Warm up

Calculate the energy released if eight hydrogen atoms and eight neutrons combine to form an ^{16}O atom.

$$m_p = 1.007825\ u$$

$$m_N = 1.008665\ u$$

$$m_O = 15.994915$$

$$1u = 1.660539 \times 10^{-27} kg$$

$$BE = \left[(Zm_p + Nm_n) - m_c \right] c^2$$

$$BE = 2.05 \times 10^{-11}\ J$$

$$\boxed{BE = 128\ MeV}$$

swapping Wednesday and Thursday plans

Bohr's Theory of the Hydrogen atom

- Planetary model of atom from Rutherford experiment.

- Niels Bohr applied the planetary model to the H atom.

- H atom characteristics had been observed, but couldn't be explained.

- H Spectra

- Postulates

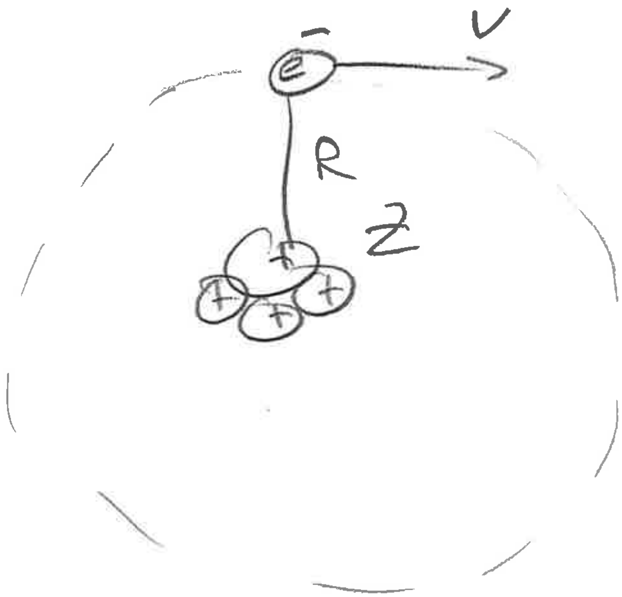
1. Angular momentum was quantized

$$L = m_e v r = n \frac{h}{2\pi} \quad n = 1, 2, 3, \ldots$$

$(h = 6.63 \times 10^{-34}\ Js)$ - Planck constant

2. Hydrogen-like (Z protons, 1 e^-)

3. Accelerating charges do not in spiral



$$\sum F = ma$$

$$a = \frac{v^2}{R}$$

$$\frac{1}{4\pi\varepsilon_0}\frac{(Ze)(e)}{R^2} = \frac{mv^2}{R}$$

(1) $$\boxed{v^2 = \frac{Ze^2}{4\pi\varepsilon_0 R m}}$$

$$L = \frac{nh}{2\pi}$$

$$mvR = \frac{nh}{2\pi}$$

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$$\boxed{v = \frac{nh}{2\pi mR}}$$

Find total energy

$$E = K - U$$

$E < 0$, bound

$E \geq 0$, unbound

$$E = \frac{1}{2}mv^2 - qV$$

$$E = \frac{1}{2}mv^2 - e\left(\frac{Ze}{4\pi\varepsilon_0 R}\right)$$

$$\boxed{E = \frac{1}{2}mv^2 - \frac{Ze^2}{4\pi\varepsilon_0 R}} \quad 3$$

Bring [illegible] in (3)

Rewrite 2 in terms of v^2

$$v = \frac{nh}{2\pi m R}$$

$$v^2 = \frac{n^2h^2}{4\pi^2 m^2 R^2}$$

Substitute 1

$$v^2 = \frac{Ze^2}{4\pi\varepsilon_0 R m}$$

$$\frac{n^2h^2}{\cancel{4}\pi^{\cancel{2}} m^{\cancel{2}} R^{\cancel{2}}} = \frac{Ze^2}{\cancel{4}\cancel{\pi}\varepsilon_0 \cancel{R}\cancel{m}}$$

$$\frac{n^2h^2}{\pi m R} = \frac{Ze^2}{\varepsilon_0} \quad (4)$$

$$\boxed{R = \frac{n^2h^2\varepsilon_0}{Ze^2 m\pi}}$$

Bohr Radius $n = 1, 2, 3, \ldots$

Rewrite 3

$$E = \frac{1}{2} m v^2 - \frac{Ze^2}{4\pi \varepsilon_0 R}$$

Sub in 1

$$E = \frac{1}{2} \cancel{m} \left[\frac{Ze^2}{4\pi \varepsilon_0 R \cancel{m}} \right] - \frac{Ze^2}{4\pi \varepsilon_0 R}$$

$$E = \frac{Ze^2}{8\pi \varepsilon_0 R} - \frac{Ze^2}{4\pi \varepsilon_0 R}$$

$$E = \frac{Ze^2}{4\pi \varepsilon_0 R} \left[\frac{1}{2} - 1 \right]$$

$$E = \frac{Ze^2}{4\pi \varepsilon_0 R} \left[-\frac{1}{2} \right]$$

$$E = - \frac{Ze^2}{8\pi \varepsilon_0 R}$$

Plug in 4

$$E = - \frac{Ze^2}{8\cancel{\pi} \varepsilon_0} \left[\frac{Ze^2 m \cancel{\pi}}{n^2 h^2 \varepsilon_0} \right]$$

$$E_n = -\frac{Z^2 e^4 m}{8 n^2 h^2 \varepsilon_0^2}$$

$$E_n = -\frac{e^4 m}{8 h^2 \varepsilon_0^2} \cdot \frac{Z^2}{n^2}$$

$$E_n = -2.17 \times 10^{-18}\,\mathrm{J}\ \frac{Z^2}{n^2}$$

$$\boxed{E_n = -13.6\,\mathrm{eV}\ \frac{Z^2}{n^2}} \qquad n = 1, 2, 3, 4, \ldots$$

Energy of electron in hydrogen-like atom.

$$E_{photon} = -\Delta E_{electron}$$

Energy of photon!

$$E = hf$$

$$E = \frac{hc}{\lambda}$$

$$E_{photon} = -\Delta E_{electron}$$

$$\frac{hc}{\lambda} = -(E_f - E_i)$$

$$\frac{hc}{\lambda} = E_i - E_f$$

$$E_i = \frac{-13.6\ eV\ Z^2}{n_i^2}$$

$$E_f = \frac{-13.6eV\ Z^2}{n_f^2}$$

$$\frac{hc}{\lambda} = \frac{-13.6eV}{n_i^2} - -\frac{13.6eV}{n_f^2}$$

$$\frac{hc}{\lambda} = Z^2 13.6\ eV \left[\frac{1}{n_f^2} - \frac{1}{n_i^2}\right]$$

$$\frac{1}{\lambda} = \underbrace{\frac{13.6\,eV\,Z^2}{hc}}_{\text{Rydberg constant}} \left[\frac{1}{n_f^2} - \frac{1}{n_i^2}\right]$$

$$\boxed{\frac{1}{\lambda} = R\left[\frac{1}{n_f^2} - \frac{1}{n_i^2}\right]}$$

$R = 1.097 \times 10^7\ m^{-1}$ (hydrogen)

Bohr model problem

Singly ionized helium ($Z = 1$) is a hydrogen-like atom.

a) Find E_2

b) Find E_4

c) Find the wavelength of the photon emitted from the $n = 4 \rightarrow 2$ e^- transition.

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$$E_n = -13.6\,eV \frac{Z^2}{n^2}$$

$$Z = 2$$

$$E_n = \frac{-54.4\,eV}{n^2}$$

a) $E_2 = -13.6\,eV$

b) $E_4 = -3.4\,eV$

c) $\Delta E = E_2 - E_4$

$$-\Delta E = E_4 - E_2$$

$$\frac{hc}{\lambda} = -\Delta E$$

$$\lambda = \frac{hc}{-\Delta E}$$

$$\lambda = \frac{hc}{E_4 - E_2} \cdot \frac{1 \; eV}{1.6\times10^{-19} J}$$

$$\lambda = 1.22\times10^{-7}\,m$$

$$\boxed{\lambda = 122\,nm}$$