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Magnetic braking: Simple theory and experiment

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A simple theory of magnetic braking in a thin metal strip is proposed. The predictions of the model are compared to experiment and good agreement is obtained. The experimental tests were conducted by spinning a thin aluminum disk of large radius between the pole pieces of an electromagnet. A field range of 0 to 150 mT was used.

I. INTRODUCTION

When a piece of metal is placed in a *time-dependent* magnetic field $B(r,t)$, an electric field is induced and circulating reaction currents, called eddy currents, are generated in the metal. This phenomenon may be explained by Faraday's law of electromagnetic induction. The reaction currents dissipate energy in the metal, in the form of heat (Joule effect). Similarly, when a piece of metal is forced to move in a nonuniform but *stationary* magnetic field $\mathbf{B}(\mathbf{r})$, there is an induction of reaction currents and energy is dissipated in the metal. This phenomenon may be explained by the Lorentz force law. Because energy is being dissipated, a magnetic drag force is induced so as to slow down the motion of the sample. The purpose of the present paper is to study the nature and origin of this drag force.

Magnetic braking forms the basis of a growing technology and devices based on the effect have been proposed and used to damp unwanted nutations in satellites[1,2] and to eliminate vibrations in spacecrafts.[3] Some brake dynamometers[4] also operate on the eddy current braking concept. Reference to magnetic braking is occasionally made in textbooks[5–7] but the approach is then maintained at the qualitative level only. Most textbook authors avoid mentioning the effect[8,9] although some authors[10,11] propose it as a problem but the book by Smythe[12] gives a comprehensive analytic treatment of some cases. We have been unable to locate a single note or paper about magnetic braking in this Journal.

To calculate the magnetic drag force on a moving metal object is generally difficult and implies solving Maxwell's equations in a time-dependent situation. This may be one of the reasons why the phenomenon of magnetic braking, although conceptually simple to understand, has not attracted the attention of textbook authors or pedagogs. A simple approximate treatment is however possible in some special cases, and the purpose of the present note is to present such a treatment for the case of a long wide strip which is moving in the air gap between the rectangular-shaped pole pieces of an electromagnet. As will be shown, the present problem may be solved approximately by considering the electrical analog of a battery, of electromotive force ϵ and internal resistance r, connected in series to an external resistive load of resistance R. These concepts are well understood by the early undergraduate and thus make a quantitative presentation of magnetic braking feasible at that stage. We have been unable to locate a model similar to ours in the literature.

To obtain the magnetic drag force, we first calculate the total current I which is induced in the metal strip by the applied field. To do so, we first assume that the speed of the strip is sufficiently small that the magnetic field generated by the induced current I is negligible in comparison with the applied field $\mathbf{B}_0$. This will occur if the strip speed is much smaller than some characteristic eddy-current-related speed v_c for the metal composing the strip. Intuitively, one can appreciate that v_c will contain the conductivity of the metal σ, the thickness of the strip δ, and the permeability μ_0, all in SI units. Indeed, it will be easier to induce strong reaction fields if the conductivity is high. Similarly, it will be easier to induce a strong reaction if the plate is thicker since there is then more volume available for damping. Finally, μ_0 should enter because it appears in Ampère's law for magnetic currents. We estimate v_c by writing it in the form $\sigma^l\delta^m\mu_o^n$, with l, m, n exponents to be determined by dimensional analysis. [The basic units of the appropriate quantities are (1) A $\mathrm{V^{-1}\,m^{-1}}$ for σ, from Ohm's law; (2) m for δ; (3) Vs $\mathrm{A^{-1}\,m^{-1}}$ for μ_0, from the voltage–current law for induction.] We then find $v_c \simeq 1/(\sigma\mu_0\delta)$; a rigorous calculation gives $v_c = 2/(\sigma\mu_0\delta)$.

Under the conditions just stated, the magnetic drag force is seen to arise from a mutual coupling between the induced current I and the applied field $\mathbf{B}_0$ in the "shadow region" of the pole pieces. The calculation is outlined in Sec. II of this paper. Section III gives an experimental proof of our model. As will be seen, agreement between theory and experiment is good.

II. MAGNETIC BRAKING OF A LONG WIDE METAL STRIP

To understand magnetic braking simply, consider a thin metal strip of thickness δ, which is moving at constant velocity $\mathbf{v} = \hat{j}v$ in the y direction of the xy plane, in the air gap between the rectangular pole pieces of an electromagnet. The cross-sectional width of each pole piece is w and its length is l, as illustrated in a view from above the plate, in Fig. 1; the electromagnet produces a field which is perpendicular to the strip. It is assumed that the north and south pole faces of the electromagnet are very close together so that the field is essentially uniform,[7] of value $\hat{k}B_0$, in the shaded area $-w/2 \leqslant x \leqslant w/2$, $-l/2 \leqslant y \leqslant l/2$ of Fig. 1, and of negligible magnitude outside that zone. To avoid having to consider the effects due to the edges of the metal strip on the distribution of currents in the strip, we assume that the latter is very wide compared to the width w of the magnet pole piece.

In the absence of other forces, the strip would move freely between the pole pieces when there is no current through the electromagnet. However, when a current is flowing through the electromagnet, a magnetic field $\hat{k}B_0$ is created in the shaded region and magnetic braking occurs. From Ohm's law and the Lorentz force law, we may write that

$$\mathbf{J} = \sigma(\mathbf{E} + \mathbf{v}\times\mathbf{B}), \qquad (1)$$

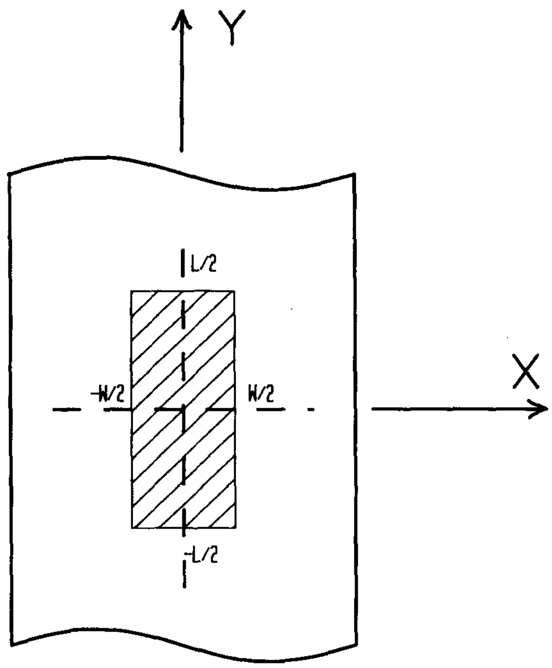

Fig. 1. Location of magnet pole piece for the moving strip case. The strip slides in the thin air gap between the pole pieces in a horizontal plane.

where **J** is the induced current density, **E** is the induced electric field, and **B** is the net magnetic field. If we assume that the cross-sectional length of the pole pieces is much larger than their cross-sectional width, $l \gg w$, then we expect that **J** will essentially be uniform, of value $\hat{i}(I/\delta l)$, except near the ends but these deviations may be neglected overall; here, I is the net current flowing in the shaded region of the metal strip. The induced field is then of the form $\mathbf{E} = -\hat{i}(V/w)$, where V is the voltage drop across the width w in the "shadow" region. Finally, we neglect the induced part of **B**, as already discussed, and easily obtain the relationship

$$V = \epsilon - rI, \tag{2}$$

where

$$\epsilon = vB_0 w \tag{3}$$

and

$$r = w/(\sigma l \delta) \tag{4}$$

by definition. Equation (4) gives the resistance to current flow of the shadow region and so Eq. (2) may be interpreted as giving the circuit-equivalent of our problem, since Eq. (2) describes the voltage across the terminals of a battery of e.m.f. ϵ and internal resistance r. As a result, if we denote the external resistance of the strip (i.e., strip less metal contained in shadow region) by R, then $V = RI$ and we can solve for the induced current I:

$$I = \alpha \sigma B_0 l \delta v, \tag{5}$$

with

$$\alpha \equiv (1 + R/r)^{-1}. \tag{6}$$

The parameter α is a dimensionless quantity which may be found if the external resistance R is known. The calculation of R is generally complicated and we do not elaborate on it here since one can readily measure this parameter in the laboratory, by first carving out an area $l \times w$ from a large plate. We have carried out a detailed calculation of R using conformal transformation techniques but an outline is inappropriate in the present context. (We will, however, supply all the details of the method upon request, by writing to the second author of this paper.)

The drag force is easily found to be

$$\mathbf{F} = \int \mathbf{J} \times \mathbf{B}_0 d\tau$$
$$= \hat{j}(\alpha \sigma l \delta w B_0^2 v) \tag{7}$$

in the present circumstances; it varies linearly with the velocity of the strip and quadratically with the applied field.

III. EXPERIMENTAL RESULTS

Equation (7) has been tested experimentally by spinning a thin aluminum disk of large radius on an air cushion. This system will produce results which are almost identical to those of a long wide strip if the magnet is positioned sufficiently far from the center of rotation that the velocity of the disk under the pole pieces is practically uniform. The outer edge of the disk is also assumed to be sufficiently far from the field (shaded area) to warrant neglecting its influence on the induced current pattern. Under these conditions, which are easily achieved experimentally, we can view the spinning disk braking problem as being equivalent to that of braking a long wide strip.

Magnetic braking was measured in a 435-mm-diam aluminum disk which was free to rotate in a 7-mm gap between the rectangular pole pieces of an electromagnet. The thickness of the disk was $\delta = 1.17$ mm, and the electrical resistivity $\rho = 1/\sigma$ was found to be 4.02×10^{-8} Ωm, using the four-terminal method described by van der Pauw.[13] The cross-sectional area of the rectangular pole pieces measured 57 mm × 29 mm; the position of the pole pieces with respect to the spinning disk is shown in Fig. 2.

The aluminum disk was coaxially attached to a horizontal 125-mm-diam aluminum plate which formed the rotat-

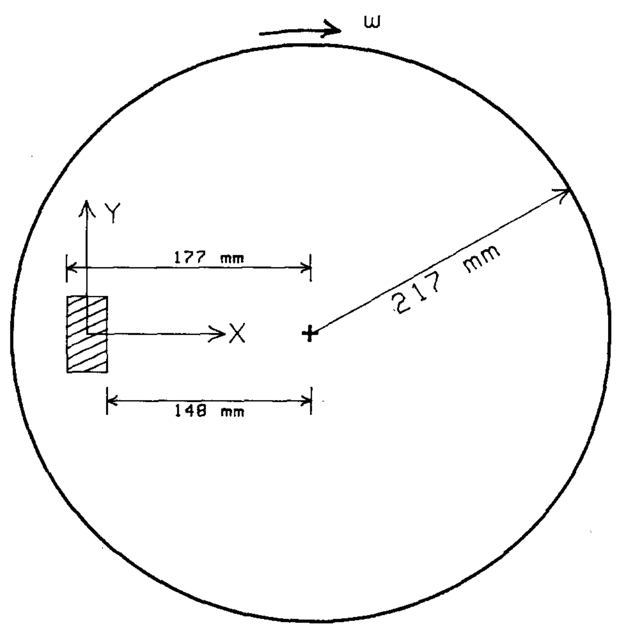

Fig. 2. Location of the magnet pole piece. The disk rotates in the air gap, in a horizontal plane.

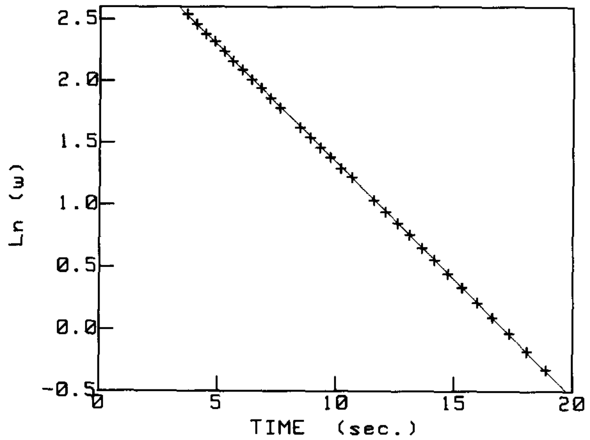

Fig. 3. Semilogarithmic plot of the angular frequency as a function of time. The slope of this graph is $-0.18/T^2$ s and the magnetic field is $B = 69$ mT.

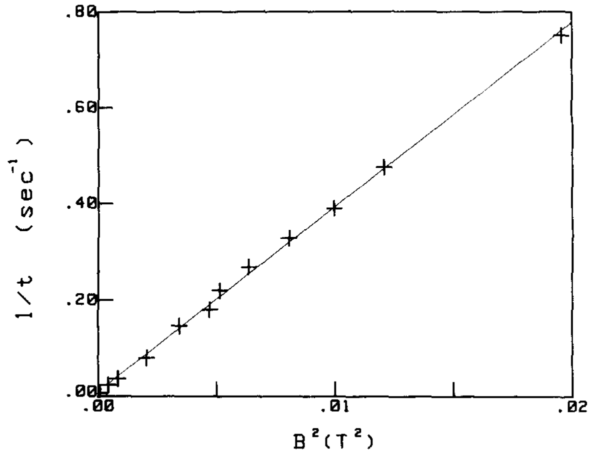

Fig. 4. Plot of the inverse time constant as a function of the square of the magnetic field.

ing element of a Pasco Scientific air bearing. This bearing is designed to support both radial and downward loads. The moment of inertia of the rotating assembly, consisting of the aluminum disk and the air bearing plate, was calculated to be 1.18×10^{-2} kg^2 ($\pm1\%$). The air bearing plate has a strip of equally spaced optical bars on its circumference, and a photoreflective optical reader was used to detect the motion of the bars past the reader.

The output of the optical reader was fed to the game port of an Apple II$^+$ computer which was programmed to measure the average frequency of the optical reader signal over successive 10-cycle intervals, and to determine the accumulated time at each of these measurements. About 20 such measurements were obtained during each revolution of the disk. The data was then converted to give the natural logarithm of the angular speed $\omega(t)$ of the spinning disk as a function of time. A typical decay curve is shown in Fig. 3. Spinning speeds were always much smaller than the skin-effect-related characteristic speed v_c introduced at the end of Sec. I. [Such effects appear at speeds of order $2/(\sigma\mu_0\delta)\simeq 55$ m s^{-1} in the case of aluminum; typical spinning speeds at the magnet location were 1 to 2 m s^{-1} in our experimental situation.]

According to the results of rotational dynamics, we have

$$K\frac{d\omega}{dt} = \Gamma_0 + \Gamma_B, \tag{8}$$

where Γ_0 is the residual ($B_0 = 0$) air-drag torque, Γ_B is the magnetic torque, K is the moment of inertia (disk plus bearing), and $d\omega/dt$ is the time rate of change of the angular velocity ω. The air drag is assumed to be linear (of viscous type) in the angular velocity; similarly, by setting $v = \omega L$ in Eq. (7), the magnetic drag is seen to be linear also. Here, L is the distance between the axis of rotation and the center of the rectangular region where the applied magnetic field penetrates the disk. Equation (8) is readily integrated and the result is

$$\omega(t) = \omega_0 e^{-t/\tau}, \tag{9}$$

where

$$\frac{1}{\tau} = \frac{1}{\tau_0} + mB_0^2, \tag{10}$$

with the parameter $1/\tau_0$ is associated with air damping alone. According to the results of the previous section, the parameter m is

$$m = \sigma\delta l w L^2/(1 + R/r)K. \tag{11}$$

A least-squares fit analysis of the data reported in Fig. 3 gives an inverse time constant $\tau^{-1} = 0.1823$ s^{-1}, with an uncertainty of 0.0004 s^{-1} (correlation coefficient: 0.9999), for an applied magnetic field of 69 mT. Measurements of the inverse time constant were performed for different magnetic fields, in the range 10 mT$\leqslant B_0 \leqslant$150 mT. The results shown in Fig. 3 are typical of all the measurements taken. The time dependence of the angular speed is very well described by an exponential behavior in time in all cases. In our arrangement, $L = 162.5$ mm.

Figure 4 shows a plot of the measured inverse time constants as a function of the square of the applied magnetic field intensity. The extrapolated intercept is $\tau_0^{-1} = 0.01$ s^{-1}; this result agrees well with the inverse time constant measured directly, without magnetic braking. The measured slope is $m = 38.5$ T^{-2} s^{-1}, with a least-squares uncertainty of ± 0.5 T^{-2} s^{-1}. For magnet parameters $l = 57$ mm and $w = 29$ mm, our conformal evaluation of R gives $0.87/\sigma\delta$. The slope calculated with the help of Eq. (11) is then 39.7 T^{-2} s^{-1}. We have not been able to determine with a high degree of confidence if our neglect of the fringing currents in the shadow region caused a decrease or an increase in the calculated value of m, and by how much. On the face of the above results however, agreement between theory and experiment is reasonable.

IV. CONCLUSION

We have presented a very simple model of magnetic braking. The model contains all the important physical features of this phenomenon and it is easily presented at an early level as an illustration of the importance of the Lorentz force law. Our second year undergraduates have been very enthusiastic about the experiment and we believe that the approach can be useful to others as well.

As already stated, the present approach does not permit to decide conclusively what effect the neglect of fringing has on the resistance ratio R/r. A better model is needed in this respect.

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Student difficulties in connecting graphs and physics: Examples from kinematics

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Some common errors exhibited by students in interpreting graphs in physics are illustrated by examples from kinematics. These are taken from the results of a descriptive study extending over a period of several years and involving several hundred university students who were enrolled in a laboratory-based preparatory physics course. Subsequent testing indicated that the graphing errors made by this group of students are not idiosyncratic, but are found in different populations and across different levels of sophistication. This paper examines two categories of difficulty identified in the investigation: difficulty in connecting graphs to physical concepts and difficulty in connecting graphs to the real world. Specific difficulties in each category are discussed in terms of student performance on written problems and laboratory experiments. A few of the instructional strategies that have been designed to address some of these difficulties are described.

I. INTRODUCTION

Many undergraduates taking introductory physics seem to lack the ability to use graphs either for imparting or extracting information. As part of our research on student understanding in physics, the Physics Education Group at the University of Washington has examined some of the graphing errors made by students. Part of the motivation for undertaking this study has been a conviction that facility in drawing and interpreting graphs is of critical importance for developing an understanding of many topics in physics. We have been especially interested in exploring whether some of the difficulties with the kinematical concepts that we identified in an earlier study might be effectively addressed through an increased emphasis on graphical representations.[1,2]

The problems students have with graphing cannot be simply attributed to inadequate preparation in mathematics. Frequently students who have no trouble plotting points and computing slopes cannot apply what they have learned about graphs from their study of mathematics to physics. Therefore there must be other factors, distinct from mathematical background, that are responsible. The analysis of graphing errors identified in this study indicates that many are a direct consequence of an inability to make connections between a graphical representation and the subject matter it represents. In this paper, we describe two categories of student difficulty that we have investigated: difficulty in connecting graphs to physical concepts and difficulty in connecting graphs to the real world. Specific difficulties in each category are identified and discussed in terms of student performance on written problems and laboratory experiments. All of the examples used as illustrations are from kinematics, although our study also included other topics in physics and physical science.

Most of the work reported here was carried out over a period of several years in the context of a year-long preparatory physics course for undergraduates intending to enroll in either algebra- or calculus-based physics.[3] We have supplemented the information obtained from this group by extending the study to include students enrolled in our special physics courses for prospective and practicing precollege teachers and in the standard introductory physics courses at the University of Washington. We have also examined responses by high school physics and physical science students to some of the same questions that we administered to the college students in the study. Although there were differences in severity, the nature of the difficul-

