Chaos Experiment Computer Setup

- Make sure the Power Amplifier and Interface (Science Workshop 750) are ON.
- Open DataStudio
- Click "Open Activity"
- On the Desktop is a folder named "Physics Experiment Resources D", in there is a folder named "Chaos" and in there is another folder named "DataStudio Files", in there is a file named "CHAOS.ds" – open that. You might not have to navigate to this folder to find "CHAOS.ds", the dialog box might already be in that folder.
- This will give you all the plots and tools you should need for this experiment.
- To connect the interface in order to control the pendulum, click on the little button "Setup" in the top left corner of DataStudio.
- In the little picture of the interface, click on the "A" port. This is where the Power Amplifier is plugged into Interface which you can see on the desk right next to you.
- Scroll down and choose "Power Amplifier" because that's what is plugged into the "A" port in the Interface.
- A little box named "Signal Generator" will pop up. This is where you control the speed of the driven pendulum by changing the voltage.
- Select "DC Voltage" from the drop down menu in the "Signal Generator" box (you'll have to scroll up from "Sine Wave" in the drop down menu). We don't want any of the other options like "Sine Wave" or "Triangle Wave" because then we would be driving the pendulum in weird ways which would be [redacted] bad.
- To set the voltage, you must first click the box that says "Auto" – this will allow you to manually control the voltage which is what you want.
- Now you can make the pendulum do its thing by clicking the "On" button and upping the voltage by clicking the little + box under the voltage. You should see the pendulum moving now.
- You can change the increments that the +/- buttons change the voltage by clicking on the </> buttons.
- Now you are set to do all sorts of cool stuff with the driven pendulum. Just do what you want to the pendulum and click the "Start" button at the top left of DataStudio. You should see stuff pop up on the different plots and a frequency show up in the "Driver Frequency" box. You can even record data using DataStudio even without supplying any voltage to supply the pendulum like if you were to move the pendulum by hand and let gravity do its thing.
- You can export your data for graphing with Python or Origin by going "File -> Export Data..." at the top left of DataStudio.
- You can also copy graphs as they look in DataStudio by clicking the plot that you want to save, such as the "Poincare Plot" box and going "Edit -> Copy" at the top left of DataStudio. You can then paste this into Paint or another program you want.
- Make sure to save your activity! And save it somewhere else on the computer that you will remember! Don't save over the original "CHAOS.ds" file please.

Chaos

Equipment

Included:		
1	Large Rod Stand	ME-8735
2	120 cm Long Steel Rod (2)	ME-8741
1	45 cm Long Steel Rod	ME-8736
2	Multi Clamps (2)	ME-9507
1	Chaos/Driven Harmonic Accessory	CI-6689A
1	Mechanical Oscillator/Driver	ME-8750
1	Rotary Motion Sensor	PS-2120
1	Photogate Head	ME-9498A
Required, but not included:		
1	850 Universal Interface	UI-5000
1	PASCO Capstone Software	UI-5400

Introduction

The chaotic behavior of driven nonlinear pendulum is explored by graphing its motion in phase space and by making a Poincare plot. These plots are compared to the motion of the pendulum when it is not chaotic.

The oscillator consists of an aluminum disk connected to two springs. A point mass on the edge of the aluminum disk makes the oscillator nonlinear. The frequency of the sinusoidal driver can be varied to investigate the progression from predictable motion to chaotic motion. Magnetic damping can ~~be~~ also be adjusted to change the character of the chaotic motion. The angular position and velocity of the disk are recorded as a function of time using a Rotary Motion Sensor. A real-time phase plot is made by graphing the angular velocity versus the displacement angle of the oscillation.

The Poincare plot is also graphed in real time, superimposed on the phase plot. This is achieved by recording the point on the phase plot once every cycle of the driver arm as the driver arm blocks a photogate.

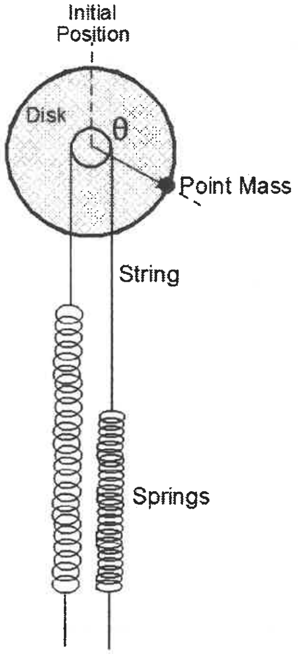


Figure 1: Pendulum and Springs

Written by Ann Hanks

Theory

The oscillator consists of an aluminum disk connected to two springs. A point mass on the edge of the aluminum disk makes the oscillator nonlinear. This nonlinearity is required to cause chaotic motion. Also, the disk is magnetically damped.

Several quantities can be varied to cause regular motion to become chaotic. These variables are the driving frequency, driving amplitude, damping amplitude, and the initial conditions.

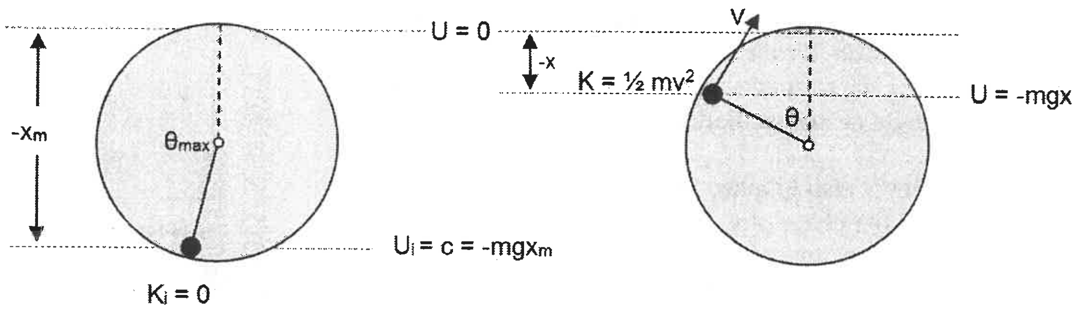
There are three different ways of plotting oscillations:

1. Angular Velocity (ω) vs. time
2. Phase Space: Angular Velocity (ω) vs. Angular Displacement (θ)
3. Poincare Plot: Angular Velocity (ω) vs. Angular Displacement (θ) plotted only once per period of the driving force.

The phase space and the Poincare plot are particularly useful for recognizing chaotic oscillations. When the motion is chaotic, the graphs do not repeat.

Potential Well

This pendulum has two equilibrium points, one on each side where the torque caused by the weight of the point mass is balanced by the torque from the springs. To map U (potential energy) versus θ (the angle the point mass is displaced from vertical), the magnetic damping and the driving force are removed and the pendulum is displaced from vertical and allowed to oscillate freely. The angular velocity is measured, and thus the kinetic energy (K) can be calculated. Then the potential energy is derived from conservation of energy:



Initial Position with Zero Velocity

Arbitrary Later Position

$$Total\ Energy = U_i + K_i = U + K$$

Since the pendulum starts from rest at maximum displacement, $K_i = 0$, and

$$U_i = U + \frac{1}{2} I \omega^2$$

Since U_i = constant = c,

$$U = c - \frac{1}{2} I \omega^2$$

Therefore, the shape of the potential energy well can be found by plotting the negative of the square of the angular speed ($-\omega^2$) versus the angular displacement (θ).

Make a calculation in the Capstone calculator: U=c-0.5*I*w²
where w = [Angular Velocity(rad/s), ▼] and I = rotational inertia of the disk and point mass. Set c = 1 and measure the radius and mass of the disk and the point mass and calculate the rotational inertia:

$$I = \frac{1}{2} M_{disk} R_{disk}^2 + M_{pt\ mass} R_{disk}^2$$

Set-Up

1. Mount the driver on a rod base and attach a photogate to the driver as shown in Figure 2. Use two vertical rods connected by a cross rod at the top for greater stability. See Figure 4.

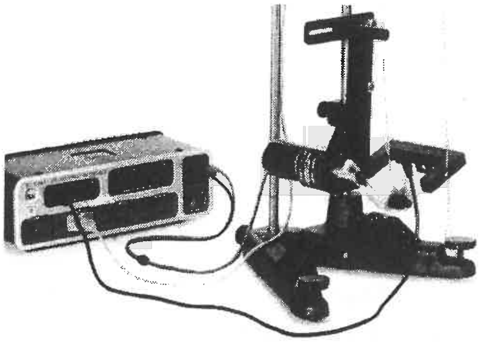


Figure 2. Driver Photogate

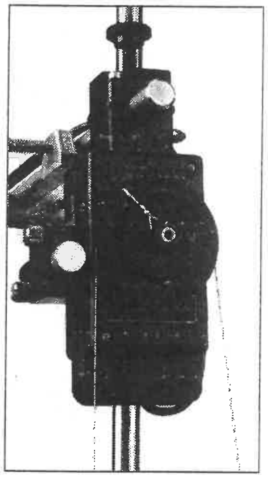


Figure 3. Tying the String

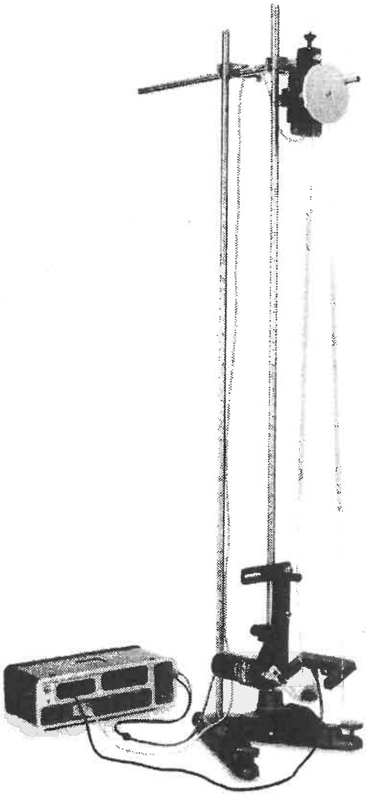


Figure 4. Complete Setup

2. Mount the Rotary Motion Sensor on the cross rod.

3. Cut a string to a length of about 1.5 m. Tie the center of the string around the smallest step of the Rotary Motion Sensor pulley. See Figure 3. Thread both ends of the string through the side hole on the largest step of the pulley. Wrap each end of the string twice around the largest step of the pulley.

4. Adjust the position of the rotating arm of the driver so the attachment screw is about in the center of the arm. Then rotate the driver arm until it is vertically downward. Attach a string to the driver arm and thread the string through the string guide at the top end of the driver. Tie

one end of one of the springs to the end of this string. Tie the end of the spring close to the driver string guide.

5. Tie a section of string (about 10 centimeters long) to the leveling screw on the base. Tie one end of the second spring to this string.

6. To complete the setup of the springs, hold the pendulum disk in place with the point mass at the top. Make sure each end of the string is wrapped around the pulley twice and then thread each of the strings from the pulley through the ends of the springs and tie them off with about equal tension is each side: The point mass should be pulled almost equally by each spring. The disk must be able to rotate one revolution in either direction without the end of either spring hitting the pulley. Also neither spring should completely close.

7. Attach the magnetic drag accessory to the side of the Rotary Motion Sensor as shown in Figure 5.

8. Plug the driver into Signal Generator #1 on the 850 Universal Interface and set the signal to 5 V DC. Leave it off.

9. Plug the Rotary Motion Sensor into Channel P1 and plug the photogate into Channel 1 on the interface.

10. Set the sample rate of the Rotary Motion Sensor to 40 Hz.

11. Create a custom timer called "Period", which is the time between blocks of the photogate.

12. To make the Poincare plot, create a calculation for the angular velocity once per revolution as the driver arm passes through the photogate:

    ω=0*[Period(s), ▼]+[Angular Velocity(rad/s), ▼].

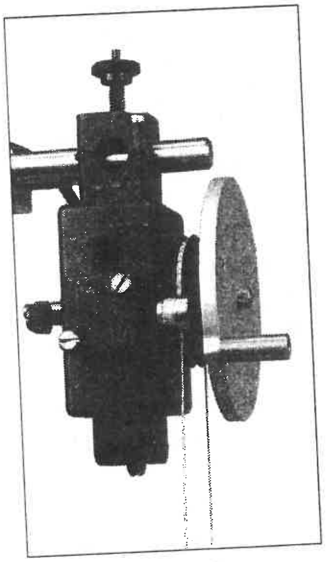


Figure 5. Magnetic Drag Accessory

Procedure

Part I: Mapping the Potential Well

1. In PASCO Capstone, make a graph of the potential energy (U) vs. angle.

2. Leave the driver power supply turned off. Screw the magnet screw all the way back away from the disk to reduce the drag. Open the properties of the Rotary Motion Sensor in the Summary and turn off "Zero on Start". Rotate the disk so the mass is at the top and zero the sensor in the properties.

3. Displace the point mass to one side far enough that the disk will oscillate all the way over to the other side when it is released.

4. Start recording, release the pendulum and let it oscillate once. Then click on STOP.
5. Examine the resulting plot of potential energy versus angle. Observe that there is a double well because there are two equilibrium points. Are the wells equally deep? Why or why not?

Part II: Resonant Frequency

1. Create a graph of angular velocity vs. time and create another graph of angular velocity vs. angle. Create a digits display of the Driver Period and one for the Output Voltage.
2. Screw the magnet toward the disk until it is about 3 mm from the disk. Without turning on the power supply that powers the driver, allow the point mass to fall into the equilibrium position on either side of the pendulum. Start recording, displace the pendulum from equilibrium and let it oscillate for a few oscillations. Click on STOP.
3. Examine the angular velocity vs. time graph. Are the oscillations sinusoidal? Are they damped?
4. Examine the phase plot (angular velocity vs. angle). What shape is it? How is affected by the amount of damping? What would it look like if there weren't any damping?
5. Measure the period of the oscillation using the Smart Tool at the top of the angle vs. time graph.

Part III: Non-chaotic Oscillations

Note about Initial Conditions: For the rest of the experiment, hold the point mass end at the top and then let go when the driver arm is at its lowest point.

1. Set the driver arm for an amplitude of about 3.3 cm. Make sure the driver arm only breaks the photogate beam once per revolution. Adjust the magnet distance to about 4 mm from the disk. Turn on the power supply and adjust the voltage to about 3.5 V so the oscillation is simply one back-and-forth motion.
2. Record data for a few minutes.
3. Examine the graph of angular velocity vs. time. Is it sinusoidal? What is the period? Is the period the same as the driving period? Why is this graph different from the graph in Part II?
4. Examine the graph of angular velocity vs. angle (the phase diagram). Why does it look the way it does? How is it different from the phase diagram in Part II?
5. Examine the Poincare plot. Why does it look the way it does? How does this plot indicate that this oscillation is regular?

Superimposing Poincare Plot on Phase Diagram:
To superimpose the Poincare plot on the phase diagram, open the Summary. Click and drag the angular velocity calculation onto the same vertical axis that has the angular velocity measurement. Drop it just to the right of the axis where the green vertical line appears. Then select the angular velocity calculation in the graph legend and click on its properties in the graph toolbar. Choose to plot data points but no connecting line.

6. Examine the FFT of the angular velocity. How many peaks are there?
7. Gradually increase the driving frequency by increasing the voltage on power supply. You will have to move the magnet closer to the disk to keep from over-driving the disk. Give the pendulum time to respond to the change in driving frequency. Increase the frequency until the motion of the pendulum is slightly more complicated: It should not simply have one back-and-forth movement but rather it should oscillate back-and-forth with an extra back-and-forth movement on one side. Re-start the oscillation, holding the point mass end at the top and letting go when the driver arm is at its lowest point.
8. Record data for a few minutes.
9. Examine the graph of angular velocity vs. time. Is it sinusoidal? What is the period? Is the period the same as the driving period? How is it different than the previous oscillation?
10. Examine the graph of angular velocity vs. angle (the phase diagram). Why does it look the way it does? Compare it to the previous phase diagram.
11. Examine the Poincare plot. Why does it look the way it does? How does this plot indicate that this oscillation is regular?
12. Examine the FFT of the angular velocity. How many peaks are there?

Part IV: Chaotic Oscillations

1. Continue to gradually increase the driving frequency to the resonant frequency by increasing the voltage on power supply. To make the motion of the pendulum very complicated, you may have to adjust the distance of the magnet from the disk. The pendulum should pause suddenly at various points in its motion and spend random times on each side of the oscillation. Re-start the oscillation, holding the point mass end at the top and letting go when the driver arm is at its lowest point.
2. Make a note of the magnet spacing and the driver voltage.
3. Record data for an hour.
4. Examine the graph of angular velocity vs. time. Is it sinusoidal? What is the period? Is the period the same as the driving period?

5. Examine the graph of angular velocity vs. angle (the phase diagram). Why does it look the way it does?
6. Examine the Poincare plot. Why does it look the way it does? How does this plot indicate that this oscillation is chaotic?
7. Examine the FFT of the angular velocity. How is it different from the non-chaotic FFT?

Further Studies

The driving frequency was varied to change the oscillation from regular to chaotic. Try adjusting the magnetic damping while holding the driving frequency at the frequency that gave chaos before.

Then try holding the damping and driving frequency constant while varying the driving amplitude.

Check the effect of initial position on the oscillations.