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Geometry factors for experiments in radioactivity

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The problem of determining the number of radioactive particles entering a detector window of radius a from a parallel and coaxial disk source of radius b is discussed. A diagram is presented which gives the effective solid angle, or geometry factor, subtended by the detector at the source for various values of b and z, the source-detector spacing. The range of parameters covered is $a = 1$, $0 < b < 8$, $1 < z < 5$.

INTRODUCTION

Gopal *et al.*[1] have recently described a simple experiment for measuring the half-life of potassium-40. Thin disks of powdered potassium chloride are mounted below an end-window geiger tube, and their activity is measured. To find the absolute activity of the disks, we need to know what proportion of the radioactive emission enters the tube window. This proportion is determined mainly by the effective solid angle, usually called the geometry factor G, subtended by the detector at the source.

Now Gopal *et al.* give a series expansion for evaluating G. However, while we were developing this experiment for one of our teaching laboratories, we considered using source diameters for which this expansion was invalid. A search through the literature brought to light several articles[2-7] discussing the calculation of geometry factors and giving series expansions with various convergence limits. But these expansions can be tedious to use in calculation, and we could find little in the way of results giving values of G for the geometries we had in mind. So it was decided to prepare a detailed diagram enabling G to be found for those geometries most often used in simple radioactivity experiments.

GEOMETRY FACTORS

The experimental arrangement to be discussed is shown in Fig. 1. A geiger tube with an end window of radius a is mounted a distance z above a disk source of radius b. The source and window are parallel and coaxial. We shall assume that the source has negligible thickness and that all elements of its area have the same activity and emit particles isotropically. We shall also at this stage ignore all absorption and scattering effects. The problem is thus a purely geometric one. What proportion of the particle flux leaving the source enters the detector?

This proportion, the geometry factor G, can be found exactly for only two special cases. Firstly, for a point source ($b = 0$), G is equal to the solid angle subtended at the source by the detector divided by 4π, or

$$G = G' = a^2/[2D(D+z)]. \qquad (1)$$

Secondly, when the source is coplanar with the detector ($z = 0$), all the particles moving upwards from the source and contained within the window area will enter the detector. This gives

$$G = G'' = 0.5 \qquad (b \leqslant a)$$

and

$$G = G'' = a^2/2b^2 \qquad (b > a). \qquad (2)$$

Neither of these two simple results can be applied to the measurement of the half-life of ^{40}K nor to many other experiments in radioactivity. Weak sources must be made large in area so as to obtain a reasonable count rate. Such, sources cannot usually be placed touching the tube window for fear of damaging or contaminating it. In consequence, the parameters a, b, and z are very often of the same order of magnitude, which makes the calculation of G much harder.

The series expansion for G used by Gopal *et al.* was first derived by Blachman and published by Burtt[2]: It may be rewritten in terms of the parameters of Fig. 1 as

$$\begin{aligned} G = {} & G' - (3/16)\,(a^2zb/D^5) \\ & + (5/128)\,(a^2zb^4/D^9)\,(4z^2 - 3a^2) \qquad (3) \\ & - (35/2048)\,(a^2zb^6/D^{13})\,(8z^4 - 20z^2a^2 + 5a^4) + \cdots, \end{aligned}$$

where G' is given by Eq. (1). But Jaffey[3] (who discusses several expansions for G in detail) points out that this expansion only converges for $b < D$, which means that it cannot be used for large sources close to the detector. A formula of wider applicability, given by Jaffey, uses the parameters $x = \cos\theta = z/L$ and $y = a/L$ and the Legendre polynomials $P_k = P_k(x)$.[8] It is

$$G = \frac{1}{2}\frac{a^2}{b^2}\left[(1-x) - \frac{1}{4}y^2(P_1 - xP_2) + \frac{1}{8}y^4(P_3 - xP_4) - \frac{10}{128}y^6(P_5 - xP_6) + \cdots\right]. \qquad (4)$$

When $z = 0$, we have $x = 0$ and hence $G = G''$ as in Eq. (2). This series converges if $z > a$ and so is applicable to a source

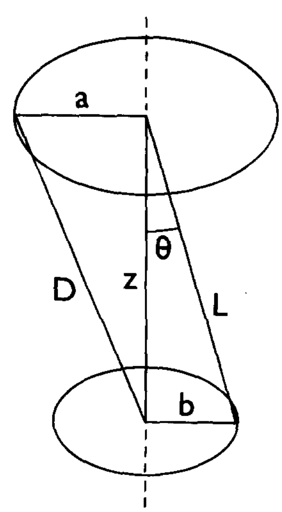

Fig. 1. A detector of radius a mounted a distance z above a source of radius b.

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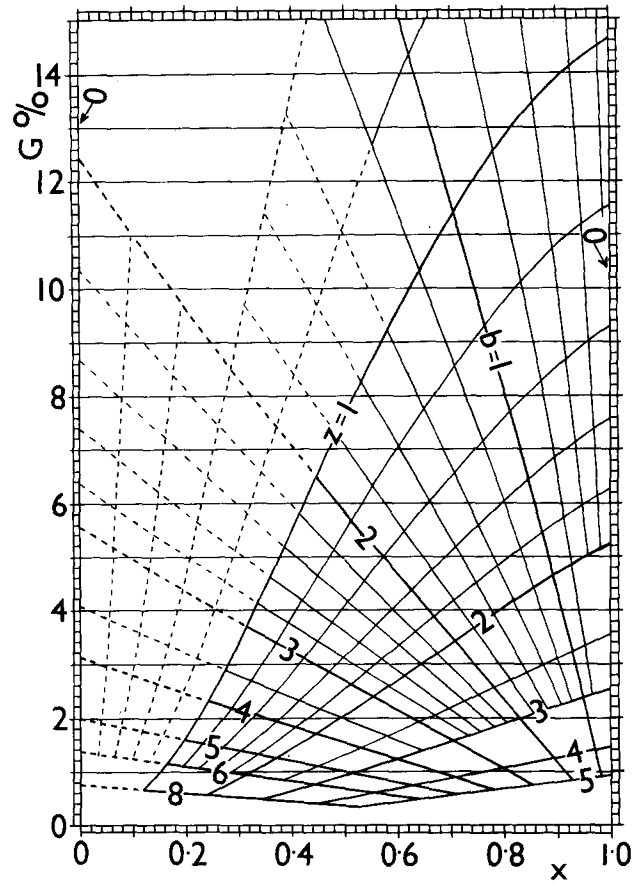

Fig. 2. The geometry factor G plotted as a function of x for various values of source diameter b and source-detector spacing z. The detector diameter $a = 1$.

of any size provided it is no closer to the detector than the window radius.

There appear to be no series expansions published for source detector geometries for which $b > D$ and $z < a$. Various methods of numerical integration have been devised,[5–7] but these usually require computer facilities for their use. However, the diagram presented here enables values of G in this difficult region to be found with moderate accuracy by extrapolation. In any case, there are, as we shall see, good reasons for not using such source-detector geometries.

THE DIAGRAM OF *G* VALUES

Figure 2 shows the geometry factor G plotted as a function of $x = \cos\theta = z/L$. All calculations have been based on a window radius of unity so that b and z are measured in units of a. In fact, many geiger tubes have window radii of about 1 cm.

The two extreme geometries for which G can be calculated exactly are represented on the diagram by the vertical lines $x = 1$ ($b = 0$) and $x = 0$ ($z = 0$). Along these lines, G is given by Eqs. (1) and (2), respectively.

Values of G in the top right-hand corner of the diagram (corresponding approximately to $G > 0.07$, $x > 0.6$) were found from Eq. (3). Equation (4) was used for the remainder of the diagram below the contour line $z = 1$. Slight discrepancies were observed at the boundary between these two regions, but the greatest of these, around the point $G = 0.1$, $x = 0.6$, did not amount to more than about $1\frac{1}{2}$% in G. As the counting errors in student radioactivity experiments are seldom this small, these discrepancies were ignored when the contours were drawn.

The network of b and z contours can be filled in by using the relationship

$$1/x^2 = (b^2/z^2) + 1, \qquad (5)$$

which can be easily deduced from Fig. 1. The b contour lines are very nearly straight and, for $b > 2$, can be extrapolated with reasonable confidence across the left-hand half of the diagram to the G axis, where the values of G are given by Eq. (2). The z contours can then be added with the aid of Eq. (5). This is, of course, not a very accurate procedure, and therefore the contours in this part of the diagram are drawn as broken lines. However, one or two values of G given in a table by Lozgachev[7] lie within this region and are in good agreement with values obtained from the diagram.

The values of G given by Fig. 2 are probably sufficiently accurate for many simple radioactivity experiments. The diagram can also be used during the design of an experiment, when, for instance, estimating the advantage of increasing the area of a weak source. A uniform disk source has a total activity proportional to the square of the radius; hence the count rate registered by the detector will vary as Gb^2 for a fixed spacing z. So, for $z = 1$ and $b = 1$, we have $Gb^2 = 0.114$. Doubling the radius reduces G to 0.065 but more than doubles the value of Gb^2. Raising b to 3 increases Gb^2 still further, but by a smaller proportion. It is also worth noting that as b increases, the error in G produced by an error of measurement in z decreases.

But it must be emphasized that Fig. 2 presents solutions to a purely theoretical problem in geometry. In practice, many other factors may alter the proportion of radioactive particles that enter the detector and hence produce what is, in effect, a systematic error in G. For instance, the detector window is usually slightly recessed in a metal tube, the edge of which may cast a "shadow" across the window if too large a source is used. Another error arising from the use of a large source is that particles from the edge of the source that enter the window must travel through a much greater thickness of source material and air than particles from the center of the source, and so are more likely to be scattered. For these and other reasons, it is probably better to keep b less than two and z greater than one. Even so, problems of self-absorption and back-scattering from the source support still remain, making the measurement of the absolute activity of a source to better than about 5% accuracy a very difficult operation.

[1]K. Gopal, H. Sanjeevaiah, and B. Sanjeevaiah, Am. J. Phys. **40,** 721 (1972).

[2]B. P. Burtt, Nucleonics **5,** 28 (1949).

[3]A. H. Jaffey, Rev. Sci. Instrum. **25,** 349 (1954).

[4]A. V. R. Sastry and S. Ramamurty, Proc. Ind. Acad. Sci. A **24,** 224 (1966).

[5]P. Komarek, Atomkernenergie **12,** 331 (1967).

[6]L. Ruby and J. B. Rechen, Nucl. Instrum. Methods **58,** 345 (1968).

[7]V. I. Lozgachev, Sov. Phys.-Tech. Phys. **5,** 1039 (1961).

[8]Tables of Legendre Polynomials are to be found in, for instance, E. Jahnke and F. Emde, *Tables of Functions* (Dover, New York, 1945).

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