Warm up

1. Consider the hydrogen atom ($Z=1$). Which initial state in the Lyman series ($n_f = 1$) will produce a photon with wavelength 97.2 nm? Hydrogen has Rydberg constant $R = 1.097 \times 10^7 \ m^{-1}$.

$$\frac{1}{\lambda} = R\left[\frac{1}{n_f^2} - \frac{1}{n_i^2}\right]$$

$$\frac{1}{\lambda R} = \frac{1}{n_f} - \frac{1}{n_i}$$

$$\frac{1}{n_i^2} = \frac{1}{n_f^2} - \frac{1}{\lambda R}$$

$$n_i^2 = \left(\frac{1}{n_f} - \frac{1}{\lambda R}\right)^{-1}$$

$$n_i = \left(\frac{1}{n_f} - \frac{1}{\lambda R}\right)^{-1/2}$$

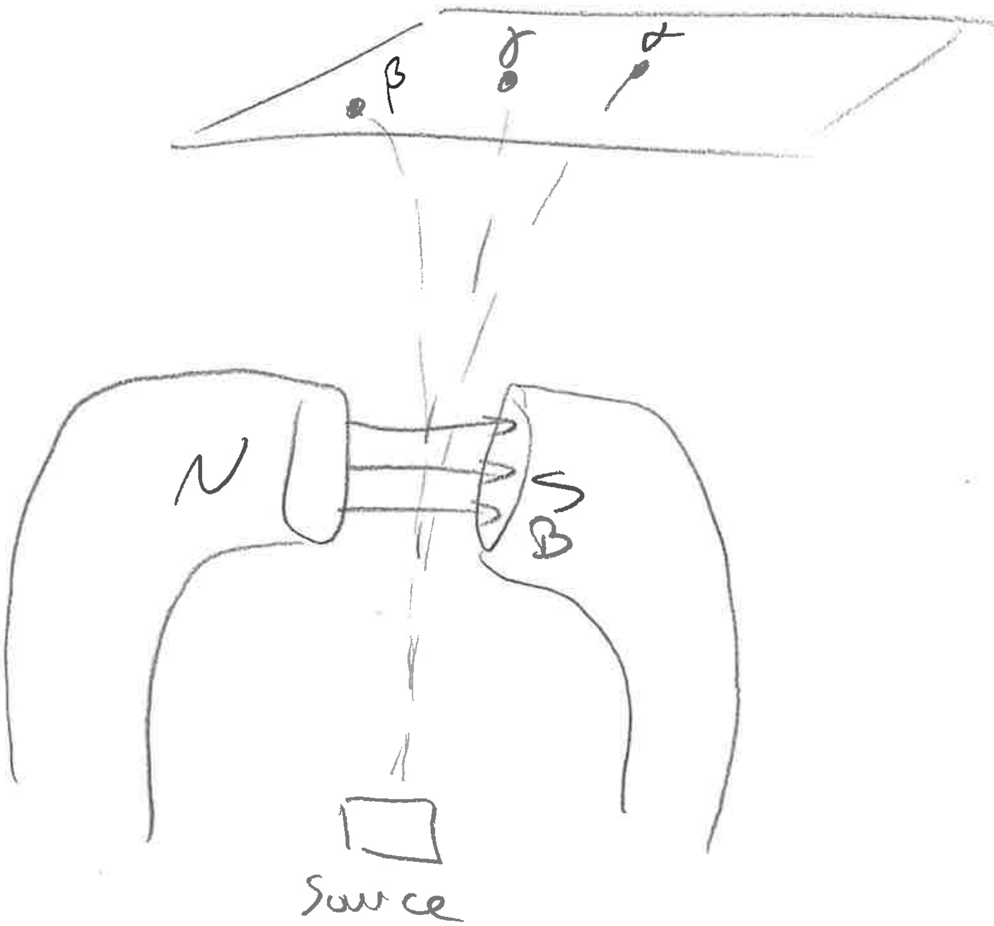
$n_f = 1$

$\lambda = 97.2 \text{ nm} = 9.72 \times 10^{-8} \text{ m}$

$R = 1.097 \times 10^7$

$$\boxed{n_i = 4}$$

Nuclear Radioactivity



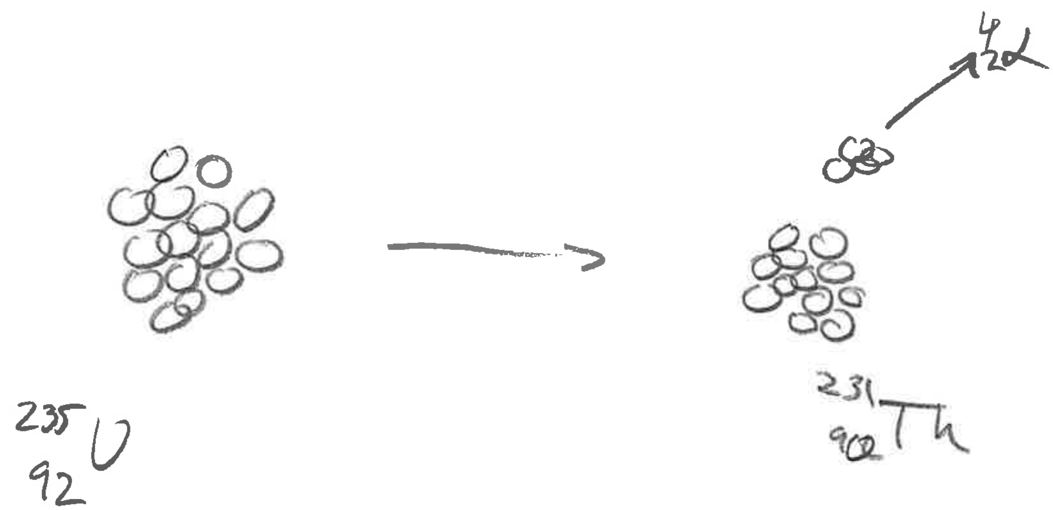
- Radioactive sources produce 3 types of particles

  1. α particles — positive. Can penetrate paper, cm of air, mm tissue.
  2. β particles — negative. Can penetrate Al plate, ~10 cm tissue.
  3. γ rays — neutral. cm of lead, ~m concrete.

- All three ionize materials.
  - ↳ cause cancer

Alpha decay

$$^{235}_{92}U \longrightarrow {}^{4}_{2}\alpha + {}^{231}_{90}Th$$



- Mass is conserved
- Charge is conserved

Beta decay

 $\longrightarrow$  $+ \bar{\nu} + \beta^-$

$^{14}_{6}C$ — 6 protons, 8 neutrons, unstable

$^{14}_{7}N$ — 7 protons, 7 neutrons

$\bar{\nu}$ — Anti-electron neutrino

β^- — β^- particle (electron)

- one proton gained, but also one electron gained (β^-)
- anti electron neutrino has no charge
- charge conserved
- Mass conserved

 $\longrightarrow$  $^{10}_{5}B + \nu + \beta^+$

6 protons, 4 neutrons, unstable — $^{10}_{6}C$

$^{10}_{5}B$ — 5 protons, 5 neutrons

ν — electron neutrino

β^+ — β^+ particle (positron)

→ 1 proton lost, one positron gained
- Mass conserved
- charge conserved

γ - EMISSION

$${}^{60}_{28}Ni^{*} \longrightarrow {}^{60}_{28}Ni + {}^{0}_{0}\gamma$$

- $*$ denotes excited nucleus
- γ has no mass, no charge
- mass conserved
- charge conserved

Half-life and Activity

- every particle has equal probability to decay by those modes.

$$\frac{dN}{dt} = -kN$$

- The rate of decay is proportional to the amount of material present.

$$\frac{dN}{dt} = -kN$$

$$\int \frac{dN}{N} = \int -k\,dt$$

$$e^{\ln|N|} = e^{-kt+C}$$

$$N = Ce^{-kt}$$

$$N(0) = N_0$$

$$N_0 = Ce^0$$

$$N_0 = C$$

$$N = N_0 e^{-kt}$$

Define half life

$$t_{1/2} = N(t_{1/2}) = \tfrac{1}{2} N_0$$

$$\frac{1}{2}\cancel{N_0} = \cancel{N_0}\, e^{-kt_{1/2}}$$

$$\ln\frac{1}{2} = -kt_{1/2}$$

$$\ln|2| = kt_{1/2}$$

$$k = \frac{\ln|2|}{t_{1/2}}$$

$$\boxed{\begin{aligned} N &= N_0 e^{-\frac{\ln|2| t}{t_{1/2}}} \\ R &= \frac{\ln|2|}{t_{1/2}} N_0 e^{-\frac{\ln|2| t}{t_{1/2}}} \end{aligned}}$$

$$N = N_0 e^{-\frac{\ln|2| t}{t_{1/2}}}$$

N – number of particles

N_0 – starting number of particles

$t_{1/2}$ – half life (units time)

t – current time (units time)

$$R = \frac{\ln|2|}{t_{1/2}} N_0 e^{-\frac{\ln|2| t}{t_{1/2}}}$$

R – decay rate (decays per time)

$1\ Bq = 1\ \text{decay}\ s^{-1}$

$1\ Ci = 3.70 \times 10^{10}\ Bq$

Half-life problem

^{14}C is produced in the upper layers of the Earth's atmosphere and is present at a fixed ratio in all living things. Calculate the age of a specimen if the measured ^{14}C ratio is 72% that of living tissue. ^{14}C has a half-life of 5730 years

$$N = N_0 e^{-t\ln 2 / t_{1/2}}$$

$t_{1/2} = 5730$ years

$\frac{N}{N_0} = .72$

$$\frac{N}{N_0} = e^{-t \ln 2 / t_{1/2}}$$

$$\ln \frac{N}{N_0} = -t \frac{\ln 2}{t_{1/2}}$$

$$t = -\frac{t_{1/2}}{\ln 2} \ln \left| \frac{N}{N_0} \right|$$

$$\boxed{t = 2720 \text{ years}}$$

Activity Problem

Calculate the activity of decaying ^{14}C in 1kg of living tissue, in both Bq and Ci.
^{12}C has a molar mass of 12.0 g/mol, the abundance of ^{14}C is 1.3×10^{-12} atoms per ^{12}C atom, and ^{14}C has a half-life of 5730 years. (Avagadro's number)

$$1 \text{ kg } ^{12}C \times \frac{1 \text{ mol } ^{12}C}{12 \text{g } ^{12}C} \times \frac{1000 \text{g } ^{12}C}{1 \text{kg } ^{12}C}$$

$$\times \frac{6.022 \times 10^{23} \text{ atoms } ^{12}C}{1 \text{ mol } ^{12}C}$$

$$N(^{12}C) = 5.02 \text{ particles } ^{12}C$$

$$5.02 \text{ particles } ^{12}C \times \frac{1.3 \times 10^{-12} \; ^{14}C}{1 \text{ particle } ^{12}C}$$

$$N(^{14}C) = 6.52 \times 10^{13} \text{ particles}$$

$$R = \frac{\ln 2}{t_{1/2}} N_0 e^{-\frac{\ln 2 t}{t_{1/2}}}$$

$t = 0$

$$R = \frac{\ln(2)}{t_{1/2}} N_0$$

$$t_{1/2} = 5730 \text{ yrs} \times \frac{\pi \times 10^7 \text{ s}}{1 \text{ yr}}$$

$$t_{1/2} = 1.80 \times 10^{11} \text{ s}$$

$$\boxed{R = 250 \text{ Bq}}$$

$$R = 250 \text{ Bq} \times \frac{1 \text{ Ci}}{3.7 \times 10^{10} \text{ Bq}}$$

$$R = 6.79 \times 10^{-9} \text{ Ci}$$

$$\boxed{R = 6.79 \text{ nCi}}$$