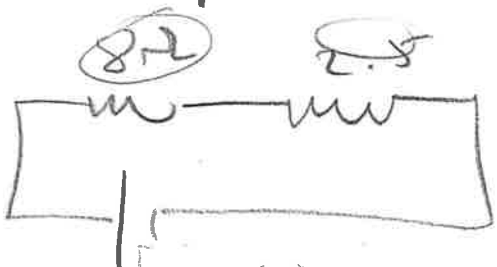


a) Find the equivalent resistance when switch is open.

No current flows in top branch, circuit simplifies.

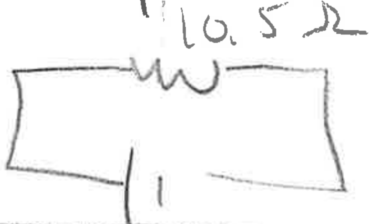
$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2}$$

$$\frac{1}{R_{eq}} = \frac{1}{5} + \frac{1}{5}$$



$$\frac{1}{R_{eq}} = \frac{2}{5}$$

$$R_{eq} = \frac{5}{2} = 2.5$$



$$R_{eq} = 8 + 2.5$$

$$R_{eq} = 10.5$$

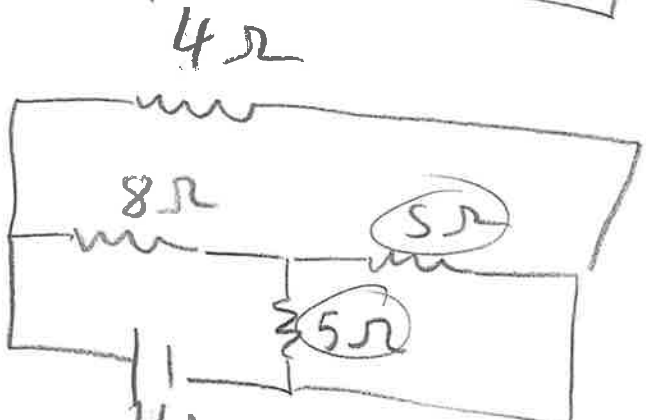
$$\boxed{R_{eq} = 10.5\,\Omega}$$

b) Now swith is closed. Current flows through all branches.

$$\frac{1}{R_{eq}} = \frac{1}{10} + \frac{1}{20} + \frac{1}{10}$$

$$\frac{1}{R_{eq}} = \frac{5}{20}$$

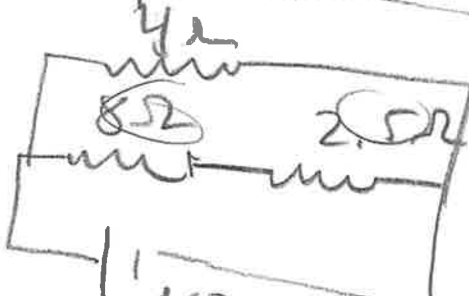
$$R_{eq} = 4\Omega$$



$$\frac{1}{R_{eq}} = \frac{1}{5} + \frac{1}{5}$$

$$\frac{1}{R_{eq}} = \frac{2}{5}$$

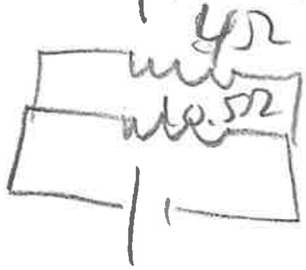
$$R_{eq} = 2.5$$



$$R_{eq} = R_1 + R_2$$

$$R_{eq} = 8 + 2.5$$

$$R_{eq} = 10.5$$



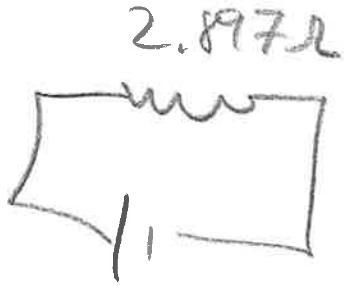
$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2}$$

$$\frac{1}{R_{eq}} = \frac{1}{4} + \frac{2}{21}$$

$$\frac{1}{R_{eq}} = \frac{21 + 8}{84}$$

$$\frac{1}{R_{eq}} = \frac{29}{84}$$

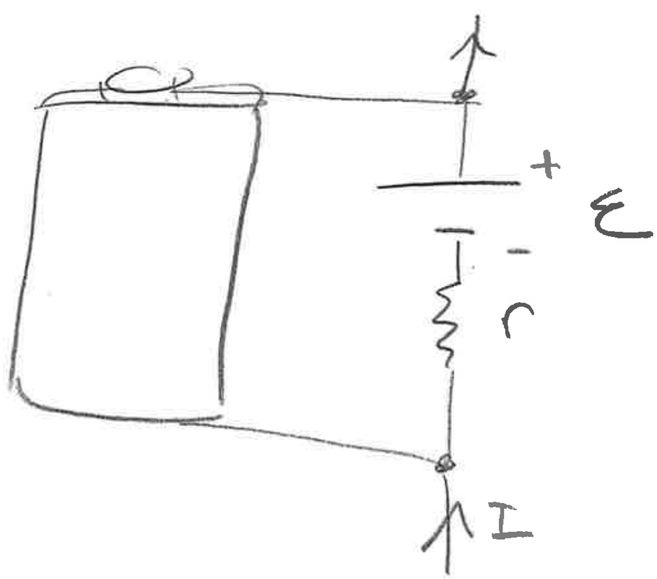
$$R_{eq} = \frac{84}{29} = 2.897\,\Omega$$



$$\boxed{R_{eq} = 2.897\,\Omega}$$

Electromotive force

- Potential difference of a voltage source when no current is flowing.



$$V = \varepsilon - Ir$$

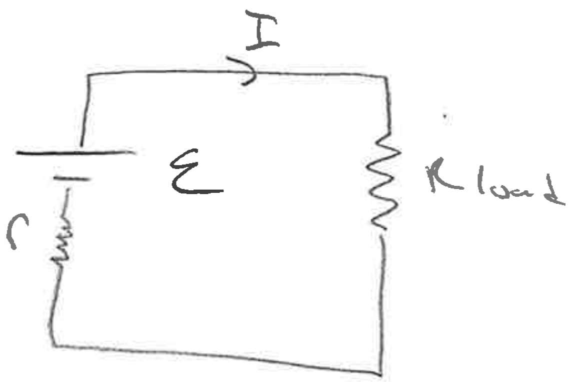
V – terminal voltage (V)

ε – electromotive force (emf).

I – current from battery (A)

r – internal resistance of battery (Ω)

Circuit with load



$$I = \frac{\varepsilon}{r + R_{load}}$$

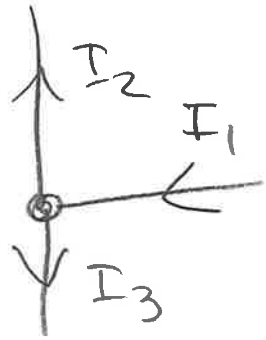
r increases as battery is depleted

↳ less I for constant R_{load}.

Kirchoff's Rules

1. Junction Rule – the sum of all the currents entering a junction equals the sum of all the currents leaving the junction.
2. Loop Rule – the sum of all of the changes in voltage around any closed loop in a circuit is zero.

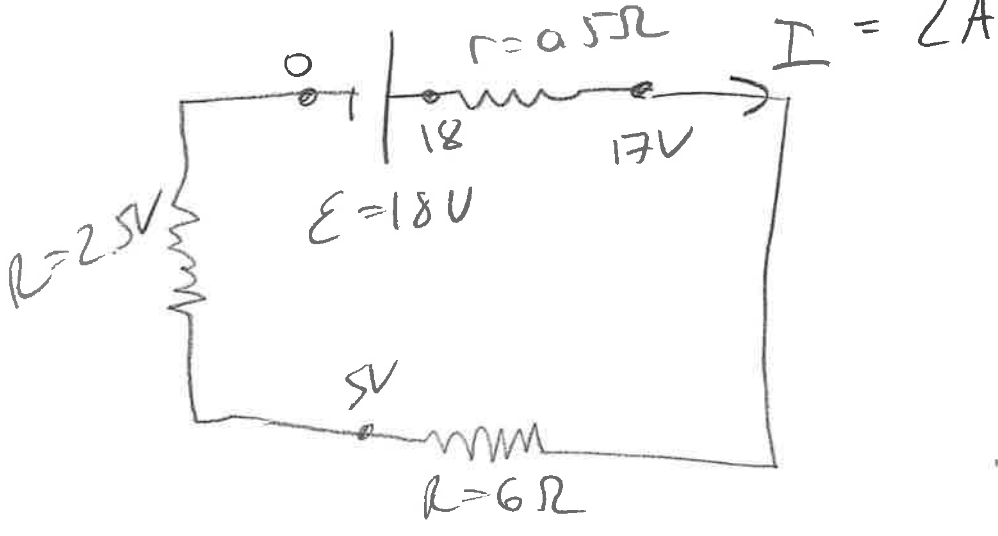
Junction Rule



$$I_1 = I_2 + I_3$$

- for unknown circuit, direction is arbitrary.
  - ↳ Assuming the wrong direction will give a negative current. When reporting your answer, flip assumed direction for all negative currents.

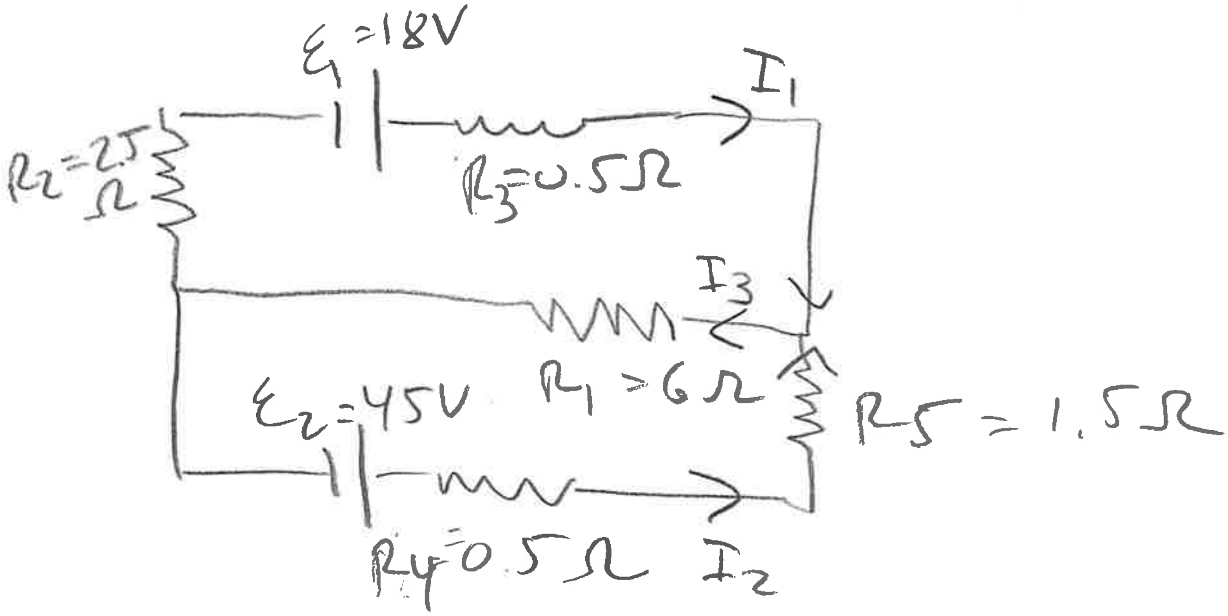
Loop Rule



- Going from − to + on battery is positive V
- Going with current through a resistor is a drop.
- clockwise

Kirchoff's Rule Example

Find the currents through all resistors.



3 unknown currents

3 equations required

① $18 - 0.5 I_1 - 6 I_3 - 2.5 I_1 = 0$

② $6 I_3 + 1.5 I_2 + 0.5 I_2 - 45 = 0$

③ $I_1 + I_2 = I_3$ Rewrite

① $-3 I_1 - 6 I_3 = -18$

② $2 I_2 + 6 I_3 = 45$

③ $I_1 + I_2 = I_3$

Plug 3 into 1 and 2

1 $-3I_1 - 6(I_1 + I_2) = -18$

2 $2I_2 + 6(I_1 + I_2) = 45$

1 $-3I_1 - 6I_1 - 6I_2 = -18$

1 $-9I_1 - 6I_2 = -18$

2 $2I_2 + 6I_1 + 6I_2 = 45$

2 $6I_1 + 8I_2 = 45$

use 2 to solve for I_2, plug into 1

$$6I_1 + 8I_2 = 45$$

$$8I_2 = 45 - 6I_1$$

$$I_2 = \frac{45}{8} - \frac{3}{4}I_1$$

1 $-9I_1 - 6I_2 = -18$

$$-9I_1 - 6\left[\frac{45}{8} - \frac{3}{4}I_1\right] = -18$$

$$-9I_1 - \frac{3 \cdot 45}{4} + \frac{18}{4}I_1 = -18$$

$$I_1\left[-9+\frac{9}{2}\right] + \frac{135}{4} = -18$$ 8

$$I_1\left(-\frac{9}{2}\right) = \frac{-72+135}{4}$$

$$I_1\left(-\frac{9}{2}\right) = \frac{63}{4}$$

$$I_1 = \frac{63}{\cancel{4}_2} \cdot -\frac{\cancel{2}}{9}$$

$$I_1 = -\frac{63}{2\cdot 9}$$

$$I_1 = -\frac{7}{2}$$

$$\boxed{I_1 = -3.5\,A}$$

From before

$$I_2 = \frac{45}{8} - \frac{3}{4} I_1$$

$$I_2 = \frac{45}{8} - \frac{3}{4}\left(-\frac{7}{2}\right)$$

$$I_2 = \frac{45}{8} + \frac{21}{8}$$

$$I_2 = \frac{66}{8}$$

$$I_2 = \frac{33}{4}$$

$$\boxed{I_2 = 8.25A}$$

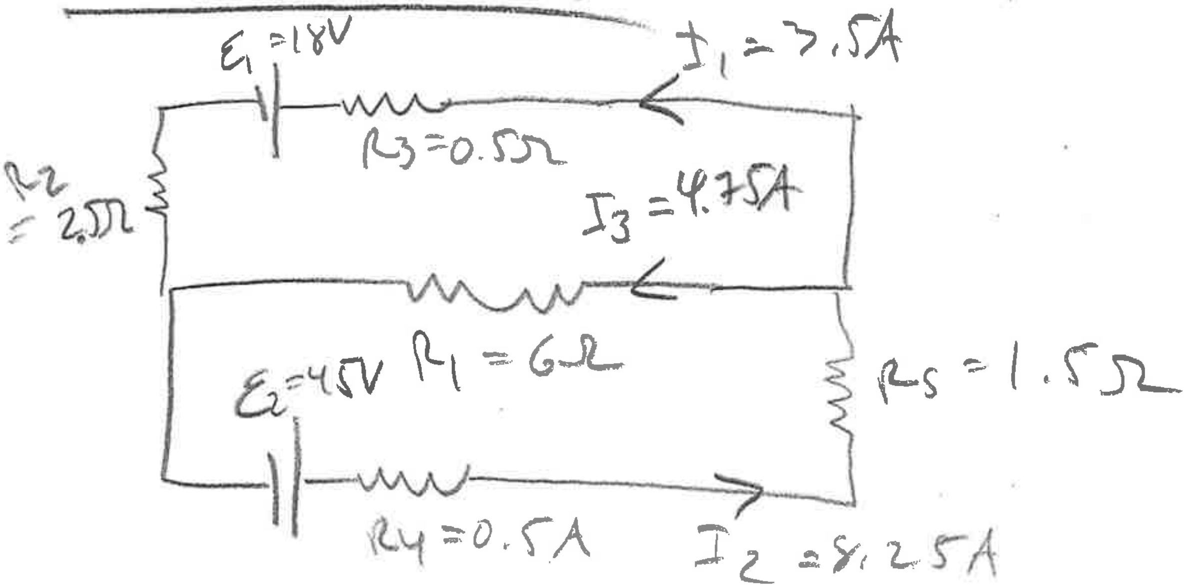
$$I_3 = I_1 + I_2$$

$$I_3 = -\frac{7}{2} + \frac{33}{4}$$

$$I_3 = \frac{33 - 14}{4}$$

$$I_3 = \frac{19}{4}$$

$$\boxed{I_3 = 4.75A}$$



Use row reduction

1 $-3I_1 + 0I_2 - 6I_3 = -18$

$0I_1 + 2I_2 + 6I_3 = 45$

$1I_1 + 1I_2 - 1I_3 = 0$

$$\begin{array}{cccc} I_1 & I_2 & I_3 & = \end{array}$$
$$\left[\begin{array}{ccc|c} -3 & 0 & -6 & -18 \\ 0 & 2 & 6 & 45 \\ 1 & 1 & -1 & 0 \end{array}\right]$$

$$\downarrow$$

$$\begin{array}{ccc} I_1 & I_2 & I_3 \end{array}$$
$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & -3.5 \\ 0 & 1 & 0 & 8.25 \\ 0 & 0 & 1 & 4.75 \end{array}\right]$$

$$\boxed{\begin{aligned} I_1 &= -3.5A \\ I_2 &= 8.25A \\ I_3 &= 4.75 \end{aligned}}$$

Same answer!

Voltmeter

- voltage is the same in parallel
- connect voltmeters in parallel

Ammeter

- current is the same in series.
- connect ammeter in series.