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A. T. Forrester

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On Coherence Properties of Light Waves

A. T. FORRESTER*
University of Southern California, Los Angeles 7, California
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The electric field of a luminous region is much more regular than is generally realized. Phases vary periodically and predictably, and the nature of the polarization remains approximately constant, for times of the order of the reciprocal of the line width in cycles per second. In a plane at right angles to the mean direction of propagation the light wave is coherent over an area approximately λ^2/Ω, where Ω is the solid angular spread in the light. These ideas, when applied to the problem of producing beats between lightwaves, lead to a signal amplitude which has been verified experimentally.

I. INTRODUCTION

CERTAIN widespread misconceptions concerning the nature of light, from the classical point of view, seem to have their origin in a rather loose usage of the term coherence and it would be profitable to begin this discussion with a definition. Two waves will be said to be coherent if there exists a correlation between their phases. Specifically, two waves are perfectly coherent if the phases of the two bear a definite relationship to each other, slightly coherent if there exists a small correlation between their phases, and incoherent only if the phase variations in each wave occur completely independently of what is happening in the other wave. Thus one can speak of the two beams of light in a Michelson interferometer as being coherent, or of induced radiation as being coherent with the inducing radiation. It is not necessary that two waves have the same frequency to be coherent. A light wave and its reflection from a moving mirror, and two radio waves whose frequency difference is controlled by a signal obtained by beating them together are examples of coherent pairs of different frequencies.

Frequently, coherence is used in another sense. A radio wave is often said to be coherent and a light wave incoherent without saying to what the coherence, or lack of it, is related. It is possible to formulate a definition consistent with usage in this respect by calling a wave coherent if its phase is predictable from the excitation and incoherent if its phase is unrelated to the excitation. However, the term is misleading in that sense, because coherence seems to imply a greater regularity than incoherence, and, as shall be discussed in the next section, it is not necessarily true that radio waves are more periodic than light waves. It would be preferable, therefore, that whenever the term coherence is used it be clear which two phenomena are correlated, e.g., a radio wave is coherent with the antenna current. For a single wave we can speak of coherence between two points on the wave (Sec. III), or between the waves existing at the same point at two different times (Sec. II).

* Now on leave at Westinghouse Research Laboratories.

II. TEMPORAL COHERENCE

For illustrative purposes consider a simplified picture of a steady light source. At any instant a very large number of radiators is active and we imagine each of them to emit a pure sine wave of a definite frequency for a duration of time T. Over a time interval $t \ll T$ only a small fraction of the oscillators have decayed and been replaced by new and randomly phased oscillators, and during this time the resultant electric field amplitude will vary in a manner which is predictable. We may say that the waves at two instants of time separated by an interval t are coherent if $t<T$ and incoherent if $t>T$. For this wave we say that there is a coherence time of the order of T.

It is easy to show that the Fourier transform of the wave amplitude from a single radiator of the type we have been discussing has the form $\sin[\pi T(f-f_0)]/(f-f_0)$ which has a width

$$\Delta f = 1/T, \tag{1}$$

if we define the line width as

$$\Delta f = \int_0^\infty [I(f)/I(f_0)]df, \tag{2}$$

where $I(f)df$ is the intensity of the components with frequency f to $f+df$. For a source made up of a large number of such oscillators emitting at random times, the spectrum is the same as for a single radiator. The only difference lies in the phase relationships between components which is quite definite in the case of a single radiator and random for the case of a large number of radiators.

For an actual spectral line, widths are determined by other factors than radiation damping. If the broadening is due to Doppler effect, for example, the situation is different from the one described in that the distribution in intensity with frequency is the result of a distribution in the frequencies emitted by the different radiators as observed in the laboratory reference system. This is, however, not important as far as it concerns the nature of the radiation, unless we are concerned with the specific line shape.[1] It is still true that there exists a coherence time $\tau \approx 1/\Delta f$, i.e., that a correlation between the phase of the wave at two different times will persist for times of the order of $1/\Delta f$. This is very simply demonstrated as follows: In a time τ such that $(\Delta f)\tau < 1$ the relative phases of the waves of the two frequencies marking the boundaries of the pass band, and even more so for the included frequencies, shift by only a small relative amount and the addition of the various components is like the addition of sine waves of the same frequency in that it generates another sine wave of the same frequency.

These ideas receive excellent quantitative expression in the correlation function[2]

$$\psi(\tau) = \lim_{T\to\infty} \frac{1}{T} \int_0^T E(t)E(t+\tau)dt. \qquad (3)$$

For a wave $E(t)$, whose spectrum is confined to a narrow band of frequencies, $\psi(\tau)$ has the character of a damped oscillation. The amplitude of this oscillation, relative to the amplitude at $\tau = 0$, can be used as a measure of the degree of coherence between the waves which exist at one point at two times separated by an interval τ. The quantity we have called the coherence time might be taken as the time required for the amplitude to fall to some small fraction, say $1/e$th, of the value for $\tau = 0$. For the idealized light source discussed the correlation function actually falls sharply to zero for τ equal to the lifetime of a single radiator, as shown in Fig. 1.

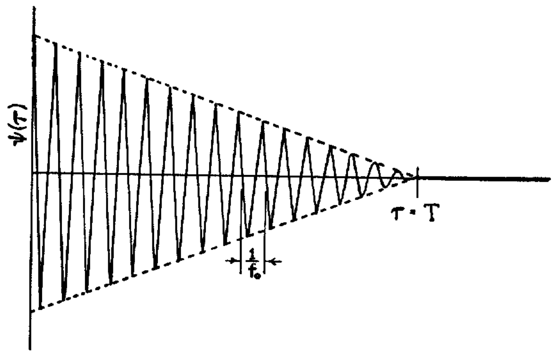

FIG. 1. Correlation function for a source comprised of radiators emitting sine waves of frequency f_0 for a duration of time T.

The use of some numbers gives an appreciation of what these ideas imply concerning the nature of light. The natural width of a spectral line for a dipole transition in the visible is approximately 10^8 cps, meaning a coherence time of 10^{-8} sec. The frequency of visible light is approximately 6×10^{14} cps so that there are approximately 6×10^6 periods in the coherence time. This is a very regular wave and, as a matter of fact, more regular than radio-frequency oscillators can be expected to be unless special precautions are taken. It would correspond to a frequency stability of 0.1 cps for a 600-kc oscillator.

These ideas about the rate at which phase can change in a light wave apply to each direction of polarization. Since any type of polarization is analyzable in terms of two plane polarized waves it follows that the nature of the polarization in an unpolarized beam of light can alter at a rate which is also limited by the coherence time. An unpolarized beam of light, one should realize, is elliptically polarized (or in special cases circularly or plane polarized) at any instant and the nature of the polarization, as described by, say, the directions and relative sizes of the axes and direction of rotation will remain constant for times small compared to the coherence time and be unrelated only at times separated by an interval large compared to the coherence time.

[1] The intensity distribution in the case of Doppler broadening is Gaussian instead of $\sin^2[\pi T(f-f_0)]/(f-f_0)^2$, as in the idealized case discussed, or Lorentzian as in the case of an actual natural line shape.

[2] S. O. Rice, Bell System Tech. J. **23**, 282 (1944).

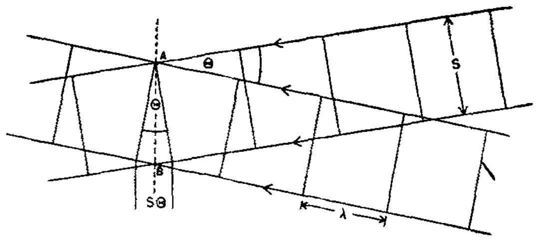

FIG. 2. Analysis of a light wave in terms of plane waves.

III. SPATIAL COHERENCE

Spatial variations in phase in a luminous region are also interesting to examine. As far as points along the direction of propagation are concerned, the coherence length is merely the velocity of light times the coherence time. The variation in phase between two points at right angles to the direction of propagation presents a more complicated problem.

For the unrealizable unidirectional wave, i.e., infinite plane wave, all points lying on a plane at right angles to the direction of propagation are on a wave front and are always exactly in phase. In a real case this is not so, but neither is it true that the light is incoherent at two points arbitrarily close together. There is in the general case an area of coherence, namely an area over which the phase of the light can be expected to be constant, or vary in a predictable fashion, which depends on the wavelength and the angular spread in the light. The problem of estimating this area can be approached in several ways.

One way of estimating the order of magnitude of the area of coherence is with respect to Fig. 2. A beam of light with angular spread can be thought of as being made up of a large number of plane waves of which the two most divergent are shown in the figure. At one edge of the area to which these beams converge, the two waves, and all those at the intermediate angle as well, add to give a wave of some resultant amplitude and phase. Clearly the phases of the resultant fields at two points such as A and B separated by a distance S are very closely the same if $S\theta \ll \lambda$, leading to an area of coherence $\ll \lambda^2/\Omega$, where Ω is the solid angular spread in the light.

This picture is rather crude and it is worth while to look at the problem from other points of view. The area of coherence at an image should be approximately the area of the diffraction pattern of a single source point. There is a phase change through the central image of a diffraction pattern of 2π but in a definite and predictable fashion. Between points A and B, Fig. 3, separated by the width of a diffraction pattern there will be a great many overlapping patterns from intermediate source points, but, neglecting the small intensity in secondary maxima, not until a point is displaced by this distance are the contributing atoms replaced by an altogether new set of radiators. There is to be expected a coherence between any two points separated by less than this distance.

Neglecting numerical factors of the order of unity

$$S \approx \phi f \approx \lambda f/D. \tag{4}$$

But D/f is the angular spread θ, so that $S \approx \lambda/\theta$, leading to an area of coherence

$$A_0 \approx \lambda^2/\Omega, \tag{5}$$

different from the previous result only in that the *much less than* has been replaced by an *approximately equal to*. This is easily understandable. It was in the nature of the first calculation to yield an area over which there is negligible phase shift and of the second to give the area over which there is a strong phase correlation.

There is still another and particularly interesting approach to the question of coherence area, through the number of degrees of freedom. The number of degrees of freedom per unit volume for radiation with wavelength in the range λ to $\lambda + d\lambda$ is given by the Rayleigh-Jeans

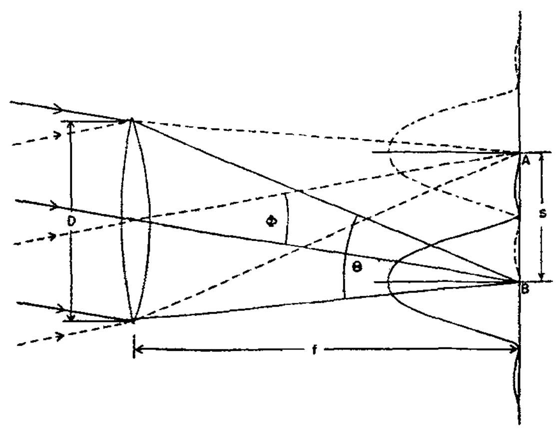

FIG. 3. Diffraction patterns separated by the diameter of the central maximum.

formula

$$n = 8\pi\Delta\lambda/\lambda^4. \quad (6)$$

The number of degrees of freedom in a column of cross sectional area A and length ct is $Actn$, and and the number of degrees of freedom associated with the radiation emitted through the area A in a time t, into a solid angle Ω is obtained by multiplying this by $\Omega/4\pi$, yielding[3]

$$N_1 = \frac{2A\Omega t\Delta f}{\lambda^2}, \quad (7)$$

where $\Delta f = c\Delta\lambda/\lambda^2$ is the frequency range. The number of degrees of freedom in a coherence time is the number of independent oscillators required to duplicate the radiation field. Since this means taking $t = 1/\Delta f$ we get simply

$$N_2 = A\Omega/\lambda^2 \quad (8)$$

as the number of independent oscillators for each plane of polarization. It seems reasonable to equate A/N_2 to the coherence area, leading once again to Eq. (5).

While the question of coherence area is here approached qualitatively, the result receives quantitative confirmation in the calculation of E. Gerjuoy referred to in the next section.

It is noteworthy that the coherence of a radiation field over an area λ^2/Ω is just as valid at a source as at an image, as evidenced by the nature of the first and last approach to the problem.

IV. APPLICATION TO LIGHT BEATS

The coherence properties discussed in Secs. II and III are well illustrated by the production of

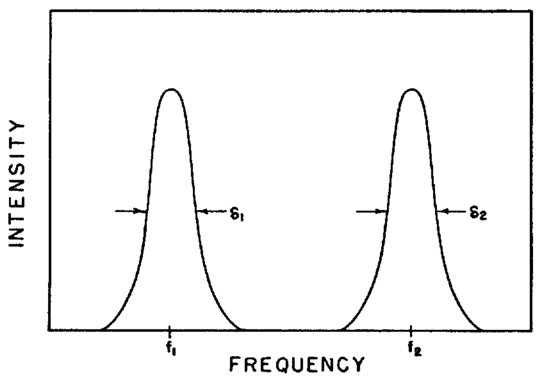

FIG. 4. Spectrum required for the production of sharp beats.

[3] A. Lande, Handbuch der Physik **XX**, 453 (1928).

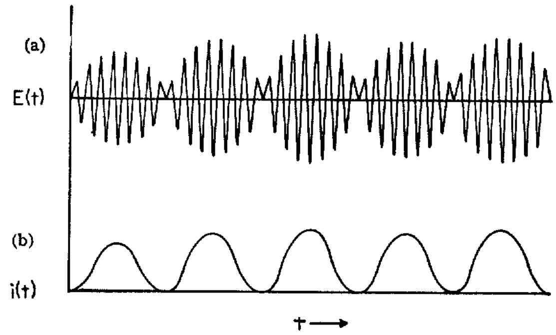

FIG. 5. Electric field variation (a) due to an admixture of two spectral lines and the accompanying photoelectric current (b). The irregularities show the effects of line widths which are not negligibly small.

beats between incoherent light waves.[4] Consider light whose intensity distribution is shown in Fig. 4, composed of two sharp incoherent spectral lines, i.e., lines in which the relative phase shifts occur in a random manner but at a rate limited by the individual line widths. These two lines will beat against each other as shown in Fig. 5(a) and the beats will be well defined providing that the coherence time $1/\delta$ is long compared to the beat period $1/(f_2 - f_1)$, or providing

$$f_2 - f_1 \gg \delta. \quad (9)$$

This is easy to achieve. Spectral line width of 10^9 cps can be obtained even under conditions of intense excitation so that the inequality requires a separation of 10^{10} cps. A convenient way of getting two lines of equal intensity, separated by the correct amount, is through the Zeeman effect, and fortunately 10^{10} cps is in a spectral region where sensitive detection has been developed to a high degree for radar purposes.

The generation of a difference frequency requires a nonlinear detector which, for light, is provided in the photoelectric effect[5] which will produce the 100% modulated current shown in Fig. 5(b). This is the current per unit area within a single area of coherence. The dc current from an area of coherence is $(I/A)(\lambda^2/\Omega)$, where I is the total photocurrent from a cathode of total area A, leading to a mean square ac current, corre-

[4] Forrester, Gudmundsen, and Johnson, Phys. Rev. **99**, 1691 (1955).

[5] For a discussion of the nature of the assumptions concerning the photoelectric process see reference 4.

sponding to the beats,

$$\langle i^2\rangle = \tfrac{1}{2}(I\lambda^2/A\Omega)^2. \quad (10)$$

The cathode area will ordinarily contain many areas of coherence for which the signals will be in random phases, leading to an increase in current proportional to the square root of the number of such areas, or

$$\langle i^2\rangle = \langle i_0{}^2\rangle A\Omega/\lambda^2 = I^2\lambda^2/2A\Omega. \quad (11)$$

The measureability of this current will be determined by a comparison with the unavoidable shot noise $\langle i_n{}^2\rangle = 2eI\Delta f$, where for proper comparison, Δf should be taken the same as the band of frequencies over which the beat signal is spread, approximately $\sqrt{2}\times$ the line width δ. This gives

$$\langle i^2\rangle/\langle i_n{}^2\rangle = \lambda^2 I/4\sqrt{2}e\delta A. \quad (12)$$

Under optimum conditions this equation leads to a signal-to-noise ratio no higher than 10^{-4}. Specifically, even for a very intense light source and very sensitive photosurface the signal is overwhelmed by noise. Nevertheless, by imposing a modulation on the signal in such a way that the shot noise remained unmodulated it has been possible[4] to detect the signal which measured to be the size indicated by Eq. (12), in verification of the ideas of Secs. II and III used in its derivation.

Equation (12) has also received quantitative theoretical confirmation through a calculation by E. Gerjuoy[6] of the expected intensity of a beat signal, in which he uses an analytic expression for electromagnetic waves spread uniformly over an area A with a solid angular spread Ω.

[6] E. Gerjuoy, unpublished work.

Mesons and Hyperons*

MAURICE M. SHAPIRO
Naval Research Laboratory, Washington, D. C.
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Basic properties of L-mesons, K-mesons, and hyperons are summarized in the light of evidence available in mid-1955. Masses, decay schemes, lifetimes, and Q-values are tabulated. The accompanying notes describe how "best values" were arrived at, and serve as a guide to a representative selection of references on unstable particles.

NOMENCLATURE

MESONS are unstable particles intermediate in mass between the electron and proton. *Hyperons* are unstable particles intermediate in mass between the neutron and deuteron. So many new types of mesons and hyperons have recently been discovered (LL, LC, RG, BR, BR1, SA, OC, BH, AR, TR, BA1, FM3, MM)[1] that confusion in nomenclature has resulted. A systematic notation, proposed (AE, TR) as the result of discussion at the International Congress on Cosmic Radiation at Bagnères-de-Bigorre, France, in 1953 (BP, SM), has gained wide acceptance, and is employed here.

I. *Generic* symbols (Latin letters) classify the particles according to *mass* and *phenomenology of decay*, respectively.

(a) *Mass categories*[2]

L-meson (light meson), $m_e < m_L \leq m_\pi$
K-meson (heavy meson), $m_\pi < m_K < m_p$
Y-particle (hyperon), $m_n < m_Y < m_d$.

(Note that neutrons and protons are excluded.)

(b) *Phenomenological categories*

V-event—Phenomenon interpretable as the *decay in flight* of a K-meson or hyperon. Subclasses: "V^0-event," decay

* Prepared for later publication in the *Handbook of Physics*, Section on Nuclear Physics, edited by Dr. F. N. D. Kurie, and published here by permission of McGraw-Hill Book Company, Inc.

[1] The bibliography at the end of this chapter includes some useful *general* references on heavy unstable particles (MR1, VP, PC, RB2, PC1, LL1, SM, WJ, BJ, PC2, TA, RG1, RU, BP, DP, PP, HT1).

[2] Symbols: e = electron; $\pi = \pi$ meson (pion); $\mu = \mu$ meson (muon); p = proton; n = neutron; d = deuteron. In addition, the following symbols are employed to denote various decay products in Table I: γ = photon; ν = neutrino; η, η' = neutral particles as yet unspecified, which may or may not be alike.