This serves to reinforce our belief that the dynamics of photons and of other particles can be brought, for some purposes at least, within the same descriptive framework. Our next step will be to suggest what that framework might be. Our argument will appeal to one's sense of what is plausible; it will not be logically inescapable. But as the old saying goes, "the proof of the pudding is in the eating," and we shall see how beautifully one can describe the transition from Newtonian to non-Newtonian behavior on the basis of our conclusions (which are indeed precisely those of special relativity).

TER AND RADIATION: THE INERTIA OF ENERGY

Are not gross Bodies and Light convertible into one another, and may not Bodies receive much of their Activity from the Particles of Light which enter their Composition?

Newton, *Opticks* (4th ed., 1730)

It would be quite wrong to suggest that Newton had really anticipated 20th-century physics to the extent that the above quotation might imply, but his provocative query is superbly appropriate as an introduction to the discussion that we shall now undertake. For we shall consider the intimate connection between the inertia of ordinary matter and the energy of radiation, and in so doing we shall develop some dynamical results that apply equally to photons and "gross bodies." We shall obtain, as one of the consequences, a full account of the relation between speed and kinetic energy for the electrons in the ultimate-speed experiment.

Our starting point will be a *gedanken experiment* (literally a "thought experiment," i.e., a fictitious, not really feasible experiment) which was invented by Einstein himself in 1906.[1] The purpose of it is to suggest that energy must have associated with it a certain inertial mass equivalent.[2] We suppose that an amount E of radiant energy (a burst of photons) is emitted from one end of a box of mass M and length L that is isolated from its surroundings and is initially stationary [Fig. 1–4(a)]. The radiation carries momentum E/c. Since the total momentum of the system remains equal to zero, the box must acquire a momentum equal

[1]A. Einstein, *Ann. Phys.*, **20,** 627–633 (1906).

[2]By inertial mass we mean the ratio of linear momentum to velocity.

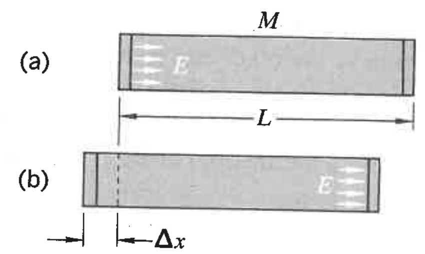

Fig. 1–4 Einstein's box—a hypothetical experiment in which a box recoils from its initial position (a) to a final position (b) as a result of a burst of radiant energy traveling from one end of the box to the other.

to $-E/c$. Hence the box recoils with a speed v, given by

$$v = -\frac{E}{Mc} \qquad (1\text{–}5)$$

After traveling freely for a time Δt ($= L/c$ very nearly, provided $v \ll c$), the radiation hits the other end of the box and conveys an impulse, equal and opposite to the one it gave initially, which brings the box to rest again.[1] Thus the result of this process is to move the box through a distance Δx:

$$\Delta x = v\,\Delta t = -\frac{EL}{Mc^2} \qquad (1\text{–}6)$$

But this being an isolated system, we are reluctant to believe that the center of mass of the box plus its contents has moved. We therefore postulate that the radiation has carried with it the equivalent of a mass m, such that

$$mL + M\,\Delta x = 0 \qquad (1\text{–}7)$$

Putting the last two equations together, we have

$$m = \frac{E}{c^2} \quad \text{or} \quad E = mc^2 \qquad (1\text{–}8)$$

For the man on the street, Einstein and relativity are probably epitomized by this result. For the physicist, its importance is not lessened by its becoming hackneyed; it asserts a fundamental inertia of energy. Although the calculation as we have presented it (which differs somewhat from Einstein's original version) points in the first instance to the mass associated with radiant energy, one quickly recognizes that the implications are much wider than this. When the radiation is emitted from one end of Einstein's box, that end must surely suffer a decrease, by

[1]If you feel that more careful account should be taken of the recoil of the box and its effect on the time and distance of transit of the radiation, see Problem 1–13.

the amount E/c^2, in its inertial mass. Likewise, the absorption of the radiation at the other end means an addition to the mass of that portion. Once the energy has been absorbed, it loses its identification as the energy of photons and ultimately becomes just an addition to the thermal energy. And we are quickly led to the idea that energy in any form has the mass equivalent defined by Eq. (1–8)—a general principle of the inertia of energy.[1]

The prime example of the mass-energy equivalence, to which we owe our continuing existence, is provided by thermonuclear reactions occurring in stars such as the sun. Observation tells us that radiant energy is reaching us from the sun at the rate of 1.35×10^3 watts/m^2. Given this figure and Eq. (1–8), we can infer that the mass of the sun is decreasing at the rate of about 4.5×10^6 tons/sec—an impressively rapid loss, even though it is only about 1 part in 10^{13} of the sun's mass per year. This comes about through chains of nuclear reactions, chief among which is the sequence by which hydrogen (^{1}H) is converted to helium (^{4}He). One must, of course, have four hydrogen atoms to end up with one helium atom, and the process takes place in several separate steps. One of these steps is particularly worth mentioning here, because it is a simple and remarkably direct example of the equivalence of the mass of ordinary matter and the energy of photons. It is this:

$$p + \mathrm{D} \rightarrow {}^3\mathrm{He} + \gamma \qquad (1\text{–}9)$$

A proton fuses with a deuteron D (the nucleus of hydrogen-2, containing one proton and one neutron), making a system of two protons and one neutron, which is the nuclear composition of ^{3}He. But, as mass-spectrometer measurements show us, the mass of this combination is greater than the mass of ^{3}He in its normal state. Here are the approximate values:

Proton	1.6724×10^{-27} kg
Deuteron	3.3432
$p + \mathrm{D}$	5.0156
^{3}He nucleus	5.0058
Mass excess	9.8×10^{-30} kg

This amount of mass is carried off by a photon (a γ ray) as indicated by Eq. (1–9). The energy of that photon is given by

[1]For a fine discussion of this question, see M. von Laue's article "Inertia and Energy" in *Albert Einstein: Philosopher-Scientist*, Vol. II, (P. A. Schilpp, ed.), Harper Torchbook, Harper and Row, New York, 1959.

Eq. (1–8):

$$\begin{aligned} E = mc^2 &= 9.8 \times 10^{-30} \times 9.0 \times 10^{16} \\ &= 8.8 \times 10^{-13}\ \text{joule} \\ &= 5.5\ \text{MeV} \end{aligned}$$

This process has been studied in the laboratory, and γ rays of the expected energy have been observed.[1] It should perhaps be added that such reactions, when they occur as thermonuclear reactions in the sun, require temperatures of the order of 10^7 °K and thus take place only in the inner regions. Gamma rays, such as those just considered, are completely absorbed before reaching the sun's surface, and their energy finally escapes in photons with individual energies of the order of only 1 eV—infrared, visible, and ultraviolet—that constitute the familiar solar spectrum.

The equation $E = mc^2$ has (at least in popular accounts) been so exclusively linked to nuclear transformations as to divert attention from its universality. But the message of Einstein's equation is that *any* change ΔE in the energy of a body implies a corresponding change Δm in its inertial mass:

$$\Delta E = c^2 \Delta m \qquad (1\text{–}10)$$

A golf ball in motion has more mass than the same golf ball at rest. The heated filament of a lamp has more mass than the same filament when cold. A charged capacitor has more mass than the same capacitor uncharged. And so on. Because, in terms of familiar magnitudes, the mass associated with a given amount of energy is exceedingly small (e.g., the energy used per day for domestic purposes in a city of a million people has a mass equivalent of only about 1 g), this intimate connection between the two was long unrecognized. Einstein regarded the discovery of this connection as being extremely important. To quote his own words[2]:

> The most important result of a general character to which the special theory has led is concerned with the conception of mass. Before the advent of relativity, physics recognized two conservation laws of fundamental importance, namely, the law of the conservation of energy and the law of the conservation of mass; these two fundamental laws appeared to be quite inde-

[1]W. A. Fowler, C. C. Lauritsen, and A. V. Tollestrup, *Phys. Rev.*, **76**, 1767 (1949).

[2]A. Einstein, *Relativity*, Crown, New York, 1961.

pendent of each other. By means of the theory of relativity they have been united into one law.

Perhaps one of the best ways to appreciate the pervasive character of the mass-energy equivalence is to consider a single, neutral atom in a piece of ordinary matter. From one point of view it is just one of a collection of what Newton called "solid, massy, hard, impenetrable, movable Particles."[1] The question of any inner structure does not arise, and it seems almost obvious that the atom's inertial property should be described by a single quantity that we call the mass. But now consider this same atom from the standpoint of present-day knowledge. It is a complicated assembly of electrons, neutrons, and protons (and if we want to probe more deeply, there is finer structure yet). The mass of the atom as a whole contains positive contributions from the kinetic energies of its swiftly moving constituents, and contributions of both signs (predominantly negative) from the potential energy of their electrical and nuclear interactions. (Note that a force of attraction between two particles automatically represents a *negative* contribution to the total mass of the system.[2]) Any change in the internal state of the atom is accompanied by a flow of energy into or out of it, with an associated increase or decrease in its mass. The ability of the constituents to cohere depends on the fact that their total energy in this configuration is less than if they were all separated from one another. In these terms, then, the mass of an atom is the result of a remarkable and subtle synthesis. Yet it serves to characterize the whole atom in every dynamical context—including gravitation—in which it moves as a single unit.

ENERGY, MOMENTUM, AND MASS

Let us now try to put together some of the results we have discussed. For photons we have

$$E = cp \tag{1-3}$$

and

$$m = \frac{E}{c^2} \tag{1-8}$$

[1]Sir I. Newton, *Opticks*, 4th ed., 1730; reprinted in revised form by G. Bell, London, 1931; Bell edition reprinted by Dover, New York, 1952.

[2]Provided the strength of the attractive force gets less with increasing separation, which is true of all such forces between elementary particles in atoms.

(the first experimental, the second based on Einstein's box). Combining these, we have

$$m = \frac{p}{c} \tag{1-11}$$

In Newtonian mechanics, however, we have

$$m = \frac{p}{v} \tag{1-12}$$

It looks as though we might regard Eq. (1–11) as a particular case of Eq. (1–12), for $v = c$. If, further, we suppose that Eq. (1–8) describes a universal equivalence of energy and inertial mass, we can combine Eqs. (1–8) and (1–12) into a single statement:

$$E = \frac{c^2 p}{v} \tag{1-13}$$

Now in classical mechanics we are never concerned with absolute energies but only with energy differences, and with the transformation between one form of energy and another. A particle suffers a change of potential energy, for example, and its kinetic energy undergoes a corresponding change, so that the total energy remains constant. The basis for analyzing all such situations is *Newton's law*. The increment of kinetic energy corresponds to the work done by external forces,[1] and we have

$$dE = F\,dx = \frac{dp}{dt}\,dx$$

i.e.,

$$dE = v\,dp \tag{1-14}$$

If we accept Eqs. (1–13) and (1–14) we can obtain from them a relationship, now proposed as a general one, between energy and momentum for a particle. We do this by multiplying together the left and right sides of the two equations, and integrating:

$$E\,dE = c^2 p\,dp$$

Therefore,

$$E^2 = c^2p^2 + E_0{}^2 \tag{1-15}$$

where $E_0{}^2$ is a constant of integration, written explicitly as the square of some constant energy.

[1]The ultimate-speed film presents evidence that, even under conditions where some of the features of Newtonian mechanics have broken down, the increase of energy (kinetic energy) of an electron is still equal to the work calculated from the electrostatic force multiplied by the distance traveled.

From here it is possible to proceed in several ways. For example, we can substitute in Eq. (1–15) the relation $cp = Ev/c$ from Eq. (1–13). This leads at once to the following result:

$$E(v) = \frac{E_0}{(1 - v^2/c^2)^{1/2}} \tag{1–16}$$

For $v \ll c$ we can approximate this exact result by the binomial expansion, neglecting terms of higher order than v^2/c^2.

$$\text{[Approximate result } (v \ll c)] \quad E(v) \approx E_0 + \tfrac{1}{2}\left(\frac{E_0}{c^2}\right)v^2 \tag{1–17}$$

If Eq. (1–17) is to harmonize with Newtonian mechanics at low velocities, we must identify E_0/c^2 with the classical inertial mass of a particle: Let us denote this by m_0. Then Eqs. (1–8) and (1–16) together lead to an explicit variation of inertial mass with speed:

$$m(v) = \frac{m_0}{(1 - v^2/c^2)^{1/2}} \tag{1–18}$$

The quantity m_0, which in Newtonian mechanics would be *the* inertial mass of a body, now assumes a new role as the *rest mass* of the body for $v = 0$; at any other speed the inertial mass is greater.[1]

An increase of inertial mass with speed is of course implied as soon as one embraces a general principle of the inertia of energy. The particular form of variation expressed by Eq. (1–18) is shown graphically in Fig. 1–5, together with some experimental results based on the electric and magnetic deflection of energetic electrons.

Equations (1–15) and (1–18) are two of the central results of the new dynamics; the first of them—the relation between energy and momentum—will prove to be of special importance and applicability. But the kinetic energy of a particle, so valuable a quantity in classical dynamics, now takes on a secondary status. It is merely the difference between the total energy E and the rest energy E_0:

$$K = m_0c^2\left[\frac{1}{(1 - v^2/c^2)^{1/2}} - 1\right] \tag{1–19}$$

Of course K remains a quantity of practical importance, because it is the measure of the extra energy conferred on a particle

[1]And the quantity E_0 $(= m_0c^2)$ is the *rest energy*. Thus for electrons (for example) we have $m_0 = 9.11 \times 10^{-31}$ kg, $E_0 = 8.2 \times 10^{-14}$ joule $=$ 0.51 MeV.

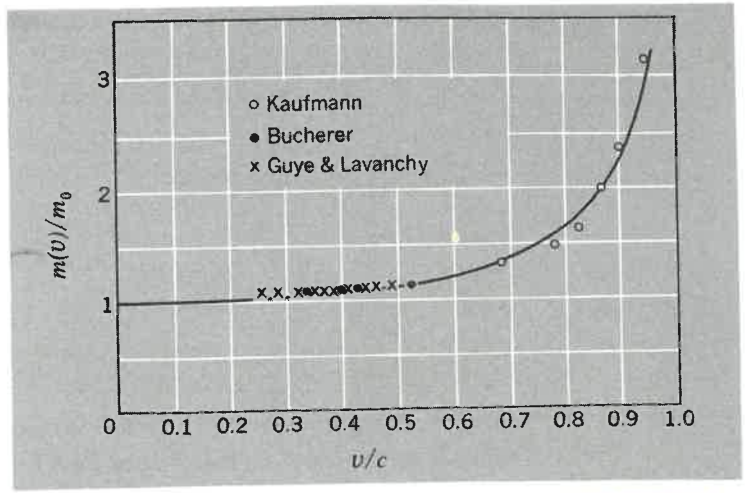

Fig. 1–5 Variation of inertial mass with speed for electrons. Based on data of Kaufmann (1910), Bucherer (1909), and Guye and Lavanchy (1915). (After R. S. Shankland, Atomic and Nuclear Physics, Macmillan, New York, 1961.)

through the work done by external forces. Note that K is *not* obtained by substituting into the expression $\frac{1}{2}mv^2$ the value of m calculated from Eq. (1–18)—a frequently made error, because the temptation to cling to the Newtonian form of the kinetic energy is very strong.

At the risk of seeming repetitious, let us reemphasize the significance of $m(v)$ as defined by Eq. (1–18). It describes the inertial property of a body moving with velocity $\mathbf{v}$, so that the momentum $\mathbf{p}$ is given by the equation

$$\mathbf{p} = m(v)\mathbf{v} \tag{1–20}$$

It also describes the total energy content of the body, so that

$$E = m(v)c^2 \tag{1–21}$$

Now it is the quantities $\mathbf{p}$ and E, rather than $m(v)$ by itself, that figure in any actual dynamical situation. In this sense the variable mass $m(v)$ is just a convenient construct which, for example, allows us to preserve the form of the Newtonian statement that momentum is mass times velocity. Many physicists prefer to reserve the word *mass* to describe the rest mass m_0, a uniquely defined property of a given particle. But this is essentially a matter of taste.[1] Whatever words one elects to use, there is no disagreement on the fact that Eqs. (1–20) and (1–21) describe the momentum and total energy of a particle, where $m(v)$ is given by Eq. (1–18).

[1]And one cannot escape the fact that, for almost any particle, even the rest mass involves contributions associated with the motions and kinetic energies of its constituents.

The denominator $(1 - v^2/c^2)^{1/2}$ appears so often in special relativity, and is so awkward to write, that nearly all discussions of relativity make use of a single symbol, γ, defined as follows. Put

$$\gamma(v) = \frac{1}{(1 - v^2/c^2)^{1/2}} \qquad (1\text{–}22)$$

Then we have

$$m = \gamma m_0 \qquad (1\text{–}23)$$

$$\mathbf{p} = \gamma m_0 \mathbf{v} \qquad (1\text{–}24)$$

$$E = \gamma m_0 c^2 \qquad (1\text{–}25)$$

where in using Eqs. (1–23) to (1–25) we must remember that γ depends on the speed v according to Eq. (1–22).

IS THE NEW DYNAMICS CORRECT?

It is important to ask whether Eq. (1–19) does indeed provide a correct account of the relation between speed and kinetic energy as observed, for example, in the linac experiment. Rearranging the result, we have

$$1 + K/m_0c^2 = (1 - v^2/c^2)^{-1/2}$$

Therefore,

$$1 - v^2/c^2 = (1 + K/m_0c^2)^{-2}$$

or

$$v^2 = c^2[1 - (1 + K/m_0c^2)^{-2}] \qquad (1\text{–}26)$$

Clearly the rest energy m_0c^2 provides a natural unit in which to measure the extra energy K that is added to a particle by means of an acceleration process. We can, in fact, draw up a table showing how the speed would depend on K for any particle whatsoever (Table 1–4).

Given that, for electrons, $m_0c^2 = 0.51$ MeV, we can readily plot a curve of v^2 in m^2/sec^2 against K in MeV. This curve has been drawn in on the graph of the data in the ultimate-speed experiment (Fig. 1–3). It may be seen that the agreement between theory and experiment is very good, and speaks strongly for the correctness of the revised dynamics, as does the measured variation of mass with speed, shown in Fig. 1–5.

If we wanted to plot a curve of v^2 versus K for protons, all

TABLE 1–4: SPEED VERSUS KINETIC ENERGY FOR PARTICLES

K/m_0c^2	$(1 + K/m_0c^2)^{-2}$	v^2/c^2	v/c	v^2, $\times 10^{16}$ m^2/sec^2
0.1	0.8264	0.1736	0.417	1.56
0.2	0.6944	0.3056	0.553	2.75
0.3	0.5917	0.4083	0.639	3.67
0.5	0.4444	0.5556	0.745	5.00
1.0	0.2500	0.7500	0.866	6.75
2.0	0.1111	0.8889	0.943	8.00
5.0	0.0278	0.9722	0.986	8.87
10.0	0.0083	0.9917	0.996	8.93
30.0	0.0010	0.9990	0.999	8.99

we would need to do would be to put $m_0 = 1.672 \times 10^{-27}$ kg, which gives $m_0c^2 = 0.938$ GeV (or 938 MeV), and Table 1–4 would provide the rest of the information needed. The fact that this does indeed give correct results for protons is amply attested in the operation of big nuclear accelerators, and there is plenty of evidence that Eq. (1–26) holds for particles of all kinds.

Among the various features of these modified laws of motion, the phenomenon of the limiting speed c is perhaps the most noteworthy. It means that energy (and mass) can be piled onto atomic particles without increasing their speed appreciably. To see in detail how this works, it is convenient to rewrite Eq. (1–16) as follows:

$$(1 - v^2/c^2)^{1/2} = E_0/E$$

Therefore,

$$v^2/c^2 = 1 - (E_0/E)^2 \qquad (1\text{–}27a)$$

and

$$v/c = [1 - (E_0/E)^2]^{1/2}$$

For $E \gg E_0$, we then have, approximately,

$$v/c \approx 1 - \tfrac{1}{2}(E_0/E)^2 \qquad (1\text{–}27b)$$

For example, the Harvard–M.I.T. electron accelerator has as its injector a linear accelerator (like the one used in the ultimate-speed film) that gives the electrons 15 MeV energy. The main accelerator brings the electrons up to about 5 GeV (= 5000 MeV).

Using these values, one finds

Injection from linac (15 MeV) $\rightarrow v/c \approx 0.9995$
Final energy (5 GeV) $\rightarrow v/c \approx 0.99999995$

Thus the change of v/c after the preliminary acceleration is only about 5 parts in 10^4. These big nuclear machines might appropriately be called "ponderators"[1] rather than accelerators, for to an excellent approximation they do just add mass to the particles injected into them, with no significant increase in the speed as such.

MOTION UNDER A CONSTANT FORCE

The simplest dynamical problem in classical mechanics is the motion of a body under a constant force. Let us see how this problem is modified in the new dynamics. Suppose a force F acts on a body for a time t (we assume one-dimensional motion); the body is assumed to be initially at rest, and ends up with a speed v. Then

$$Ft = mv = \frac{m_0 v}{(1 - v^2/c^2)^{1/2}} \tag{1-28}$$

Therefore,

$$1 - v^2/c^2 = (m_0 v/Ft)^2$$
$$c^2 = v^2[1 + (m_0 c/Ft)^2]$$

and

$$v(t) = \frac{c}{[1 + (m_0 c/Ft)^2]^{1/2}} \tag{1-29}$$

This is a rather complex-looking result. Let us consider two extreme cases:

(a) $Ft \ll m_0 c$:

$$(m_0 c/Ft)^2 \gg 1$$

Therefore,

$$v(t) \approx \frac{c}{(m_0 c/Ft)} = \frac{F}{m_0} t$$

(b) $Ft \gg m_0 c$:

$$(m_0 c/Ft)^2 \rightarrow 0$$

[1]This name was first proposed around 1945 by Prof. A. G. Hill of M. I. T.

Therefore,

$$v(t) \approx c$$

Case (a) corresponds to ordinary Newtonian mechanics. Case (b) displays the now-familiar property of a limiting constant speed c for motion under any force, no matter how large it is or for how long it is applied.

"EINSTEIN'S BOX UNHINGED"

According to our present beliefs as expressed by special relativity, the speed of light in free space represents an upper limit, not only to the speed of material particles such as electrons, but also to the speed with which an interaction of any kind can be propagated—gravitational, nuclear, electric, etc. Were this not so, it would be possible (as we shall discuss later) to arrive at a paradox involving the interchange of the roles of cause and effect, according to one's point of view (see the discussion of causality near the end of Chapter 4).

One particular consequence of the physical speed limit equal to c is that the classical concept of an ideal rigid body finds no place in special relativity. (And strictly speaking, it cannot be justified in classical mechanics either.) For by a rigid body we mean an object along which physical information can be transmitted in an arbitrarily short time, so that the object is set in motion instantaneously, as a single unit, when a force is applied to any point in it. For any ordinary box the information that one end has been struck is transmitted as an elastic wave, which we know is many orders of magnitude slower than a light signal. Thus the Einstein box argument in its original form cannot be maintained. At the receiving end of the box, the first intimation that anything had happened at the other end would be the arrival of the radiation itself. We can, however, rehabilitate the argument as follows.

Ignore completely any connection between the ends of the box, and regard it as two separate masses, m_1 and m_2 (Fig. 1-6). Just suppose that one end, of initial mass m_1, emits energy E at $t = 0$ and suffers a mass change to m_1'. It acquires a velocity v_1 given by

$$v_1 = \frac{-E/c}{m_1'}$$

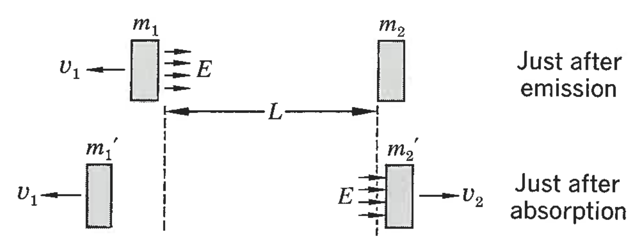

Fig. 1-6 "Einstein's box unhinged." The recoil processes in two unconnected masses in consequence of a burst of radiant energy emitted from one (m_1) and absorbed in the other (m_2).

If m_1 were originally at $x = 0$, its position at any later time is thus given by

$$x_1(t) = -\frac{E}{m_1'c}t \tag{1-30}$$

When the energy arrives at m_2 (at $t = L/c$) it causes a recoil and a change of mass so that we have, for the position of m_2,

$$x_2(t) = L + \frac{E}{m_2'c}(t - L/c) \tag{1-31}$$

Let the total mass be M, and let the position of the center of mass be $\bar{x}$ before the radiation was emitted from m_1 and $\bar{x}'$ after it was absorbed in m_2. Then

$$M\bar{x} = m_1 \cdot 0 + m_2 \cdot L \tag{1-32}$$

and

$$M\bar{x}' = m_1'\left(\frac{-E}{m_1'c}t\right) + m_2'\left[L + \frac{E}{m_2'c}(t - L/c)\right]$$

i.e.,

$$M\bar{x}' = -\frac{E}{c}t + m_2'L + \frac{E}{c}t - \frac{E}{c^2}L \tag{1-33}$$

Hence, if $\bar{x}' = \bar{x}$,

$$\Delta m_2' = m_2' - m_2 = \frac{E}{c^2} = -\Delta m_1' \tag{1-34}$$

Thus the principle of inertia of energy finds a sounder theoretical basis, but by this stage we have seen its real vindication in the experimentally observed behavior of particles.

SOME COMMENTS

In this chapter we have presented evidence to show that the behavior of particles at very high speed simply does not conform to Newtonian dynamics. By analyzing this behavior, and by following Einstein in the assumption that the center of mass of an isolated system does not spontaneously shift, we have developed some relations (which appear experimentally to be valid for all attainable speeds) connecting energy, momentum, and mass. This has allowed us to arrive rather quickly at some important dynamical results. On the other hand, it is clear that the arguments we have used involve a good deal of conjecture; they are suggestive but by no means irresistible. Furthermore, one may well ask what all this has to do with the things one normally thinks of when relativity is mentioned—such things as the Lorentz contraction, frames of reference, space-time, the Michelson-Morley experiment. The answer is that the connection is very, very close. But apart from one small hint in our discussion of the results of the ultimate-speed experiment, we have so far not tried to deal with these very fundamental aspects of relativity. There is a good reason for that; each of the experiments that we cited was conducted within a single frame of reference—the experimenter's laboratory. But the concepts of distance, time, and velocity were involved at every turn; without them it is impossible to formulate or discuss dynamics.

It was in the attempt to explain optical phenomena that the need for some drastic revision of our ideas about space and time finally became overwhelming. The development of this problem, culminating in the Michelson-Morley experiment, is the subject of Chapter 2. And then we shall see how Einstein, through his insistence on a fundamental reexamination of the bases of dynamical measurement, made it possible to fit everything together within a single dynamical scheme. The same concepts of space and time are found to be appropriate to the facts of optics and electromagnetism and to the non-Newtonian dynamical behavior that we have been discussing in this chapter. Our program, then, will be to describe the predicament engendered by the facts of optics, to show how Einstein eliminated the apparent conflict between optics and Newtonian mechanics, and then to illustrate some of the applications of Einstein's formulation of the principle of relativity.

PROBLEMS

1-1 A burst of 10^{14} electrons accelerated to an energy of 15 MeV per electron is stopped in a copper target block of mass 100 g. If the block is thermally insulated, what is its temperature rise? The specific heat of copper is 0.09 cal/g·°K.

quite independent of any motion of the source itself. In our first discussion of the Michelson-Morley experiment (in Chapter 2) we stated that this was indeed the case. For a long time it was believed that this was proved by observations on the light from close binary stars. The two members of any such binary system have large relative velocities, and when one star has a component of velocity toward the earth the other will be moving away. It was argued that if these velocities were communicated to the emitted light, the *apparent* motions of the stars would be distorted away from the Newtonian orbits required by the law of gravitation. No such distortions were observed. It has been more recently argued, however, that since these binary star systems are usually surrounded by a gas cloud, which absorbs and then reradiates the light from the stars, the speed of the light that crosses interstellar space may in any case be independent of any possible influence of the original moving sources.[1] Subsequently, however, experiments have been made on rapidly moving terrestrial sources of radiation which verify this aspect of Einstein's second postulate in a convincing way. In one such experiment made with high-energy photons, not visible light, the source consisted of unstable particles (neutral π mesons) traveling at 99.975% of the speed of light. The measured speed of the photons emitted forward with respect to this motion was $(2.9977 \pm 0.0004) \times 10^8$ m/sec.[2] Reference to Table 1–2 will show that this is in excellent agreement with the best values of c obtained for stationary sources. In Chapters 5 and 6 we shall discuss in more detail the radiation from moving sources, in connection with the relativistic law of addition of velocities and related phenomena.

THE RELATIVITY OF SIMULTANEITY

An immediate consequence of Einstein's prescription for synchronizing clocks at different locations is that simultaneity is relative, not absolute. Let us see how this follows.

Suppose that three observation stations A, B, and C are equally spaced along the x axis of an inertial frame S in which they are all at rest. We can construct a simple x-t coordinate system, on which we draw "world lines" (to use the accepted phraseology) showing the development of the system in space

[1]J. G. Fox, *Am. J. Phys.*, **30**, 297 (1962).

[2]T. Alväger, F. J. M. Farley, J. Kjellman and I. Wallin, *Phys. Letters*, **12**, 260 (1964).

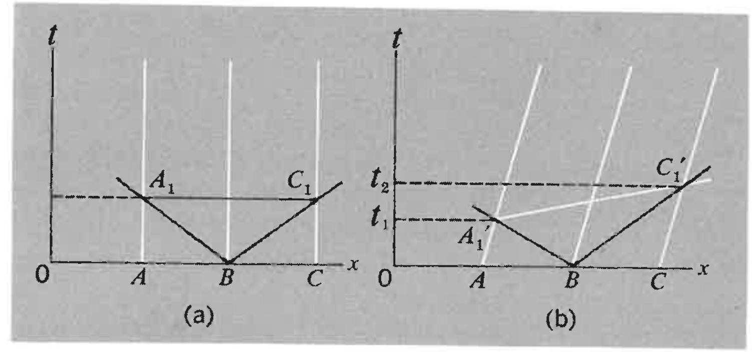

Fig. 3–2 (a) Space-time diagram showing experiment to define simultaneity at stations A and C (at rest in this reference frame) by light signals emitted from a station B midway between them. (b) Equivalent experiment for the case in which A, B, and C all have a velocity with respect to the reference frame.

and time [Fig. 3–2(a)]. The world line of any given particle is just a graph of its position as a function of time; it provides a complete picture of the history of the particle as observed within a given frame of reference. The world lines of A, B, and C are of course just vertical lines parallel to the t axis, corresponding to $x =$ constant. Suppose that a light or radio signal is sent out from B at $t = 0$. It travels at the same speed c forward and backward along the x axis—an assertion that embodies the universality of c. This signal is described by two sloping lines $x = x_B \pm ct$. The arrival of the signal at the positions of A and C is thus given by the intersections A_1, C_1, and simultaneity at the positions of A and C is *defined* by the line A_1C_1, parallel to the x axis, which joins a series of points possessing the same value of t.

But now suppose that A, B, and C are at rest in an inertial frame S' which is moving with respect to S at a speed v along the x direction [Fig. 3–2(b)]. The world lines of A, B, and C are now inclined as shown. A signal sent from B at $t = 0$ is again described (in S) by the lines $x = x_B \pm ct$, and the arrival of the signal at the positions of A and C is now given by the intersections A_1' and C_1'. These are clearly not simultaneous for S, because the line $A_1'C_1'$ is manifestly not parallel to the x axis. Or, to put it more concretely, the signal reaches A before it reaches C because, as observed in S, A is running to meet the signal pulse whereas C is running away from it. But we require