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Citation: **10**, 186 (1939); doi: 10.1063/1.1751523
View online: http://dx.doi.org/10.1063/1.1751523
View Table of Contents: http://aip.scitation.org/toc/rsi/10/6
Published by the American Institute of Physics

On the Theory and Use of Ross Filters

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(Received March 18, 1939)

Resolving power, preferred thickness and balance errors of Ross filters for x-ray monochromatization are discussed. It is theoretically impossible to balance two filters of elementary composition because of scattering, L limits and K jump, even if the absorption coefficients of the two filters vary similarly with wave-length between the limits, which is not known to be the case. It suffices, however, to obtain good balance for wave-lengths near the pass band since others longer and shorter may be relatively reduced by pre-filtration through a filter whose composition is the same as that of the high atomic number member of the balanced pair. The K jump balance error may be completely removed by adding a slightly absorbing filter of low atomic number to one member of the pair. By employing a specially designed ionization chamber Ross filters may be used simultaneously instead of alternately and a considerable gain in speed and accuracy achieved.

AMONG the contributions of the late P. A. Ross to x-ray physics none excites more immediate admiration or illustrates better the character of his inventive gifts than the double filter method which he devised in 1926[1] for the investigation of polarization in the continuous x-ray spectrum. This method of analysis, utilizing what the inventor called balanced foils, here entitled Ross filters, has become a standard tool in many lines of x-ray investigation as a list of several dozen published papers from widely scattered laboratories bears witness.

METHOD

The purpose of the filters to produce a monochromatizing effect is accomplished by using two filters composed of elements of adjacent atomic number whose thicknesses have been so adjusted that the transmitted spectra are identical for all wave-lengths except those lying within the narrow band between the two K absorption limits. The filters are allowed to intercept alternately the beam of heterochromatic radiation under observation and measurements of the intensity or effectiveness of the filtrate in any respect are recorded. Any difference between the observations with the two balanced filters must be ascribed to the effect of the unbalanced band between the absorption limits, called occasionally hereafter the "pass band." The degree of monochromatism thus obtained is sufficient for many studies and as to intensity there is of course a great gain over what is ordinarily possible by crystal diffraction methods.[2]

The wave-length to be isolated is controlled by the experimenter's choice of filters. The K critical absorption wave-lengths of the elements range from one-tenth to several hundred angstroms; so the range of choice is wide. In practice, however, the metals have been most often employed and pass band wave-lengths have not greatly exceeded 1A.

RESOLVING POWER

The ratio, $\lambda_K/\Delta\lambda$, of the wave-length at the pass band to the band width serves as a convenient measure of the resolving power of a pair of Ross filters. Moseley's law for the K limits states that approximately $\lambda_K Z^2 = M$, a constant. Differentiating and putting $dZ = -1$, since adjacent elements in the atomic series are used, we have $\Delta\lambda = 2MZ^{-3}$ and $\lambda_K/\Delta\lambda = Z/2$. The resolving power, evidently, improves as the wave-length of the filtrate decreases. When desirable this resolving power may be decreased with a compensating gain in intensity of the band radiation by using filters with atomic numbers differing by two or more.

[1] P. A. Ross, Phys. Rev. **28**, 425 (1926); J. Opt. Soc. Am. and Rev. Sci. Inst. **16**, 433 (1928).

[2] In a case worked out by Ross (reference 1) the advantage of balanced filters over crystal reflection was represented by a factor of 1440.

Preferred Thickness

That filter thickness is best which yields the most accurate value of the intensity of the pass band. If the intensity measurements are made with an electrometer we have to reckon with the fact that errors in electrometer measurements are chiefly due to drift and are, as to absolute magnitude, nearly independent of the magnitude measured. For band measurements of high *percentage accuracy*, therefore, we desire to have the band intensity (measured by the difference between observed intensities transmitted by the two filters) absolutely large. The transmitted band intensity, as one sees by reference to the shaded area of Fig. 1, is proportional to $I_0(e^{-\mu_L t} - e^{-\mu_s t})\Delta\lambda$, where $I_0\Delta\lambda$ is the unfiltered intensity within the band of width $\Delta\lambda$, μ_L and μ_s are total linear absorption coefficients of either filter for wave-lengths close to the discontinuity but on the long and short wave sides, respectively, and t is the thickness of the same filter.

The expression for transmitted band intensity has a maximum value when the filter thickness has the value $t_{\max}=\log r/\mu_L(k-1)$, where r is the ratio of μ_s to μ_L, commonly called the K jump. With silver, as one member of a pair of filters, for example, we have by reference to a table of absorption coefficients $t_{\max}=0.00334$ cm. This is close to the thickness 0.0040 cm adopted by Ross without analysis. The thickness of the companion filter will be dictated by the requirement that the transmission of the two filters be alike for wave-lengths without the band. When a silver filter of optimum thickness is used as one of a balanced pair the transmitted band intensity will be about 0.6 of the incident or original band intensity.

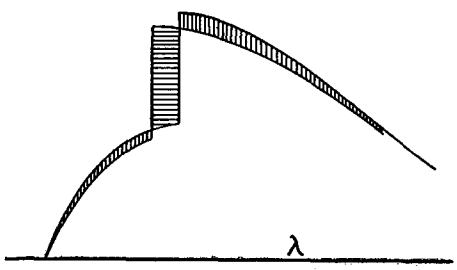

Fig. 1. Spectral intensity distributions separately transmitted by two imperfectly balanced Ross filters. Pass band shaded horizontally, error area vertically.

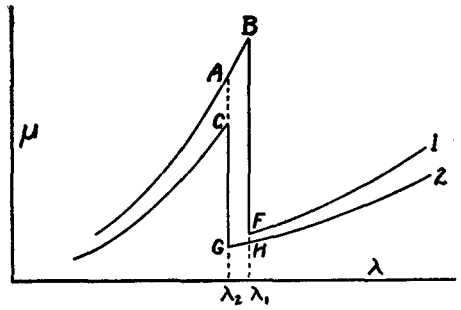

Fig. 2. Schematic plot of linear absorption coefficients *vs.* wave-length for a pair of filters. The dissimilarity of the K jumps is exaggerated.

Possibility of Balance

It is necessary to investigate the extent to which the requirement of balance may be fulfilled. Ross attacked this question directly by observing the forms of the transmitted continuous spectra and found it possible to adjust thicknesses of filters of adjacent atomic numbers so as to obtain the satisfactory agreement shown in Fig. 2 of the second paper referred to in reference 1. Approaching the question from the theoretical direction one notes that perfect balance requires that $\mu_1 t_1=\mu_2 t_2$, where the subscripts refer to elements Z_1 and Z_2, of which the respective filters are composed. We assume $Z_1<Z_2$. Since the t's are independently variable it suffices to find μ_1/μ_2 constant for all wave-lengths lying outside the pass band.

This desired result is possible if and only if μ_1 and μ_2 are representable by a function of λ identical in the two cases except for possible differences in the coefficient of the λ-dependent part of the function. This functional identity necessarily breaks down for wave-lengths within the pass band but should obtain for all longer or shorter wave-lengths. This does not mean that for either element the function of λ which describes μ on the short wave side of the band must also apply on the long wave side; on the contrary, new functions may appear at any place on the wave-length scale so long as the two filter elements change together. We are under no restrictions concerning the nature of the function of λ; it is not required, for example, that μ be proportional to some power of λ. It is also immaterial how Z enters into the functional representation of μ, so long as the Z-dependent part

of the function appears simply as a coefficient of the λ-dependent portion.

It will be found that the requirements of balance are almost, though not quite, satisfied. It was customary at one time to represent absorption coefficients by an empirical expression $C\lambda^n$ where C is a function of Z but not of λ. Such a law of absorption would permit perfect balancing even though n might have different values on opposite sides of the K limits, provided the proportional change in C at the K limit were the same for both filters of a pair. This formula, however, can at best be only a representation of the *fluorescent part* of the total absorption and as soon as we add to it a scattering coefficient *not proportional to* λ^n the possibility of rigorous balance disappears.

As a matter of fact the case is worse than this, for $C\lambda^n$ is now known to be an inexact simplification of the unknown wave-length dependence, and certain it is that the K jump varies from element to element.

It is not profitable to try to isolate the effect of scattering upon balance since existing absorption data do not permit us to separate photoelectric and scattering effects with enough confidence to justify reliance upon conclusions so obtained. It is necessary, though, to consider the effects of the K jumps and to discuss the possibility of rectifying the errors of balance which they introduce.

Discussion of Balance Errors

The measure of success in balancing a pair of filters is the smallness of the difference between the areas under the curves (as in Fig. 1) representing the spectra transmitted by the separate filters, this areal difference to be evaluated over all wave-lengths except those within the pass band. This difference, called the "error area," is the integral with respect to wave-length of $I_0e^{-\mu t}\Delta(\mu t)$ over all wave-lengths lying without the pass band. I_0 is the incident intensity, a function of wave-length, and $\Delta(\mu t)$ also a function of wave-length is the (small) difference between the values of the product μt for the two filters. For filters in perfect balance $\Delta(\mu t)$ is of course zero (outside the pass band) and the error area vanishes.

The effect of the K jump upon error area may be approached by reference to Fig. 2 where the linear total absorption coefficients of a pair of filter substances of consecutive atomic numbers are schematically represented as functions of wave-length through the region of the K limits. The upper-case letters beside the curves will appear as subscripts when the corresponding absorption coefficients are introduced. Assuming the filters to be in balance just to the long wave side of the pass band we investigate the state of balance on the short wave side. From the supposed balance we have $\mu_F t_1=\mu_H t_2$, t_1 and t_2 being the thicknesses of the filters of atomic numbers Z_1 and Z_2, respectively. By definition of the K jumps, r_1 and r_2, we have $r_1=\mu_B/\mu_F$ and $r_2=\mu_C/\mu_G$. From the approximate proportionality of μ to λ^3, which is regarded as sufficiently accurate over a small interval of wave-length, we have $\mu_A/\mu_B=(\lambda_2/\lambda_1)^3=\mu_G/\mu_H$.

The ratio at λ_2 (or just to the short wave side of λ_2) of intensity transmitted by Z_2 to intensity transmitted by Z_1 is $e^{\mu_A t_1-\mu_C t_2}=1+\mu_A t_1-\mu_C t_2$ $=1+p_2$. The small quantity p_2 thus defined is $\Delta(\mu t)$ close to but on the short wave side of λ_2. Using relations set up above we have at once

$$p_2=\mu_H t_2(\lambda_2/\lambda_1)^3(r_1-r_2). \qquad (1)$$

From Moseley's law approximately $(\lambda_2/\lambda_1)^3$ $=Z/(Z+6)$. Müller[3] in an investigation of the K jumps of twenty-eight elements finds $r=1+63.868Z^{-0.6207}$, from which with sufficient exactness $r_1-r_2=39.6Z^{-1.62}$. In an earlier paragraph it was shown that $\log r/(r-1)$ is a desirable value of $\mu_H t_2$. Making these insertions there results $p_2=0.62 \log r/(Z+6)$. The resulting error area may not be evaluated without knowledge or assumptions about the intensity distribution in the incident spectrum but its order of magnitude and variation with atomic number may be shown by plotting the ratio of a section of the error area embracing a wave-length breadth equalling that of the pass band to the pass band area itself. This ratio, which has the value

$$\frac{p_2e^{-\mu_s t}}{e^{-\mu_L t}-e^{-\mu_s t}}=\frac{0.62 \log r}{(Z+6)(r-1)}$$

is plotted as Fig. 3. Clearly, filters of high atomic number, passing short wave radiation, possess

[3] I. Müller, Ann. d. Physik **32**, 625 (1938).

the advantage of small jump error, but at best, since the error area may actually extend over a wave-length range many times that contemplated in Fig. 3, errors of a few percent in the pass band determination are to be expected.

Although in deducing Fig. 3 we have adopted Müller's rule for the magnitude of the K jumps, in practical cases this rule is not entirely trustworthy. Published data on the magnitude of the K jumps do not lie on *any* smooth curve when plotted against atomic number. It is not necessary to determine at the moment whether this scattering of points is accurately descriptive of atomic characteristics or the result of errors of measurements, since in either case the presence of impurities of either higher or lower atomic number in a filter will reduce the jump by an amount depending upon the amount present. As a case in point a satisfactory Ag-Pd filter pair may be cited in which, contrary to the supposed trend of the jump with atomic number, the Ag jump was found to be definitely the larger. For precise balancing it is therefore necessary to examine the jumps of the proposed filters individually in dealing with the K jump error.

It is, of course, possible to bring a pair of filters into balance with respect to radiation of wave-length just *shorter* than that of the pass band, in which case the error area appears on the long wave side. There arises the question of the relative undesirability of this situation and that of the foregoing paragraphs, in answer to which it may be shown that if the filters are thin the error area is much greater if it occurs on the short wave side of the pass band, while with thick filters an opposite situation obtains. If the filters have the thickness referred to above as optimum the magnitude of the K jump error area is almost independent[4] of its location relative to the pass band and Fig. 3 applies again.

Experiments with filters of intermediate atomic number have shown that if the pair be brought into balance for a wave-length just

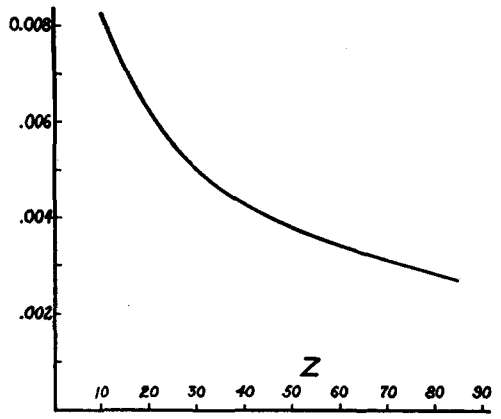

FIG. 3. Variation with atomic number of balance error resulting from dissimilarity of the K jumps of a pair of filters. With filters balanced at the long wave side of the pass band, ordinates represent ratio of error area to band area for equal wave-length intervals.

short of the pass band no marked divergence from balance will be found on passing to shorter wave-lengths. In a typical search in this region, for instance, the powers transmitted by the two filters were identical to a fifth of one percent, the limiting precision of observation, down to the disappearance of the spectrum at a wave-length less than one-half that at the band. It must be remarked that a few observations of this kind cannot establish a general assurance relative to filters as yet untried; existing measurements of absorption coefficients are very far from being sufficiently reliable to show that the various elements do or do not possess the same functional dependence of absorption coefficient upon wave-length to the degree of precision desired here. Then, too, actual filters may possess unsuspected impurities capable of affecting both the wave-length variation and the magnitude of the observable K jump.

Turning to the direction of greater wave-lengths the possibly disturbing effects of the scattering coefficients fade to insignificance with increasing wave-length while the L limits loom as a possible source of concern. If the filter transmission is appreciable in the wave-length region of the L limits there will be three undesired pass bands corresponding to the three L limits, but it may be shown that any filter whose thickness renders it at all usable for observation of the K band will transmit in the region of L wave-lengths a fraction of the order of e^{-30} or

[4] More rigorously the error area (per unit wave-length interval) with optimum filter thickness and with a flat continuous spectrum incident is slightly larger when it is found on the long wave side than when on the short wave side of the pass band, the ratio being

$$1+[1-(\lambda_2/\lambda_1)^3][1-\log r/(r-1)].$$

In what follows it will appear that this slight argument for balancing on the long wave side of the pass band has little value.

less, which will be negligible under almost all conceivable circumstances.

Removal of Balance Errors by Additional Filters

The addition of a filter of the same composition as the higher atomic number element of the balanced pair will serve to reduce the error areas from whatever cause all along the line. Such a filter possesses a transmission window just at the pass band and diminishes the intensities elsewhere so that departures from perfect balance elsewhere are of reduced importance. The third filter, transmitting from one-half to three-fourths of the band intensity, should be installed at any point in the beam of radiation and left in place throughout operations with the balanced pair. With it in use one can concentrate upon adjustment of the filters to balance in the neighborhood of the pass band, with little or no concern for the state of balance for other wave-lengths.

There remains the problem of obtaining simultaneous balance at both sides of the pass band. Though impossible with two simple filters it will now be shown to be possible through the permanent addition of a slightly absorbing layer of a substance of low atomic number to one member of the balanced pair.

Given two filters composed respectively of elements of atomic numbers Z_1 and Z_2 where $Z_2=Z_1\pm1$ we find upon spectrometric examination that their K jumps, r_1 and r_2, are unequal in the sense that the difference $r_1-r_2=\Delta$, a positive quantity. With equal K jumps a balance on either side of the pass band would insure perfect balance at the other; *with unequal jumps we attempt to combine with the filter possessing the larger jump a layer of a third absorber possessing no jump* (in the observed region) *and having a thickness such that the jump of the combination equals the jump of that original filter whose jump was the less.* Let the atomic number of the third filter be Z_3 and the three filter thicknesses t_1, t_2 and t_3. For each filter we are concerned with two absorption coefficients relating respectively to wave-lengths just out of the pass band on the long and short wave sides, and designated, for Z_1 and Z_2, by the subscript notation used earlier in discussing Fig. 2. For Z_3 the symbols μ_{3L} and μ_{3S} will be used for coefficients at the long and short wave sides of the pass band. The conditions of the desired balance are stated by the equations

$$\mu_F t_1+\mu_{3L}t_3=\mu_H t_2,$$
$$\mu_A t_1+\mu_{3S}t_3=\mu_C t_2.$$

Using previously stated definitions of the K jumps and assuming that the absorption coefficients of all three filters vary similarly with wave-length (except for the jumps) in the band region the balance equations yield

$$\mu_F t_1=\mu_H t_2(r_2-1)/(r_1-1)$$
and (2)
$$\mu_{3L}t_3=\mu_H t_2\Delta/(r_1-1).$$

Therefore starting with a filter Z_2 of any thickness we may obtain the desired balance at both sides of the pass band by giving filters Z_1 and Z_3 the readily calculated thicknesses of Eqs. (2). Of course the final balance must be attained or confirmed by actual spectrometer observations. It may be seen from Eqs. (2) that the actual absorption by Z_3 is slight and its effect in reducing the working intensities negligible. In balancing up the Ag-Pd pair mentioned above Z_3 was 0.0035 cm of aluminum built up of three layers of commercial foil. While aluminum is convenient and generally satisfactory for this purpose, a substance of higher atomic number would have the slight theoretical advantage of following the main filters more closely in respect to the wave-length dependence of absorption coefficient, a desirable characteristic tending to prevent the appearance of error areas in wave-length domains apart from the pass band. Chromium applied by electroplating to one member of the pair to be balanced would probably serve well.

Balancing Procedure and Use of Filters

Nearly all of the filters assembled by Ross are metal foils which have been brought to approximately the desired thickness by rolling. They are mounted by pairs in frames in such a way that either filter may be adjusted by rotation about an axis lying in its own plane for the purpose of increasing its effective thickness. In obtaining a balance the present procedure is to clamp the mount with its filters to a fixed support before a telescope and scale. A bit of

plane mirror is held against the frame of one of the filters by means of a rubber band and an observation of the filter's angular position made with the telescope. The two transparencies to the chosen radiation are then measured, using a Bragg spectrometer, after which the angular position of the filter is altered in the direction required to equalize the filters and the spectrometer measurement repeated. After two or three approximate adjustments, a curve relating angular position to relative transparency can be plotted and by its use the final adjustment to identical transmission within 1/5 percent is hastened.

In the use of the filters they have heretofore been applied to the x-ray beam successively. For some measurements now in progress in this laboratory a simultaneous method has been originated. The two balanced filters are placed side by side before two windows in a double ionization chamber having a single collecting plate disposed between the incoming beams. The electrostatic fields in the chamber are so directed that the plate collects positive ions from one beam and negative ions from the other, and hence the net charge measured is a direct measure of the intensity of the pass band. This method of electrical subtraction has the definite advantage that fluctuations of the target current in the x-ray tube are without effect upon the accuracy, while the effect of tube voltage fluctuations is partially eliminated also. Filters may be brought into a preliminary or approximate balance very easily with this chamber by dropping the tube voltage below the K critical potentials of the filters and adjusting the inclination of one filter until the collected ion current drops to zero.

JUNE, 1939 R. S. I. VOLUME 10

Switching Action of the Eccles-Jordan Trigger Circuit

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(Received March 13, 1939)

THE Eccles-Jordan trigger circuit (Fig. 1) has been made the basis of a number of vacuum tube instruments.[1] Of fundamental importance in some of these applications is its ability to carry on its switching operation when both tubes are simultaneously excited with a series of identical pulses. This type of operation is similar to that exhibited by the parallel type of Thyratron switching circuit, and, consequently, the high vacuum tube circuit has replaced these tubes in a number of high speed counting circuits.[2–5] The limitation imposed upon speed by de-ionization time is thereby eliminated. In Fig. 1 the switching action is much improved by shunting two small condensers C_1 and C_2 across the coupling resistors R_{c1} and R_{c2}. These condensers increase the sensitivity of the circuit to excitation because they tend to offset the stabilizing action of the interelectrode capacitances.[5] They also provide much closer coupling between the grids and plates of opposing tubes.[4] Thus a small change in plate potential of either tube strongly influences the grid voltage, and, consequently, the plate current of the other tube.

It is known, however, that successive impulses applied simultaneously to the grids of the two tubes will cause the plate current to switch from one tube to the other on each impulse. This action cannot be adequately explained by either the close coupling afforded by the condensers or the balancing of interelectrode capacitances. Experiment shows that the introduction of these small condensers is necessary for reliable operation. Since the plate currents of both tubes are reduced to zero during a negative pulse, and the circuit is entirely symmetrical, the explanation must lie in a transient unbalance of electrode

[1] W. H. Eccles and F. W. Jordan, Radio Rev. **1**, 143 (1919).
[2] W. B. Lewis, Proc. Camb. Phil. Soc. **33**, 549 (1937).
[3] E. C. Stevenson and I. A. Getting, Rev. Sci. Inst. **8**, 414 (1937).
[4] H. Lifschutz and J. L. Lawson, Rev. Sci. Inst. **9**, 83 (1938).
[5] H. J. Reich, Rev. Sci. Inst. **9**, 222 (1938).