AAPT PHYSICS EDUCATION
AMERICAN JOURNAL of PHYSICS

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Citation: American Journal of Physics **39**, 1396 (1971); doi: 10.1119/1.1976670
View online: http://dx.doi.org/10.1119/1.1976670
View Table of Contents: http://scitation.aip.org/content/aapt/journal/ajp/39/11?ver=pdfcov
Published by the American Association of Physics Teachers

A High-Resolution Gamma Ray Spectrum of Background Radiation*

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(Received 11 February 1971; revised 26 April 1971)

When teaching courses in the physical sciences the instructor often either demonstrates the use of gamma ray detectors or has the students themselves perform simple gamma ray counting experiments in a laboratory session. As part of the discussion that usually accompanies these introductions to radioactivity, the matter of "room background" is considered. In simple counting experiments this "room background" is measured as a certain number of gamma ray interactions within the detector per unit time when "no source" is present and the explanation is put forth that these counts are associated with the decay of naturally occurring radioactive isotopes that are always present in our surroundings.

Most students already are aware of the fact that natural (and man made) radioactivity is part of our environment, and many know further that the gamma radiation associated with the decay of a radioactive isotope is a "fingerprint" for that isotope. With the advent of high-resolution, large-volume lithium-drifted germanium gamma ray detectors (see Ref. 1) it is now possible to show in a dramatic fashion just how good a "fingerprint" the gamma ray spectrum of an isotope really is and to also verify in a graphic way what the student has been told about background radiation, i.e., that the counts recorded by his apparatus are associated with the decay of such naturally occurring radioactive isotopes as potassium-40 and bismuth-214.

The figure which accompanies this note shows the spectrum of background radiation found in one of the rooms of the recently constructed University of Michigan physics and astronomy building. This spectrum was taken over a period of about 2.5 days using a lithium-drifted germanium gamma ray detector having an active volume of 29 cm³. All peaks (with the exception of a weak peak of unknown origin at about 1 MeV) can be associated with the decay of potassium-40, or one of the elements of the uranium series, or one of the elements of the thorium series. The peaks are labeled with the nucleus in which the gamma de-excitation actually occurs, i.e., the so-called daughter nucleus. Peaks not labeled with an energy value are present as a consequence of pair production within the detector by gamma rays having energies in excess of 1022 keV and the subsequent escape of one or both of the 511 keV

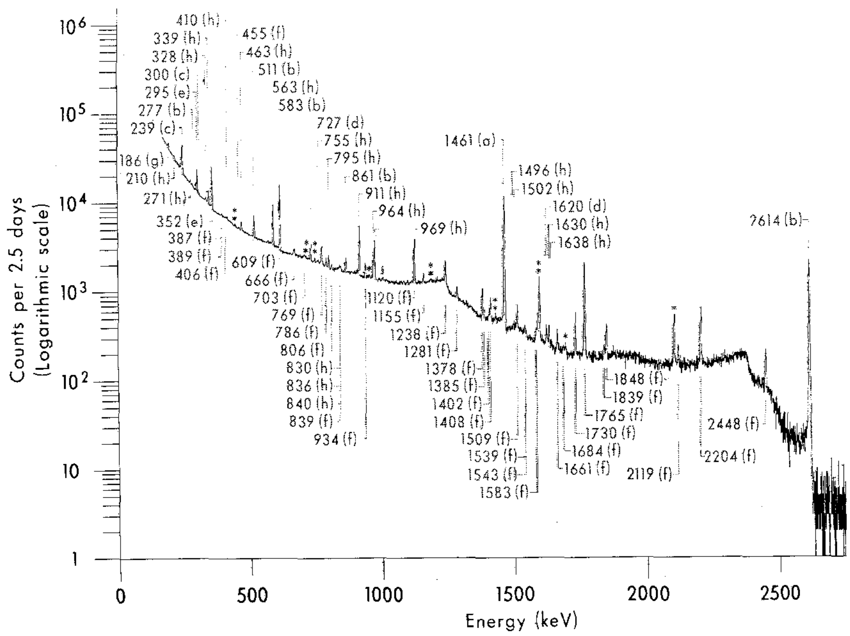

FIG. 1. A gamma ray spectrum of "room background" taken with a lithium-drifted germanium gamma ray detector. The scheme utilized in the labelling is as follows: (a) Ar-40; (b) Pb-208; (c) Bi-212; (d) Po-212; (e) Bi-214; (f) Po-214; (g) Rn-222; (h) Th-228; *, single escape; **, double escape. A weak peak of unknown origin can be seen at about 1000 keV. Calcomp plot made at the University of Michigan computing center using the plot program of I. P. Auer.

gamma rays associated with the annihilation of the positron member of the pair.

* Work supported in part by the U. S. Atomic Energy Commission.

[1] A very readable discussion of modern gamma ray spectroscopic techniques can be found in the article by P. J. Ouseph and M. Schwartz, Phys. Teacher **8**, 374 (1970).

Numerical Differentiation of Data

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It is frequently the case that scientists must differentiate numerical data to obtain a complete analysis. If the phenomena observed is thought to be described by a formula containing adjustable parameters, the data may be least squares fitted over the interval of observation and the resulting formula differentiated to obtain the derivative of interest. This procedure can be unsatisfactory because (a) the model over the complete interval of data is uncertain, (b) deviations from the model may be of interest, or (c) emphasis is needed on certain details of the data. In this case a point by point numerical method must be used to obtain a table of derivatives, and several simple differentiation formulas[1] are available for use with smooth data. If the data contains much scatter, the derivative values obtained with the simple formulas will contain large oscillations that may preclude confident analysis. We have found that the solution to the oscillatory problem is not to smooth[2] the original data with additional formulas but instead to increase the number of data points used to evaluate the derivative at a point. The simple differentiation formulas are readily exhausted, and one writes a computer program which makes a least square fit[2] to a suitable polynomial over a data subinterval and determines the derivative at the midpoint of the subinterval using the fitted formula. We have used such a procedure to differentiate specific heat data[3] in the vicinity of a second-order phase transition. In the process we have discovered an interesting property of numerical derivatives obtained by least squares polynomial fitting that is not discussed in standard treatments on numerical analysis.[2,4] The discussion also includes several practical findings for the numerical differentiation of data.

The observations we wish to stress are the following: Suppose a set of data is least squares fitted to polynomials of degree $2k$ and $2k-1$, where $k=1, 2, \cdots$. If the derivatives of the fitted polynomials are evaluated at the midpoint of the interval, they will be found to be equal, provided only that the data intervals are distributed symmetrically about the midpoint. The result applys equally when fitting a polynomial of degree $2k$ with a polynomial of degree $2k-1$. The result of polynomial least squares fitting to numerically differentiate data will be identical for linear and quadratic fits, cubic and quartic fits, etc., provided only that the data spacings are symmetric about the midpoint of the fitting intervals, a condition satisfied by equally spaced data. The equality of the pairs of derivatives can be used to check the validity of a computer program and permits a 50% savings of calculations if one wishes to investigate the numerical derivatives as a function of the degree of the fitting polynomial.

In order to understand the origin of the above observation, let us examine the details of the least squares method. Following the notation of Hildebrand,[2] we wish to fit data $f(x_i)$ with the polynomial $y(x_i)$ of degree n over a range consisting of $2m+1$ symmetrically spaced data points $(x_{MP}=x_{m+1}=0)$. Thus

$$f(x)\approx y(x)\equiv\sum_{j=0}^{n}a_j\phi_j(x), \tag{1}$$

where

$$\phi_j(x)=x^j, \tag{2}$$

and the a_j are $n+1$ expansion coefficients to be determined $(2m+1\geq n+1)$. Assuming a weighting function of unity, the sum of the squares of the deviations is given by

$$E=\sum_{i=1}^{2m+1}[f(x_i)-\sum_{j=0}^{n}a_j\phi_j(x_i)]^2. \tag{3}$$

Minimizing (3) with respect to the a_j yields the set of linear equations

$$\sum_{k=0}^{n}a_k\sum_{i=1}^{2m+1}\phi_r(x_i)\phi_k(x_i)=\sum_{i=1}^{2m+1}\phi_r(x_i)f(x_i), \tag{4}$$

where $r=0, 1, \cdots, n$. Thus the linear least squares fitting problem is of the form