NAME Modern Physics LAB PARTNER LOCKER NUMBER

DATE EXPERIMENT NUMBER EXPERIMENT

– The systematic of classical physics.

mass → mechanics
temperature → stat. mech + thermodyn.
waves
charge → electricity, magnetism } electrodyn.
waves → electromagnetic waves, optics

mechanics: N1 $\sum \vec{F} = 0$
N2 $\sum \vec{F} = m_{iner} \cdot \vec{a}$
N3 $\vec{F}_{A\to B} = -\vec{F}_{B\to A}$

mom. cons. $\vec{p}_{sys,1} = \vec{p}_{sys,2}$

if external forces negligible

E cons. $E_{1,tot} = E_{2,tot}$

L cons. $L_{1,sys} = L_{2,sys}$

put E&M here first

if external torque is neglig.

th. dyn: 0^{th} law defines Th. eq.
1^{st} law E cons: $\Delta U = Q - W$
2^{nd} law provides direction for E flow
3^{rd} law zero [K] cannot be reached

waves: $\vec{E}(x,t) = E_{max} \cdot \cos(kx - \omega t)\,\hat{j}$
$\vec{B}(x,t) = B_{max} \cdot \cos(kx - \omega t)\,\hat{k}$
$c = 1/\sqrt{\varepsilon_0 \mu_0}$, $n = \frac{c}{v_{wave}}$
$y(x,t) = A \cdot \cos(kx - \omega t)$
$v = \lambda \cdot f$

NAME ______ LAB PARTNER ______ LOCKER NUMBER ______

DATE ______ EXPERIMENT NUMBER ______ EXPERIMENT ______

$y = y(x,t)$: $\frac{\partial^2 y}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 y}{\partial t^2}$

wave superposition : add amplitudes

electromagnetism : 4 Maxwell eqn

$\oint \vec{E}\cdot d\vec{A} = \frac{Q_{encl}}{\varepsilon_0}$ Gauss

$\oint \vec{B}\cdot d\vec{A} = 0$

$\oint \vec{B}\cdot d\vec{l} = \mu_0 \left(i_{cond} + \varepsilon_0 \frac{d\phi_E}{dt}\right)_{encl}$ Ampere

$\oint \vec{E}\cdot d\vec{l} = -\frac{d\phi_B}{dt}$ Faraday

more useful form : 'vector differential calc'

gradient $\vec{\nabla}\vec{E} := \frac{\partial \vec{E}}{\partial x}\hat{i} + \frac{\partial \vec{E}}{\partial y}\hat{j} + \frac{\partial \vec{E}}{\partial z}\hat{k}$

"del" points in direction of max. increase of $\vec{E}$

divergence $\vec{\nabla}\cdot\vec{E} := \frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} + \frac{\partial E_z}{\partial z}$

a measure how much a vector spreads out from a point

curl $\vec{\nabla}\times\vec{E} := \left(\frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z}\right)\hat{i} + \left(\frac{\partial E_x}{\partial z} - \frac{\partial E_z}{\partial x}\right)\hat{j} + \left(\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y}\right)\hat{k}$

a measure of how much a vector curls around a point

↳ Maxwell eqn :

$\vec{\nabla}\cdot\vec{E} = \frac{\rho}{\varepsilon_0}$ Gauss

$\vec{\nabla}\cdot\vec{B} = 0$

$\vec{\nabla}\times\vec{E} = -\frac{\partial \vec{B}}{\partial t}$ Faraday

$\vec{\nabla}\times\vec{B} = \mu_0 \cdot \vec{J}$ Ampere

How long to reach ~~c~~ w/ $\vec{a} = 10\,\frac{m}{s^2}\,\hat{i}$ = const?

$3\cdot 10^8 \frac{m}{s} = v_x = \cancel{v_0} + a\cdot t$

$3\cdot 10^7\,[s] \cong 347.2$ days (86,400[s] per day)

NAME ______ LAB PARTNER ______ LOCKER NUMBER ______

DATE ______ EXPERIMENT NUMBER ______ EXPERIMENT ______

(.) Without absolute space and time the law
inertia would be meaningless

a coord. system at absolute rest is
meaningless (because absolute space cannot
ascertained) we talk of 'inertial ref frames'.

"...re is an infinite number of equivalent, inertial
...tems, in rectilinear uniform motion relative to
... another, in which the laws of classical
mechanics hold true."

...d thus, the problem of space is connected
intimately with mechanics.

Galileo transformations

powerpoint slide 3 b)

The Ether from: Einstein ~ Sidelights On Relativity

the absolute space was called the ether/aether.

it was an absolute necessity for E&M
→ the medium in which the light wave
travels → Maxwell's eqn.

otoh in Mechanics we have the
"action at a distance" principle
of Newton: $F_G \sim \frac{1}{r^2}$

w/o! physical contact

cause and effect

ether medium: contact by movement or
deformation of that medium

properties of the medium: - E&M → elastic
& inert

- Light can be polarized ⇒ the medium must be a solid

(b/c there are no transverse waves in a fluid)

- quasi-rigid, b/c the motion of the earth does not drag the ether (its parts can not move, except in vibration)

stationary ether

Fizeau
aberration

- "the luminiferous ether" (must be transparent)

⇒ Maxwell's eqn : e-m clear + simple
but its mechanical interpret. is contradictory

fundamental dualism that was not supportable

: since E&M fields occur in vacuo (read: in pure, un-filled ether)

the ether is also a bearer of E&M fields

thus ether is indistinguishable in its functions from matter!

↳ within matter it takes part in the motion of matter and in empty space it has everywhere a velocity

NAME ______ LAB PARTNER ______ LOCKER NUMBER ______

DATE ______ EXPERIMENT NUMBER ______ EXPERIMENT ______

HA Lorentz: brought theory into harmony w/ experience by means of a simplification:

taking from ether its mechanical and from matter its e-m qualities

↳ assigned <u>all</u> em-fields to the ether

leaving the particles as only em-activity the carrying of charge

→ the only mechanical property of ether: immobility

Special Relativity: take away that last mechanical property ~ the immobility

making space altogether imperceptible.

R3

slide 3 relativity of rectilinear motion
↳ solution roll-up game
↳ disseminate roll-up game point by point

poll everywhere what does Newton's law require for const. velocity?
1 – straight line
2 – curvy line, const. curvature
3 – any curvy line
4 – does not deal w/ this case

here: rotation = acceleration (recall water-can't circle)

Galilean transformation: 1-d → $x \pm vt$; y, z, t 'const'

read for F: p. 39-50, Ferrel

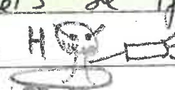
the absolute space is necessary "ether"
required of Maxwellian ether – invisible
– imponderable unweighable
– must move rel. to object

more about it next time: stellar aberration, Fizeau, Jup. moons

for now, more about c – expts + Einstein's premise
"c is always the same"

Postulates:
(1) physics the same is all inert. sys.
(2) c the same for all everywhere + always, obs.

sounds easy :) let's see if we understand it

poll everywhere Q1 consider H  torch
→ 0.9c = const!

with what v does light leave Henry?
1 – 0.1c
2 – < 0.9c, > 0.1c
3 – 0.9c
4 – > 0.9c, < c
5 – c
6 – > c

A: 5

now, consider Albert watching A 
Q2 same question, same A: 5

blue DVD: #10 skateboard + simultaneity
2' to write up tape $1^{15} - 3^{25}$
: #11 light clocks + simultaneity
2' to write up tape $1^{15} - 2^{40}$

#13 Spacetime diagram tape $1^{20} - 4^{00}$ (no contr. / during clean party)
3'' to write up

If space time : double slit expt.

replay #17 this time w/ length contraction R4

the issue of the ether : stellar aberration, Fresnel, Jup. moons
gave mixed evidence → expt. worked is order $\frac{v}{c}$

$$\text{b/c} \quad \Delta t = \frac{l}{c-v} - \frac{l}{c+v} \approx \underbrace{\frac{2l}{c}}_{t_0} \cdot \frac{v}{c}$$

where (−) due to rays traveling is same dir. all the time

what is needed (Maxwell) is expt. on earth b/c times add up
b/c change of sign due to chg of direction

$$t_2 + t_1 = \frac{l}{c+v} + \frac{l}{c-v} = \frac{2lc}{c^2-v^2}$$

for v orb. speed earth : $\frac{v}{c} \approx 10^{-4}$, $\frac{v^2}{c^2} \approx 10^{-8}$ i.e. $1 : 10^8$

Michelson : I can do this
2 slit expt
→ opt. path is Michelson
→ ether drift (video #8, 1:28, #9) ⇒ no ether

Einstein Postulates : (1) laws of phys the same is all inert. ref. fr.
: (2) speed of light is vac. same for all observers

- Lorentz Transformations

$$x' = \gamma(x - vt) \qquad x = \gamma(x' + vt')$$
$$y' = y \quad z' = z$$
$$t' = \gamma\left(t - \frac{vx}{c^2}\right) \qquad t = \gamma\left(t' + \frac{vx'}{c^2}\right)$$

Lorentz contraction: $l = \frac{l_0}{\gamma}$

hw: read p. 76-80

l as measured in $S' = x_2' - x_1'$

$$x_1 = \gamma(x_1' + vt')$$
$$x_2 = \gamma(x_2' + vt')$$

$$x_2 - x_1 = \gamma x_2' + \gamma vt' - \gamma x_1' - \gamma vt'$$
$$= \gamma(x_2' - x_1')$$

not semantics but reality in S'

Time Dilation:

$$t_1' = \gamma\left(t_1 - \frac{v}{c^2}x_0\right)$$
$$t_2' = \gamma\left(t_2 - \frac{v}{c^2}x_0\right)$$

$$\underbrace{t_2' - t_1'}_{\tau} = \gamma(\underbrace{t_2 - t_1}_{\tau_0})$$

$$\tau = \gamma\tau_0$$

// measured length greater in rest frame   video #12, 0:18

" time diff. less in rest frame

evidence: μ-meson ($\to$ M)

exp.: parallel clock handout, Rindler

example Lorentz - Transf - Schaum 3.2 R5

" Length contraction - Schaum 4.3

" Time Dilation - Schaum 5.5 + 5.1

handout simultaneity (Rindler 'Spec Rel' fig 4)

↳ polleverywhere 1 set synchr time to set of std clocks in S and in S', as seen in S", which moves such that S is moving tow. it w/ +v and S' w/ -v,

1 - same times on all clocks in S
2 - " " S'
3 - diff times on all clocks in S and S' at same x
4 - none of the above
5 - ~~all of the above~~ 1 + 2

4 : each clock in S shows diff time in S" → see handout
" S' " in S"
some clocks in S and S' show same time in S"
but most are at diff x positions

: video #11 (repeat) 2:40
#12 (") 2:18
#13 (part repeat) 1:20 to 6:22

· if time μ-meson video

hw read
μ-meson p99-105

extra thoughts regarding hw1, 3c/d:

assume: pole must stop at end of garage (→ going rest frame garage)
as it does so ≙ rotates in spacetime t', x'

"paradox" — the symmetry of the problem
→ what happens in the rest frame of the pole?
: open garage comes toward it, it is 5 ft long — yes, indeed
: it keeps on going even after first impact, taking ~~rest~~ front end and pole with it
<u>but</u> the back end of the pole : is still at rest
b/c it cannot know of the impact yet
≙ finite speed of prop. of all signals
<u>here</u> signal ≙ elastic shock wave
even if it travels w/ c → 20 feet to travel against garage, path 15 ft before reach end of pole
⇒ this race would be a dead heat for some certain v

<u>moral</u> whatever result is true in one frame must be true viewed in any other frame
, as long as the phys law is Lorentz-invariant → must have an explanation
in every frame, although it may be quite a diff. one

another exp (Rindler, Essential, p. 52)
: rigid rod, rest length L slides over hole diameter $\frac{1}{2}L$
on smooth table
for $\gamma = 10$, $L_{10} = \frac{1}{5}\left(\frac{1}{2}L\right)$ ⇒ it will fall in
: must be true also in the rest frame of the rod
<u>but</u> diameter hole is only $\frac{1}{20}L$! $< \frac{1}{10}L$
: the only way this can happen : the front of the "rigid" pole bends under the force into the hole
even after the front end strikes the far edge of the hole, the back end keeps coming in!
⇒ rigidity is impossible in relativity: each body has inf. many deg. of freedom
incompress. fluids are equally impossible

expt. proof of time dilation

R6

i pollowing video, ready quiz ~ μ mesons registered ~~at~~ w/o road sea level
4- all 3- only μ^- 2- μ below a certain E 4- μ in certain ΔE

unstable particles are a kind of clock → predictability of decay
we measure half-life in rest frame (laboratory)
- cosmic rays produce μ-mesons, then: $\mu^- \rightarrow e^- + \nu_1 + \bar{\nu}_2$ in atmosph.
- μ-travel at relativistic speeds, predomin. downward
- charge detector → if 2nd, delayed event (e^-) μ^- was stopped/decayed
- predict for $t \approx \frac{l}{c}$ how many should decay
- measure rates of arrival $\frac{\mu^{\pm}}{hr}$
- at 2,000(m) ≈ 563 $\frac{\mu}{hr}$

at $v \approx c$ → 6.5 μsec to reach sea level
predict: 25 μ/hr w/o time dilation
expt sea level: > 400 μ/hr
time dil. rest frame clock $\mu^{\pm}$: less than 1 μsec ~0.7 μ

$$1 - \frac{v^2}{c^2} = \left(\frac{\tau_0}{\tau}\right)^2 \approx \frac{1}{81} \qquad \frac{v}{c} \approx 0.994$$

scintillator selects μ of certain E range

describe in
: frame at rest of μ's S'
decay of $v = 0$, atm. is rushing upward tow. μ at $v \approx c$
pov μ's: distance mountain top - sea level is length contracted
acc. to time dil.: as meas. in S journey takes $\Delta t' = \Delta t\sqrt{1 - \frac{v^2}{c^2}}$ in S'
is measurable as modified distance $H \rightarrow H'$

↳ $\Delta t = \frac{H}{v}$ $\quad \Delta t' = \frac{H'}{v}$ $\rightarrow$ $\frac{H'}{H} = \frac{\Delta t'}{\Delta t} = \sqrt{1 - \frac{v^2}{c^2}}$

i.e. the dist. is modified by length contraction
dep. on pov fraction reaching sea level attributable to dilat. or contract.

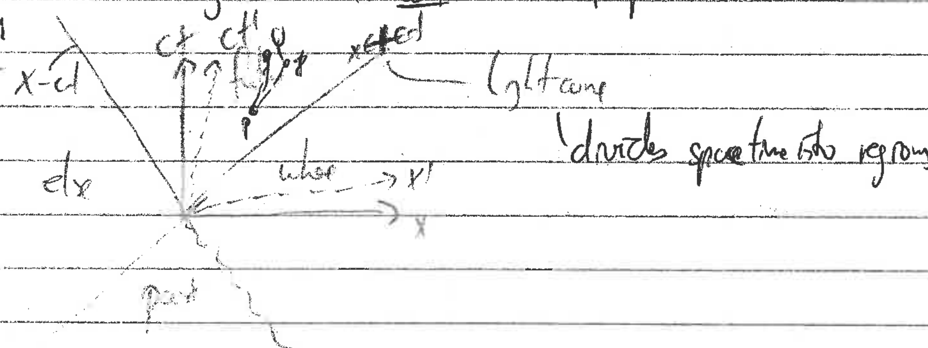
video #15 ~ 2 $\frac{31}{4}$

• proper length + proper time ⊥ as meas. in rest frame ⊥ l_0
same clock, same location
the problem of meas. time simultaneously (p 107 - 109, won't expand in lecture)
also 111 - 117 'understandly'

- space time intervals

$$s^2 = (ct')^2 - (x')^2 - (y')^2 - (z')^2$$ is an invariant under LT
i.e. agreed on in all inertial frames!

1-dim'



$(\Delta s)^2$, interval $\sqrt{(\Delta s)^2}$ b/w 2 events
>0 dep. on $c\Delta t > \Delta x$
<0 $<$
$=0$ $=$
light line connects point events b/w which $\sqrt{(\Delta s)^2} = 0$
if >0 can transform to ct' axis // P-Q
meaning P and Q occur at the same place in S'
sep. by time only = time-like interval

if <0 can transform P, Q to simult. events at diff. places
'space-like'
to connect to such events by space-like interval req. $v > c$
(causality!)

HW!

Mod. Physics

inertial frames → video tag 9/5:05 – 8:15 → Galilean transf., $x \pm vt$
rel. princ. of class mech. : dep. on abs space + time
rel. princ. of mod. phys : postulates (1) + (2)

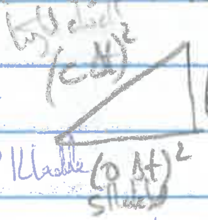
LS/RY lecture 4: 21:15 – 22:38 → video tag 4/6:40 –

just how large is this c? calc: $a = 10\frac{m}{s^2} = $ const, $v_0 = 0$
how long until $v = 3\cdot10^8 \frac{m}{s}$?
↳ $3\cdot10^8 \frac{m}{s} = 0 + 10\frac{m}{s^2} t$ ⟹ 30,000,000 s
86,400 s per day → 347.22 days ?
abs time → gone → meaning of simultaneity, video

lecture 'c' topics

casey (1) + (2)
• watched 4/6:40 – 5/1 – 0:12
• watched 5/0:28 – 2:45
• " 5/3:25 – 5:11 – Lorentz
• " 5/6:11 – 7/ – 3:05 sp-t diagr.
• " 7/0:20 – 3:05 measuring

how would rulers have to contract + clocks slow down: tag 5/0:28 – 2:28



$$(c^2 - v^2)\Delta t^2 = c^2 \Delta t'^2$$
$$\frac{c^2 - v^2}{c^2}\Delta t^2 = \Delta t'^2$$
$$1 - \frac{v^2}{c^2}\,\Delta t^2 = \Delta t'^2$$
$$\sqrt{\quad}\,\Delta t = \Delta t'$$
$$\Delta t = \frac{1}{\sqrt{\quad}}\Delta t' = \gamma\,\Delta t'$$

simple as some parallels
→ must take light longer to travel

w/s D. mod. is last

to get Gal Tr → Lor Tr   tag 5/3:45 – 5:13   slows down + contracts ✗

$x' = \gamma(x - vt)$, $y' = y$, $z' = z$
$t' = \gamma\left(t - \frac{vx}{c^2}\right)$ join t and space tog.

clock pos + end pos! when + where dep. measuring

1 move sp-t. diagram
rest frame: diff. slit + DR
tag 5/6:25 – 8:13, tag 6/ – 3:10

path

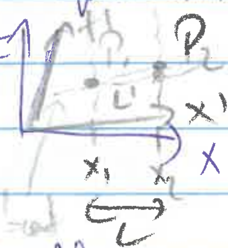
4: ~~HH~~ 10:39 – 12:12 = 4/0:08 – 2:18
$\frac{v^2}{c^2}$ $1\cdot10^{8}$ 4/3:19 – 4:12

0.01c 1.00005
0.1c – 1.005
0.25c – 1.03
0.5c ~1.15
0.7c ~1.4
0.9c ~2.29
0.9c, .99c ~7.08

addy, vol 1 3:15
vol 5 5:25 – 7:05 + 6/ – 0:12
μ meson decay 7/0:25 – 3:08
twin paradox 7/4:10 – 7:52, 8/ – 0:30

6/ ~~6/1:08:00~~ rec. 1:25 video sph dist + time

time dilation + Lor. contr. Fr[e]nch p. 94–
intro – each frame has its own criterion of simultaneity
length contr. –



$$x_1 = \gamma(x_1' + vt')$$
$$x_2 = \gamma(x_2' + vt')$$
$$L = x_2 - x_1 = \gamma(x_2' - x_1' + \cancel{vt'} - \cancel{vt'})$$

at the center of 2 sph. F81f

+ inverse, elastic cone (ASIS)

simultaneity

– $t_1' = \gamma\left(t_1 - \frac{vx_1}{c^2}\right)$

$t_2' = \gamma\left(t_2 - \frac{vx_2}{c^2}\right)$, duration $t_2' - t_1' = \gamma\left(t_1 - t_1 + \cancel{\frac{vx_1}{c^2}} - \cancel{\frac{vx_2}{c^2}}\right)$

or $\gamma \Delta t_2 = t_1 - \frac{\Delta t}{\sqrt{\cdot/\cdot}}$

⇒ length in rest frame is greatest, Δt shorter than in any other frame

– μ-mesons $\mu^{\pm} \to e^{\pm} + \nu_1 + \bar{\nu}_2$ μ are charged
travel predom. downward at $v \approx c$
expt: measure the energy of the $\underbrace{e^-}$ and μ
≡ detect μ at rest

time dilation: 6.5 μsec in rest frame
moving frame: 0.7 μsec ⇒ $1 - \frac{v^2}{c^2} \approx \frac{1}{81}$, $\frac{v}{c} \sim 0.994$ –
with cosmic rays produce μ with E range, a thin scint. to select E
a total in moving frame: decay at rest, earth is rushing upward at $v \approx c$
μ pov: dist. Lor. contracted
time of travel $\Delta t' = \Delta t \frac{1}{\gamma}$, $\Delta t = \frac{H}{v}$, $\Delta t' = \frac{H'}{v}$
or $\frac{H'}{H} = \frac{\Delta t'}{\Delta t} = \frac{1}{\gamma}$

Note that if the two events occur at the same spatial location, only *one* clock is needed by each observer to determine if the events are simultaneous. On the other hand, if the two events are separated spatially, then each observer needs *two* clocks, properly synchronized, to determine whether or not the two events are simultaneous.

Solved Problems

3.1. Evaluate $\sqrt{1-(v^2/c^2)}$ for (*a*) $v = 10^{-2}c$; (*b*) $v = 0.9998c$.

Ans. In the following we make use of the binomial expansion,

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2}x^2 + \cdots$$

(*a*) Setting $x = -10^{-4}$ and $n = \frac{1}{2}$ in the binomial expansion, and, because x is so small, keeping only the first two terms of the expansion, we obtain

$$(1-10^{-4})^{1/2} \approx 1 + \tfrac{1}{2}(-10^{-4}) = 1 - 0.00005 = 0.99995$$

(*b*)

$$\sqrt{1-(v^2/c^2)} = \sqrt{1-(0.9998)^2} = \sqrt{1-(1-0.0002)^2}$$

To evaluate $(1-0.0002)^2$ we employ the binomial expansion to obtain

$$(1-0.0002)^2 \approx 1 - 2(0.0002) = 1 - 0.0004$$

Using this in the above expression we obtain

$$\sqrt{1-(v^2/c^2)} \approx \sqrt{1-(1-0.0004)} = \sqrt{0.0004} = 0.02$$

3.2. As measured by O, a flashbulb goes off at $x = 100\,\text{km}$, $y = 10\,\text{km}$, $z = 1\,\text{km}$ at $t = 5\times10^{-4}\,\text{s}$. What are the coordinates x', y', z', and t' of this event as determined by a second observer, O', moving relative to O at $-0.8c$ along the common x–x' axis?

hw var.

Ans. From the Lorentz transformations,

$$x' = \frac{x - vt}{\sqrt{1-(v^2/c^2)}} = \frac{100\,\text{km} - (-0.8\times3\times10^5\,\text{km/s})(5\times10^{-4}\,\text{s})}{\sqrt{1-(0.8)^2}} = 367\,\text{km}$$

$$t' = \frac{t - \dfrac{v}{c^2}x}{\sqrt{1-(v^2/c^2)}} = \frac{5\times10^{-4}\,\text{s} - \dfrac{(-0.8)(100\,\text{km})}{3\times10^5\,\text{km/s}}}{\sqrt{1-(0.8)^2}} = 12.8\times10^{-4}\,\text{s}$$

$$y' = y = 10\,\text{km}$$
$$z' = z = 1\,\text{km}$$

3.3. Suppose that a particle moves relative to O' with a constant velocity of $c/2$ in the $x'y'$-plane such that its trajectory makes an angle of $60°$ with the x'-axis. If the velocity of O' with respect to O is $0.6c$ along the x–x' axis, find the equations of motion of the particle as determined by O.

Ans. The equations of motion as determined by O' are

$$x' - u'_x t' = \frac{c}{2}(\cos 60°)t' \qquad y' = u'_y t' = \frac{c}{2}(\sin 60°)t'$$

subtraction technique of Section 3.3; the correct answer will *not* be obtained by multiplying or dividing the original spatial separation by $\sqrt{1-(v^2/c^2)}$.

Solved Problems

4.1. How fast does a rocket ship have to go for its length to be contracted to 99% of its rest length?

Ans. From the expression for length contraction (*4.1*),

$$\frac{L}{L_0} = 0.99 = \sqrt{1-(v^2/c^2)} \qquad \text{or} \qquad v = 0.141c$$

4.2. Calculate the Lorentz contraction of the earth's diameter as measured by an observer O' who is stationary with respect to the sun.

Ans. Taking the orbital velocity of the earth to be 3×10^4 m/s and the diameter of the earth as 7920 mi, the expression for the Lorentz contraction yields

$$D = D_0\sqrt{1-(v^2/c^2)} = (7.92 \times 10^3\ \text{mi})\sqrt{1-\left(\frac{3 \times 10^4\ \text{m/s}}{3 \times 10^8\ \text{m/s}}\right)^2} \approx (7.92 \times 10^3\ \text{mi})(1-0.5 \times 10^{-8})$$

Solving, $D_0 - D = 3.96 \times 10^{-5}$ mi $= 2.51$ in. It is seen that relativistic effects are very small at speeds that are normally encountered.

4.3. A meterstick makes an angle of 30° with respect to the x'-axis of O'. What must be the value of v if the meterstick makes an angle of 45° with respect to the x-axis of O?

Ans. We have:

$$L'_y = L' \sin\theta' = (1\ \text{m}) \sin 30° = 0.5\ \text{m} \qquad L'_x = L' \cos\theta' = (1\ \text{m}) \cos 30° = 0.866\ \text{m}$$

Since there will be a length contraction only in the x-x' direction,

$$L_y = L'_y = 0.5\ \text{m} \qquad L_x = L'_x\sqrt{1-(v^2/c^2)} = (0.866\ \text{m})\sqrt{1-(v^2/c^2)}$$

Since $\tan\theta = L_y/L_x$,

$$\tan 45° = 1 = \frac{0.5\ \text{m}}{(0.866\ \text{m})\sqrt{1-(v^2/c^2)}}$$

Solving, $v = 0.816c$.

4.4. Refer to Problem 4.3. What is the length of the meterstick as measured by O?

Ans. Use the Pythagorean theorem or, more simply,

$$L = \frac{L_y}{\sin 45°} = \frac{0.5\ \text{m}}{\sin 45°} = 0.707\ \text{m}$$

4.5. A cube has a (proper) volume of $1000\ \text{cm}^3$. Find the volume as determined by an observer O' who moves at a velocity of $0.8c$ relative to the cube in a direction parallel to one edge.

Ans. The observer measures an edge of the cube parallel to the direction of motion to have the contracted length

$$l'_x = l_x\sqrt{1-(v^2/c^2)} = (10\ \text{cm})\sqrt{1-(0.8)^2} = 6\ \text{cm}$$

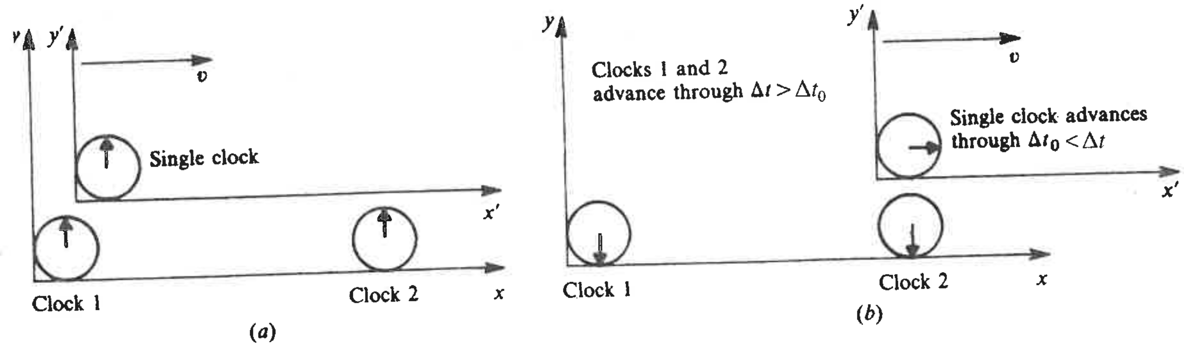


Fig. 5-1. Time Dilation as Viewed by Observer O

Time dilation is a very real effect. Suppose in Fig. 5-1 cameras are placed at the location of clock 2 and at the location of the single clock, and a picture is taken by each camera when the single clock passes clock 2. When the pictures are developed, each picture will show the same thing—that the single clock has advanced through Δt_0 while clock 2 has advanced through $\Delta t > \Delta t_0$, with Δt and Δt_0 related by the time dilation expression.

A Warning!

It is important to keep clear the distinction between the "time separation" of two events and the "proper time interval" between two events. If observers O and O' measure the time separation between two events that, for both observers, occur at different spatial locations, then these time separations are *not* related by simply multiplying or dividing by $\sqrt{1-(v^2/c^2)}$.

Solved Problems

5.1. The average lifetime of μ-mesons with a speed of $0.95c$ is measured to be 6×10^{-6} s. Compute the average lifetime of μ-mesons in a system in which they are at rest.

Ans. The time measured in a system in which the μ-mesons are at rest is the proper time.

$$\Delta t_0 = (\Delta t)\sqrt{1-(v^2/c^2)} = (6 \times 10^{-6}\ \text{s})\sqrt{1-(0.95)^2} = 1.87 \times 10^{-6}\ \text{s}$$

5.2. An airplane is moving with respect to the earth with a speed of 600 m/s. As determined by earth clocks, how long will it take for the airplane's clock to fall behind by two microseconds?

Ans. From the time dilation expression,

$$\Delta t_{\text{earth}} = \frac{\Delta t_{\text{plane}}}{\sqrt{1-\dfrac{v^2}{c^2}}} = \frac{\Delta t_{\text{plane}}}{\sqrt{1-\left(\dfrac{6\times 10^2\ \text{m/s}}{3\times 10^8\ \text{m/s}}\right)^2}} \approx \frac{\Delta t_{\text{plane}}}{1-2\times 10^{-12}}$$

$$(2\times 10^{-12})\Delta t_{\text{earth}} \approx \Delta t_{\text{earth}} - \Delta t_{\text{plane}} = 2\times 10^{-6}\ \text{s}$$

$$\Delta t_{\text{earth}} \approx 10^6\ \text{s} = 11.6\ \text{days}$$

This result indicates the smallness of relativistic effects at ordinary speeds.

5.3. Observers O and O' approach each other with a relative velocity of $0.6c$. If O measures the initial distance to O' to be 20 m, how much time will it take, as determined by O, before the two observers meet?

Ans. We have

$$\Delta t = \frac{\text{distance}}{\text{velocity}} = \frac{20\,\text{m}}{0.6 \times 3 \times 10^8\,\text{m/sec}} = 11.1 \times 10^{-8}\,\text{s}$$

5.4. In Problem 5.3, how much time will it take, as determined by O', before the two observers meet?

Ans. The two events under consideration are: (A) the position of O' when O makes his initial measurement, and (B) the coincidence of O and O'. Both of these events occur at the origin of O'. Therefore, the time lapse measured by O' is equal to the proper time between the two events. From the time dilation expression,

$$\Delta t_0 = (\Delta t)\sqrt{1 - (v^2/c^2)} = (11.1 \times 10^{-8}\,\text{s})\sqrt{1 - (0.6)^2} = 8.89 \times 10^{-8}\,\text{s}$$

This problem can also be solved by noting that the initial distance as determined by O' is related to the distance measured by O through the Lorentz contraction:

$$L' = L_0\sqrt{1 - (v^2/c^2)} = (20\,\text{m})\sqrt{1 - (0.6)^2} = 16\,\text{m}$$

Then

$$\Delta t' = \frac{L'}{v} = \frac{16\,\text{m}}{0.6 \times 3 \times 10^8\,\text{m/s}} = 8.89 \times 10^{-8}\,\text{s}$$

5.5. Pions have a half-life of 1.8×10^{-8} s. A pion beam leaves an accelerator at a speed of $0.8c$. Classically, what is the expected distance over which half the pions should decay?

Ans. We have:

$$\text{distance} = v\,\Delta t = (0.3 \times 3 \times 10^8\,\text{m/s})(1.8 \times 10^{-8}\,\text{s}) = 4.32\,\text{m}$$

5.6. Determine the answer to Problem 5.5 relativistically.

Ans. The half-life of 1.8×10^{-8} s is determined by an observer at rest with respect to the pion beam. From the point of view of an observer in the laboratory, the half-life has been increased because of the time dilation, and is given by

$$\Delta t = \frac{\Delta t_0}{\sqrt{1 - (v^2/c^2)}} = \frac{1.8 \times 10^{-8}\,\text{s}}{\sqrt{1 - (0.8)^2}} = 3 \times 10^{-8}\,\text{s}$$

Therefore, the distance traveled is

$$d = v\,\Delta t = (0.8 \times 3 \times 10^8\,\text{m/s})(3 \times 10^{-8}\,\text{s}) = 7.20\,\text{m}$$

For an observer at rest with respect to the pion beam, the distance d_p the pions have to travel is shorter than the laboratory distance d_l by the Lorentz contraction:

$$d_p = d_l\sqrt{1 - (v^2/c^2)} = d_l\sqrt{1 - (0.8)^2} = 0.6 d_l$$

The time elapsed when this distance is covered is

$$\Delta t_0 = \frac{d_p}{v} \qquad \text{or} \qquad 1.8 \times 10^{-8}\,\text{s} = \frac{0.6 d_l}{0.8 \times 3 \times 10^8\,\text{m/s}}$$

Solving, $d_l = 7.20$ m, which agrees with the answer determined from time dilation.

by the luminiferous ether. But Einstein discovered the equations quite independently a year later with the help of his fresh and radical approach to the whole problem.

MORE ABOUT THE LORENTZ TRANSFORMATIONS[1]

In deriving the Lorentz transformations in the last section, we considered only the requirements imposed by light signals traveling along the x direction. A more general approach would have developed them by applying the requirements of Einstein's second postulate to a light signal traveling in an arbitrary direction. Having already set up the transformations, however, we can use them to illustrate a seeming paradox which is contained in Einstein's postulate. It is this: Suppose that a burst of light begins spreading out (in vacuum) from the origin of frame S at $t = 0$. At any later time t the light will have reached all points on a sphere of radius r, centered on the origin of S, such that $r = ct$. Then if this same phenomenon is observed with respect to a frame S', moving with respect to S with any velocity v, the description of the expanding burst of light is again a sphere, in this case centered on the origin of S'—even though, by definition, the origins of S and S' coincide only at the instant $t = t' = 0$.

at the centre of the same sphere

The center of two spheres

To see how this result emerges, we take the equation $r = ct$ and rewrite it in terms of position and time coordinates measured in S'. By first squaring both sides of the equation we get

$$r^2 = x^2 + y^2 + z^2 = c^2t^2$$

vector!

Now use the right-hand set of equations (3–16). The above equation then becomes the following:

$$\gamma^2(x' + vt')^2 + (y')^2 + (z')^2 = \gamma^2c^2(t' + vx'/c^2)^2$$

It may be noted that the cross terms in $x't'$ on the two sides of the equation are equal, and so disappear. Collecting the other terms, we have

$$\gamma^2(x')^2(1 - v^2/c^2) + (y')^2 + (z')^2 = \gamma^2(t')^2(c^2 - v^2)$$

[1]Having once recognized that these transformations were arrived at by both Lorentz and Einstein, we shall usually in future refer to them by this briefer and more customary title.

But

$$\gamma^2(1 - v^2/c^2) = 1$$

Therefore,

$$(x')^2 + (y')^2 + (z')^2 = c^2(t')^2$$

which defines a sphere of radius r' such that

$$r' = ct'$$

Center of the sphere [illegible]

This result, which at first sight appears to do violence to one's commonsense ideas, is bound up with the relativity of simultaneity. Points which, as measured in S, are reached at the *same* time t, are reached at *different* times as measured in S', in such a fashion that the light is properly described as lying on a spherical shell expanding at speed c in both frames.

MINKOWSKI DIAGRAMS: SPACE-TIME

A valuable aid to the arguments in this chapter has been the use of graphs, with axes representing position and time, which allow one to display the complete history of a one-dimensional motion. The use of such graphs in special relativity was introduced by H. Minkowski in 1908, and they are customarily referred to as Minkowski diagrams. On any such diagram, as we have seen, any individual event—e.g., a light signal striking a detector, or one tick of a watch—is uniquely represented by some point P (Fig. 3–6). The detailed specification of this event, however, in terms of numerical values of x and t, can be made in infinitely many different ways according to the particular reference frame chosen. The description of a point event is described in frame S by the coordinates (x, t) and in S' by the coordinates (x', t'). If the origins of S and S' are chosen so as to coincide at $t = t' = 0$, then the relation between (x, t) and (x', t') is contained in the Lorentz transformations of equations (3–16).

It is very convenient to use ct, rather than t, to describe the time coordinate. Both coordinates, ct and x, are then expressed as distances, and if the scale of distance is chosen to be the same for both, the world line of a light signal starting out at $x = 0$, $t = 0$, is a bisector of the angle between the axes. This holds good in all reference frames. We can represent any one such frame (say, S) by drawing the axes of x and ct at right angles to one

ad hoc the null-
except that he
rase "*relative to*
ıanism responsi-
ıent in electrical
c structure. In
.s a purely geo-
to looking at a
ıe plane of the
relativity corre-
spacetime" (see
igned to receive
ɔeen trained to
atial dimension.
:ening of a spa-
were small, and
arise at "ordi-
·ned to sense the
bringing the eye
:e the full length
e eye along with
istic length con-
ɔk at this effect

ident that if two
ıg their common
:k to be shorter
) be the shorter.
ɔrising: a corre-
:ively stationary
angle relative to
:h contraction is
e of Section 25,
how fortuitous

ph are intended to be
:d for at this stage.

where it can only be discovered by ... *cause* of length contraction resides in the preferred ether frame.

The relativistic length contraction is no "illusion": it is real in every way. Though no direct experimental verification has yet been attempted, there is no question that in principle it could be done. Consider the admittedly unrealistic situation of a man carrying horizontally a 20-foot pole and wanting to get it into a 10-foot garage. He will run at speed $v = .866c$ to make $\gamma = 2$, so that his pole contracts to 10 feet. It will be well to insist on having a sufficiently massive block of concrete at the back of the garage, so that there is no question of whether the pole finally stops in the inertial frame of the garage, or vice versa. Thus the man runs with his (now contracted) pole into the garage and a friend quickly closes the door. In principle we do not doubt the feasibility of this experiment, i.e., the reality of length contraction. When the pole stops in the rest frame of the garage, it is, in fact, being "rotated in spacetime" and will tend to assume, if it can, its original length relative to the garage. Thus, if it survived the impact, it must now either bend, or burst the door.

At this point a "paradox" might occur to the reader: what about the symmetry of the phenomenon? Relative to the runner, won't the garage be only 5 feet long? Yes, indeed. Then how can the 20-foot pole get into the 5-foot garage? Very well, let us consider what happens in the rest frame of the pole. The open garage now comes towards the stationary pole. Because of the concrete wall, it keeps on going even after the impact, taking the front end of the pole with it. But the back end of the pole is still at rest: it cannot yet "know" that the front end has been struck, because of the

it's real

man 20 ft pole
10 ft garage
=> $\gamma = 2$ when
$v = 0.866c$

inertial garage

stops → rest frame
garage
! rotates in spacetime!

"paradox"
rel. to pole, garage
is 5 ft long!
how can 20 ft pole get in
5 ft garage??
rest frame pole) garage
comes toward rest pole
concrete makes it keep
garage going after

but finite vel. of signal! we are being told! heads
being transmitted, at most c/c

finite speed of propagation of *all* signals. Even if the "signal" (in this case the elastic shock wave) travels along the pole with the speed of light, that signal has 20 feet to travel against the garage front's 15 feet, before reaching the back end of the pole. This race would be a dead heat if v were $.75c$. But v is $.866c$! So the pole *more* than just gets in. (It could even get into a garage whose length was as little as 5.4 feet at rest and thus 2.7 feet in motion: the garage front would then have to travel 17.3 feet against the shock wave's 20 feet, requiring speeds in the ratio 17.3 to 20, i.e., .865 to 1 for a dead heat.)

There is one important moral to this story: whatever result we get by correct reasoning in any one frame, *must* be true; in particular, it must be true when viewed from any other frame. As long as the physical laws we are using are Lorentz-invariant, there *must* be an explanation of the result in every other frame, although it may be quite a different explanation from that in the first frame. Recall Einstein's "hunch" that the force experienced by an electric charge when moving through a magnetic field is equivalent to a simple electric force in the rest frame of the charge.

Consider, as another example, a "rigid" rod of rest length L sliding over a hole of diameter $\frac{1}{2}L$ on a smooth table. When its Lorentz factor is 10, the length of the rod is $\frac{1}{5}$ of the diameter of the hole, and in passing over the hole, it will fall into it under the action of gravity* (at least slightly: enough to be stopped). This *must* be true also in the frame of the rod—in which however, the diameter of the hole is only $\frac{1}{20}L$! The only way in which this can happen is that the front of the "rigid" rod *bends* into the hole. Moreover, even after the front end strikes the far edge of the hole, the back end keeps coming in (not yet "knowing" that the front end has been stopped), as it must, since it does so in the first description.

*We are here violating our resolve to work in *strict* inertial frames only! The conscientious reader may replace the force of gravity acting down the hole by a sandblast from the top—the result will be the same. For a full discussion of this paradox, see W. Rindler, *Am. J. Phys.* **29**, 365 (1961).

i.e. signal has 20 ft to travel along pole against 15 ft garage front
→ dead heat speed < v ($\gamma = 2$)
pole more than gets in
; find min. garage rest length and its length in motion $\gamma = 2$
→ lets 20 ft vs. new 15 → ? ft
⇒ low dead heat v

moral:
correct result in any frame must be true when viewed from any other frame
(post. 2, laws of phys. the same)
but may req. quite diff. explan.

e.g. rigid rod length L
hole $\varnothing$ $\frac{1}{2}L$, $\gamma = 10$
L to $\frac{1}{10}L$ or $\frac{1}{5}$ $\varnothing$
→ falls in (front end)
• must be true in rest frame of rod → $\varnothing$ $\frac{1}{20}L$!
can only happen if rod bends

Length Contraction Paradox

W. RINDLER
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(Received January 29, 1961)

A certain man walks very fast—so fast that the relativistic length contraction makes him very thin. In the street he has to pass over a grid. A man standing at the grid fully expects the fast thin man to fall into the grid. Yet to the fast man the grid is much narrower even than to the stationary man, and he certainly does not expect to fall in. Which is correct? The answer hinges on the relativity of rigidity.

SOME two or three years ago I proposed to colleagues at Cornell a simple paradox on relativistic length contraction which I had already proposed several years earlier to students at London University. It seemed the kind of paradox that must occur to anyone concerned with the subject, but I failed to find it mentioned in the literature. At a recent professional meeting it still aroused some interest, and I therefore offer it now.

A 10-in. long "rigid" rod moves longitudinally over a flat table. In its path is a hole 10 in. wide. Suppose the rod moves so fast that its Lorentz contraction factor is 10. To an observer B moving with the rod the hole is only 1 in. wide, and the rod, being "rigid," might be expected to pass unhindered over the hole. To an observer A at rest relative to the table, however, it is the rod that is only 1 in. long; in passing over the 10-in. hole it is bound to fall somewhat under gravity, and it will consequently strike the far edge of the hole and so be stopped. Which description is correct?

The resolution of the paradox has already been hinted at by setting the word *rigid* in quotation marks. There is no doubt that A's description of events is correct. The rod simply cannot remain rigid in B's inertial frame (see Fig. 1). This illustrates well the difficulties encountered in the search for a satisfactory definition of rigidity in relativity.

Before proving our assertion, let us make the experiment more concrete. The hole shall be filled with a trap door which will be removed (downward, and with sufficient acceleration to allow the rod to fall freely) by the observer A at the instant when to him the hind end of the rod passes into the hole. This precaution eliminates the tendency of the rod to topple over the edge. All points of the rod will then fall equally fast, and the rod will remain horizontal, in the frame of A. The gravitational field can be replaced by a magnetic field acting on an iron rod, or even by a uniform vertical sand blast from above, if it be held that special relativity is inapplicable to gravitation. It must be stressed, however, that special relativity is perfectly applicable to accelerated bodies: what it cannot do is cope with nonflat space times.

Now let it be understood that the rod is originally a rectangular parallelepiped and that the observer B uses an internal frame fixed to the hind end of the rod. Call this frame S', call

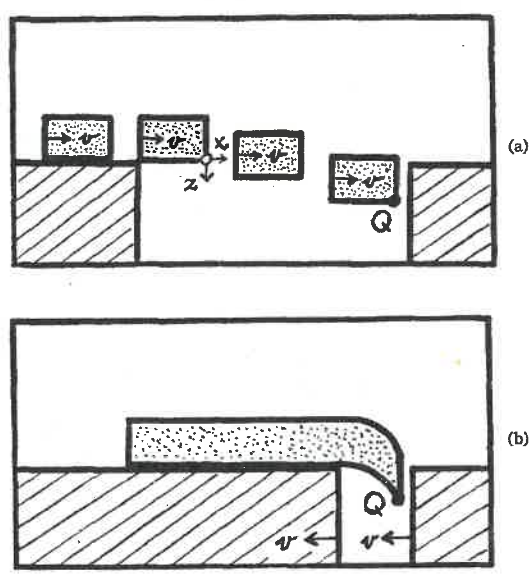


FIG. 1. (a) Sequence of four observations made by A at equal intervals of time t. (b) Observation made by B at one particular instant t'. (For convenience these diagrams are drawn for the case $\gamma=4$, not $\gamma=10$ as in the text.)

A's frame S, and let their relative velocity be v. Take as common origin event a front-bottom corner Q of the rod at the instant when the trap door separates from Q, measure z, z' down from the top of the table, and x, x' along the initial path of Q. Then the standard Lorentz transformation equations

$$z=z', \quad t=\gamma(t'+vx'/c^2), \quad \gamma=(1-v^2/c^2)^{-\frac{1}{2}} \quad (1)$$

apply to S and S'. The equations of the bottom edge of the rod in S are

$$z=0 \quad \text{when} \quad t<0, \quad z=\tfrac{1}{2}at^2 \quad \text{when} \quad t\geqslant 0, \quad (2)$$

where a is the acceleration produced by the field or sandblast. (A uniform field in relativity will only approximately produce uniform acceleration, but the small error is quite irrelevant here.) By use of (1), we can immediately transform Eqs. (2) into

$$z'=0 \quad \text{when} \quad x'<-c^2t'/v,$$
$$z'=\tfrac{1}{2}a\gamma^2(t'+vx'/c^2)^2 \quad \text{when} \quad x'\geqslant c^2t'/v. \quad (3)$$

The interpretation of Eqs. (3) is as follows. In S', imagine a parabola with vertex at Q, axis vertically down, and latus rectum $2c^4/a\gamma^2v^2$. The vertex of this parabola moves along the rod with velocity c^2/v starting at $t'=0$; and the rod, as it passes over that vertex, "flows" down the parabola. Its horizontal extent clearly remains constant until it hits the far edge of the hole. It can easily be shown that the near edge of the hole at time t' is at $x'=-L/\gamma^2-vt'$, where L is the rest length of the rod. Consequently, this edge, moving with velocity v along the rod, leads the vertex of the parabola and is overtaken by the latter exactly at $x'=-L$, i.e., at the hind end of the rod. A sizable compression of the rod must eventually occur in S' because, as can be seen from the description in S, the hind end of the rod passes well into the hole.

ORINS Summer Symposium

The eighth summer symposium of the Oak Ridge Institute of Nuclear Studies will be held August 28–30, 1961, in Gatlinburg, Tennessee. This year's topic will be the university use of subcritical assemblies.

Cosponsoring the symposium are the education committee of the American Nuclear Society, Oak Ridge National Laboratory, and the U. S. Atomic Energy Commission.

Leading representatives of universities, industry, and government will discuss the various types of subcritical assemblies, the techniques of their use, their applications in university research and education programs, and other facets of obtaining and operating subcritical assemblies on university campuses.

Further information about the meeting is available from the Symposium Office, University Relations Division, Oak Ridge Institute of Nuclear Studies, P. O. Box 117, Oak Ridge, Tennessee.

* SR

- proper length + proper time : as meas. in rest frame : l_0
  same clock, same location

the problem of meas. time simultaneously (p 107 - 109, won't expand in lecture)
also 111 - 117 understandly

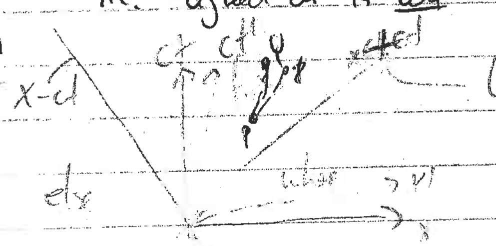
- space time intervals

$$s^2 = (ct')^2 - (x')^2 - (y')^2 - (z')^2 \text{ is an}$$

invariant under LT

i.e. agreed on in all inertial frames!

1-dim'



'divides spacetime into regions

$(\Delta s)^2$, interval $\sqrt{(\Delta s)^2}$ b/w 2 events

>0 dep. on $c\Delta t > \Delta x$

<0 $\quad <$

$=0$ $\quad =$

light line connects point events b/w which $\sqrt{(\Delta s)^2} = 0$

if >0 can transform to ct' axis $\parallel$ P-Q

meaning P and Q occur at the same place in S'

sep. by time only = time-like interval

if <0 can transform P, Q to simult. events at diff. places

'space-like'

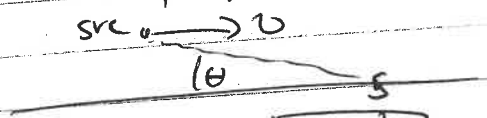
to connect to such events by space-like interval req. $v > c$

(causality!)

video 6/13/50

– the relativistic Doppler Effect

a source emits em-rad. of freq. ν_0
in motion w/ respect to S who meas. ν



$$\nu = \nu_0 \frac{\sqrt{1-\frac{v^2}{c^2}}}{1-\frac{v}{c}\cos\theta}$$

Special cases • src + obs. move toward each other, $\theta = 0$

$$\nu = \nu_0 \sqrt{\frac{c+v}{c-v}} \quad \text{and} \quad \nu > \nu_0$$ proof next pg *

• move away from each other, $\theta = 180^\circ$

$$\nu = \nu_0 \sqrt{\frac{c-v}{c+v}} \quad \text{and} \quad \nu < \nu_0$$

• rad. obs. transverse to dir. of motion, $\theta = 90^\circ$

$$\nu = \nu_0 \sqrt{1-\frac{v^2}{c^2}}$$

Schaum 7.[illegible] — star recedes from earth at $5 \cdot 10^{-3}\,c$, what $\Delta\lambda$ for sodium D_2 spectral line? (at rest $\lambda_{Na,D_2} = 5890$ Å)

↳ $\frac{c}{\lambda} = \frac{c}{\lambda_0}\sqrt{\frac{c-v}{c+v}} \rightarrow \lambda = \lambda_0 \cdot \sqrt{\frac{1+\frac{v}{c}}{1-\frac{v}{c}}}$

$= 5890\text{ Å} \cdot \sqrt{\frac{1+0.005}{1-0.005}} = 5920\text{ Å}$

$\Delta\lambda = 5920 - 5890 = 30\text{ Å}$ to greater values ⇒ red shift

nuc. emits e^- w/ 0.9c $\perp$ to dir. mot (lab frame)
rel. at rest to nuc
find θ_{e^-} in $5 \mid$ lab frame

Ans. With the same association as in Problem 7.6, one has

$$u_x = \frac{u'_x + v}{1 + (v/c^2)u'_x} = \frac{0 + 0.5c}{1 + 0} = 0.5c$$

$$u_y = \frac{u'_y\sqrt{1 - (v^2/c^2)}}{1 + (v/c^2)u'_x} = \frac{(0.9c)\sqrt{1 - (0.5)^2}}{1 + 0} = 0.779c$$

whence

$$u = \sqrt{u_x^2 + u_y^2} = \sqrt{(0.5c)^2 + (0.779c)^2} = 0.926c$$

and

$$\tan\phi = \frac{u_y}{u_x} = \frac{0.779c}{0.5c} = 1.56 \qquad \text{or} \qquad \phi = 57.3°$$

7.8. At $t = 0$ observer O emits a photon traveling at speed c in a direction of 60° with the x-axis. A second observer, O', travels with a speed of $0.6c$ along the common x–x' axis. What angle does the photon make with the x'-axis of O'?

S$\sqcap$ 10.7c

Ans. We have

$$u_x = c\cos 60° = 0.500c \qquad u_y = c\sin 60° = 0.866c$$

$$u'_x = \frac{u_x - v}{1 - (v/c^2)u_x} = \frac{0.5c - 0.6c}{1 - \dfrac{(0.6c)(0.5c)}{c^2}} = -0.143c$$

$$u'_y = \frac{u_y\sqrt{1 - (v^2/c^2)}}{1 - (v/c^2)u_x} = \frac{(0.866c)\sqrt{1 - (0.6)^2}}{1 - \dfrac{(0.6c)(0.5c)}{c^2}} = 0.990c$$

Thus

$$\tan\phi' = \frac{u'_y}{u'_x} = \frac{0.990c}{-0.143c} = -6.92$$

and $\phi' = 81.8°$ above the negative x'-axis. The magnitude of the velocity of the photon as measured by O' is

$$u' = \sqrt{u'^2_x + u'^2_y} = \sqrt{(-0.143c)^2 + (0.990c)^2} = c$$

as is necessary.

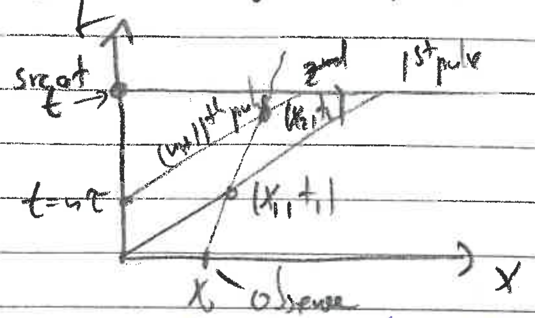
7.9. The speed of light in still water is c/n, where the index of refraction for water is approximately $n = 4/3$. Fizeau, in 1851, found that the speed (relative to the laboratory) of light in water moving with a speed V (relative to the laboratory) could be expressed as

$$u = \frac{c}{n} + kV$$

where the "dragging coefficient" was measured by him to be $k \approx 0.44$. Determine the value of k predicted by the Lorentz velocity transformations.

R8 — space time diagram of rel Doppler Effect

F p.136



src starts at origin, S
obs moves w/ v rel to S
his rest frame S'
each pulse travels w/ c
(n+1)th pulse sent at t = nτ
covers n periods of vibn
$\Rightarrow \quad \nu = \frac{1}{\tau}$

$x_1 = ct_1 = x_0 + vt_1$
$x_2 = c(t_2 - n\tau) = x_0 + vt_2$ $\Big\}$ $v(t_2-t_1) = c(t_2 - t_1 - n\tau)$

$\cancel{t_2 - t_1} \quad t_2 - t_1 = n\tau - v$

$*$

$S: \quad \tau = t_2 - t_1 = \dfrac{cn\tau}{c - v}$ (from sys) $\cancel{= \frac{\tau}{n} \ldots}$ $\frac{c}{c} \cdot \frac{n\tau}{1 - \frac{v}{c}}$

$\tau = \frac{1}{\nu}$ b/c n periods of osc

$x_2 - x_1 = v \dfrac{cn\tau}{c - v}$ ($t_2 - t_1$)

$S': \quad \tau' = t_2' - t_1' = \gamma\left[(t_2 - t_1) - \dfrac{v(x_2 - x_1)}{c^2}\right]$ LT

$= \gamma\left(\dfrac{cn\tau}{c - v} - \dfrac{v^2}{c^2}\dfrac{cn\tau}{c - v}\right)$

for n periods $\Rightarrow$ for 1 period divide by n

$\tau' = \dfrac{\gamma c\tau}{c - v}\left(1 - \dfrac{v^2}{c^2}\right)$ let $\beta = \frac{v}{c}$

$= \gamma\tau\dfrac{(1-\beta^2)}{(1-\beta)}$ b/c $\dfrac{c}{c - v} = \dfrac{1}{1 - \frac{v}{c}}\dfrac{c}{c}$

$= \gamma\tau(1 + \beta)$ Simonal

now, $\gamma := (1 - \beta^2)^{-\frac{1}{2}}$

b/c $\dfrac{1+\beta}{\sqrt{1-\beta^2}} = \dfrac{1+\beta}{\sqrt{(1+\beta)(1-\beta)}} = \dfrac{\sqrt{1+\beta}}{\sqrt{1-\beta}}$

$\Rightarrow \quad \tau' = \left(\dfrac{1+\beta}{1-\beta}\right)^{\frac{1}{2}}\tau$

and $\nu' = \left(\dfrac{1-\beta}{1+\beta}\right)^{\frac{1}{2}}\nu$ b/c $\nu = \frac{1}{\tau}$

$\sqrt{\dfrac{1 - \frac{v}{c}}{1 + \frac{v}{c}}} = \sqrt{\dfrac{c}{c}\dfrac{c - v}{c + v}}$

Schaum 8.14 — At what fraction of c must a particle be so that $K = 2E_0$?

→ mv

$$K = \gamma m_0 c^2 - m_0 c^2 \overset{!}{=} 2 m_0 c^2 \Rightarrow \gamma = 3$$

$$v \approx 0.943\,c$$

Schaum 8.15 — students: e^- velocity is $5 \cdot 10^7$ [m/s]

how much E req. to double the speed?

n/b e^-, $m_0 c^2$, 0.511 MeV

sol

$$E_i = \gamma_v m_0 c^2 = \frac{0.511\ \text{MeV}}{\sqrt{1-\left(\frac{0.5 \cdot 10^8}{3 \cdot 10^8}\right)^2}} = 0.518$$

$$E_f = \gamma_{2v} m_0 c^2 = \frac{0.511}{\sqrt{1-\left(\frac{1}{3}\right)^2}} = 0.542$$

$$\Delta E = E_f - E_i = 0.542 - 0.518 = 0.024\ \text{MeV}$$

RW — video space billiard, #18, #19

setup E + p cons. for collisions

Schaum 8.26 — Two identical bodies, each m_0, approach each other w/ eq. vel. u. and collide in perfect inel collis. (i.e. they stick tog.)

→ det. rest mass M_0 of the composite body

here: final mom. zero in frame

$$E_i \overset{!}{=} E_f \,, \quad \frac{2 m_0 \cancel{c^2}}{\sqrt{1-\frac{v^2}{c^2}}} = M_0 \cancel{c^2}$$

note, $\gamma \geq 1 \Rightarrow M_0 > 2 m_0$

– Relativistic Kinematics (ch. 5) p.125 (P)

b/w frames only one uniquely def. direction involved

↳ that of the rel. motion of the in. frames; $S, S' \to v$

any displacement → compon. ∥, ⊥ to it

Relation b/w comp. of vel meas. in S and in S', w/ v b/w frames?

1-d

LT: $x = \gamma(x' + vt')$ $\quad t = \gamma\left(t' + \frac{vx'}{c^2}\right)$

$y = y'$

video 6:13:50 – 7:36 H-ar

⟹ ~~$dx = \gamma($~~ suppose meas u_x', u_y' vel. in S'

$dx = \gamma(u_x' + v)\,dt'$ differential, not derivative – $u_x'\,dt' = \frac{dx'}{dt'}\,dt'$

$dy = u_y'\,dt'$

$dt = \gamma\left(1 + \frac{vu_x'}{c^2}\right)dt'$ no → from dx', dt' as two $\frac{dx'}{dt'}$ → separately

$u_x\left(1 + \frac{vu_x'}{c^2}\right) - v = u_x$

then $u_x = \frac{dx}{dt} = \frac{u_x' + v}{1 + \frac{vu_x'}{c^2}}$ $\qquad u_x' = \frac{u_x - v}{1 - \frac{vu_x}{c^2}}$

$u_y = \frac{dy}{dt} = \frac{u_y'/\gamma}{1 + \frac{vu_x'}{c^2}}$ $\frac{dt'}{dt'}$ $\qquad u_y' = \frac{u_y/\gamma}{1 - \frac{vu_x}{c^2}}$

b/c $v \to -v$

if at least one of the velocities ≈ c

→ result diff from Galilean kinematics of vector add.

e.g. $u_x' = c$, $v = c$; $u_x = \frac{c + c}{1 + 1 = \frac{c^2}{c^2}} = \frac{2c}{2} = c$

$u_x' = 0.9c$, $v = 0.8c$; $\frac{1.7c}{1 + 0.72} = \frac{1.7}{1.72}\,c = 0.988c$

– Relativistic Dynamics

– the inertia of energy (ch. 1, p. 16)

"$\frac{p}{v} = \frac{mv}{v}$"

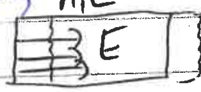
$p = \frac{E}{c} = \frac{h}{\lambda} = p$

photons : $E = c\,p$ b/c $E = h f$

Einstein's box (expt) particle

'E has mass equivalent'

M, L



photons emitted → rad. carries mom. $\frac{E}{c}$

mom. cons. : recoil box, mom $-\frac{E}{c}$, $v = -\frac{E}{Mc}$

after $\Delta t \approx \frac{L}{c}$ ($v \ll c$) → box comes to rest

+ moved by $\Delta x = v\,\Delta t = -\frac{E}{Mc}\,\frac{L}{c}$

assume the rad. carried eq. mass, m, so that CM unmoved : $mL + M\Delta x = 0$

together, $mL - \frac{M}{M}\frac{E}{c^2}L = 0$ or $m = \frac{E}{c^2}$

for photons $E = cp$, $m = \frac{E}{c^2}$ ⟹ $m_{ph} = \frac{p}{c}$

Newt. mech : $m = \frac{p}{v}$ w/ $m = \frac{E}{c^2}$

↳ Relat. : ~~$m = \frac{p}{v}$~~ $E = \frac{pc^2}{v}$

Next : $dE = F\,dx = \frac{dp}{dt}\,dx = v\,dp$

combine w/ $\frac{pc^2}{v}$:

$$E\,dE = c^2 p\,dp \quad , \text{ integrate}$$

$$\boxed{E^2 = c^2p^2 + \underbrace{E_0^2}_{\text{int. const}}}$$ the relativ. relation b/w E and p

substitute $E = \frac{pc^2}{v}$ as $cp = \frac{E v}{c}$, for $v \ll c$

use bin. expansion : $\left(1 - \frac{v^2}{c^2}\right)^{-\frac{1}{2}} \simeq 1 + \frac{1}{2}\frac{v^2}{c^2} + \dots$

$$\hookrightarrow E(v) = E_0 + \frac{1}{2}\left(\frac{E_0}{c^2}\right)v^2 + \dots$$

to work w/ N, relativity $m_0 = \frac{E_0}{c^2} \to \frac{1}{2}mv^2$

m_0 'rest mass'

$E_0 = \frac{m_0 \times c^2}{\cancel{c^2}}$ 'rest energy'

e.g. e^-: $m_0 = 9.11 \cdot 10^{-31}$ kg

$E_0 = 8.2 \cdot 10^{-14}$ J $= 0.511$ MeV

then, $\boxed{m(v) = \frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}}}$ mass is rel.

n/b $K = E - E_0$ not by substituting $m(v)$ in $\frac{1}{2}mv^2$

$$\boxed{\vec{p} = m(v)\,\vec{v}, \qquad E = m(v)c^2}$$

R9 → video #20: $E=mc^2$ demo

pulley everywhere & is relativistic (is elastic) collision of m_0 w/ m_0 (at rest) the resulting rest mass of the composite particle is

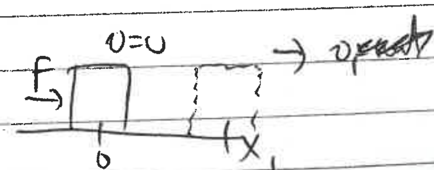
1- one can't tell

2- $< 2m_0$

→ 3- $> 2m_0$

4- $= 2m_0$

$\vec{F} = \frac{d\vec{p}}{dt}$



$W = \int_{x=0}^{x_1} F\,dx \to K_2 - K_1 = W = K_2 - 0 \equiv K$

$\vec{p} = m\vec{v} = \gamma m_0 \vec{v} = \frac{m_0 \vec{v}}{\sqrt{1-\frac{v^2}{c^2}}}$

↙ p208 ff

$$\Rightarrow \boxed{\begin{aligned} p_x' &= \gamma_v\left(p_x - \frac{vE}{c^2}\right) \\ p_y' &= p_y\ ,\ p_z' = p_z \\ E' &= \gamma_v\left(E - v p_x\right) \end{aligned}} \quad \boxed{\begin{aligned} p_x &= \gamma_v\left(p_x' + \frac{vE'}{c^2}\right) \\ &\text{''} \\ E &= \gamma_v\left(E' + v p_x'\right) \end{aligned}}$$

where $\gamma_v = \left(1 - \frac{v^2}{c^2}\right)^{-\frac{1}{2}}$

<u>a new invariant</u> $E^2 - (cp)^2 = E_0^2$ <u>in all</u> inertial frames

and v is the vel. of S' as meas. by S

important exp: read p. 176 – 180 ; absorption and emission

later on: p 184 ff creation of particles

p 189 pair prod. in nucl.

p 191 – 194 scatt. of part.

p. 194 ff Compton eff. in part.

<u>putting it all together</u>

– use 4-vectors (p. 214 ff)

the 3 comp. of $\vec{p}$ transform like the 3 comp. of $\vec{r}$

tot E, a scalar, " " t

↳ quantity $E^2 - (cp)^2$ is an invariant – same in all inertial frames

$= E_0^2$

- Newtonian thinking: p, E represent different prop. of a body
- Relativistic ": the distinctions are blurred b/c

p and E mix like x and t mix under LT

(there is a third such mixing: $\vec{E}, \vec{H}$)

this mixing is the deeper reason for giving up the idea of space and time being separate prop.: think of single 4 dim space-time structure (already used in space-time diagrams) related hw problem! mom. cons. too | here! for eq. moves

<u>cons. of mom</u> (dep. on obs. frame! choose the close one)

$$p_{i,sys} = p_{f,sys}$$

$$\frac{m_0 u_A'}{\sqrt{1-\frac{u_A'^2}{c^2}}} + \frac{m_0 u_B'}{\sqrt{1-\frac{u_B'^2}{c^2}}} \stackrel{!}{=} \frac{M_0 u_C'}{\sqrt{1-\frac{u_C'^2}{c^2}}}$$

SIS
Serm. 8.7 + 8.8

<u>E cons.</u> $E_{sys,i} = E_{sys,f}$

$$\frac{m_0 c^2}{\sqrt{1-\frac{u_A'^2}{c^2}}} + \underbrace{\frac{m_0 c^2}{\sqrt{1-\frac{u_B'^2}{c^2}}}}_{\text{what happens here, if } u_B'=0?} \stackrel{!}{=} \frac{M_0 c^2}{\sqrt{1-\frac{u_C'^2}{c^2}}}$$

→ 2 eqn for 2 unknowns (M_0, u_C')

– <u>Lorentz Tr. for E and p</u>

particle vel. in S : u
" S' : u'

$$u_x' = \frac{u_x - v}{1-\frac{v u_x}{c^2}} \qquad u_y' = \frac{\frac{u_y}{\gamma}}{1-\frac{v u_x}{c^2}}$$

<u>in S</u>

$$\left.\begin{aligned} E &= \gamma_u m_0 c^2 \\ p_x &= \gamma_u m_0 u_x \\ p_y &= \gamma_u m_0 u_y \end{aligned}\right\} \text{ where } \gamma_u = \frac{1}{\sqrt{1-\frac{u^2}{c^2}}}$$

<u>in S'</u> same, but with u' and $\gamma_{u'} = \frac{1}{\sqrt{1-\frac{u'^2}{c^2}}}$

to relate the expressions in S and S', express $\gamma_{u'}$ in terms of meas. in S (and vice versa) while the mass of the bullet as measured by O, since $u_x' = v$, is

$$m = \frac{m_0}{\sqrt{1 - \dfrac{u^2}{c^2}}} = \frac{m_0}{\sqrt{1 - \dfrac{u_x^2 + u_y^2}{c^2}}} = \frac{m_0}{\sqrt{1 - \dfrac{v^2 + u_y^2}{c^2}}}$$

If we now apply the Lorentz transformation to the quantity inside the last square root, we find

$$1 - \frac{v^2}{c^2} - \frac{u_y^2}{c^2} = 1 - \frac{v^2}{c^2} - \frac{1}{c^2}\left(u_y'\sqrt{1 - \frac{v^2}{c^2}}\right)^2 = \left(1 - \frac{v^2}{c^2}\right)\left(1 - \frac{u_y'^2}{c^2}\right)$$

so that

$$m = \frac{m_0}{\sqrt{1 - (v^2/c^2)}\sqrt{1 - (u_y'^2/c^2)}} = \frac{m'}{\sqrt{1 - (v^2/c^2)}}$$

Hence

$$p_y = m u_y' \sqrt{1 - (v^2/c^2)} = \left(\frac{m'}{\sqrt{1 - (v^2/c^2)}}\right) u_y' \sqrt{1 - (v^2/c^2)} = m' u_y' = p_y'$$

8.2. From the rest masses listed in the Appendix calculate the rest energy of an electron in joules and electron-volts.

Ans. We have $E_0 = m_0 c^2 = (9.109 \times 10^{-31}\,\text{kg})(2.998 \times 10^8\,\text{m/s})^2 = 8.187 \times 10^{-14}\,\text{J}$, and

$$(8.187 \times 10^{-14}\,\text{J})\left(\frac{1\,\text{eV}}{1.602 \times 10^{-19}\,\text{J}}\right)\left(\frac{1\,\text{MeV}}{10^6\,\text{eV}}\right) = 0.511\,\text{MeV}$$

S13

8.3. A body at rest spontaneously breaks up into two parts which move in opposite directions. The parts have rest masses of 3 kg and 5.33 kg and respective speeds of $0.8c$ and $0.6c$. Find the rest mass of the original body.

Ans. Since $E_{\text{initial}} = E_{\text{final}}$,

$$m_0 c^2 = \frac{m_{01} c^2}{\sqrt{1 - (v_1^2/c^2)}} + \frac{m_{02} c^2}{\sqrt{1 - (v_2^2/c^2)}} = \frac{(3\,\text{kg})c^2}{\sqrt{1 - (0.8)^2}} + \frac{(5.33\,\text{kg})c^2}{\sqrt{1 - (0.6)^2}}$$

$$m_0 = 11.66\,\text{kg}$$

Observe that rest mass is not conserved (see also Problem 8.26).

S134

8.4. What is the speed of an electron that is accelerated through a potential difference of 10^5 V?

Ans. Since $K = e\,\Delta V = 10^5\,\text{eV} = 0.1\,\text{MeV}$, we have

$$0.1\,\text{MeV} = K = \frac{m_0 c^2}{\sqrt{1 - (v^2/c^2)}} - m_0 c^2$$

Substituting $m_0 c^2 = 0.511\,\text{MeV}$ (Problem 8.2) and solving, we find $v = 0.548c$.

8.5. Calculate the momentum of 1 MeV electron.

Ans.

$$E^2 = (pc)^2 + E_0^2$$
$$(1\,\text{MeV} + 0.511\,\text{MeV})^2 = (pc)^2 + (0.511\,\text{MeV})^2$$
$$p = 1.42\,\text{MeV}/c$$

But $u'_A = 0$.

$$\frac{m_0 \dfrac{-2u}{1(u^2/c^2)}}{\sqrt{1-\left[\dfrac{2u/c}{1+(u^2/c^2)}\right]^2}} = \frac{M_0(-u)}{\sqrt{1-(u^2/c^2)}}$$

$$M_0 = \frac{2m_0}{\sqrt{1-(u^2/c^2)}}$$

in agreement with the value found from energy considerations by observer O (Problem 8.26).

S13I4 m_0 @ 0.8c, $6m_0$

8.28. A particle of rest mass m_0 moving with a speed of $0.8c$ makes a completely inelastic collision with a particle of rest mass $3m_0$ that is initially at rest. What is the rest mass of the resulting single body?

HW Sug

Ans. From $p_{\text{final}} = p_{\text{initial}}$

$$\frac{M_0 u_f}{\sqrt{1-(u_f^2/c^2)}} = \frac{m_0 u_i}{\sqrt{1-(u_i^2/c^2)}} = \frac{m_0(0.8c)}{\sqrt{1-(0.8)^2}} = \tfrac{4}{3}m_0 c$$

From $E_{\text{final}} = E_{\text{initial}}$,

$$\frac{M_0 c^2}{\sqrt{1-(u_f^2/c^2)}} = \frac{m_0 c^2}{\sqrt{1-(u_i^2/c^2)}} + 3m_0c^2 = \frac{m_0 c^2}{\sqrt{1-(0.8)^2}} + 3m_0c^2 = 4.67 m_0 c^2$$

Solving these two equations simultaneously, we get

$$u_f = 0.286c \qquad M_0 = 4.47 m_0$$

8.29. Find the increase in mass of 100 kg of copper if its temperature is increased 100 °C. (For copper the specific heat is $\mathscr{C} = 93\ \text{cal/kg}\cdot{}^\circ\text{C}$.)

Ans. The energy added to the copper block is

$$\Delta E = m\mathscr{C}(\Delta T) = (100\ \text{kg})(93\ \text{cal/kg}\cdot{}^\circ\text{C})(100\ {}^\circ\text{C})(4.184\ \text{J/cal}) = 39\times10^5\ \text{J}$$

If this energy appears as an increase in mass, then

$$\Delta m = \frac{\Delta E}{c^2} = \frac{39\times10^5\ \text{J}}{(3\times10^8\ \text{m/s})^2} = 4.33\times10^{-11}\ \text{kg}$$

This increase is far too small to be measured.

Supplementary Problems

8.30. From the rest masses given in the Appendix calculate the rest mass of one atomic mass unit in joules. *Ans.* 1.49×10^{-10} J

8.31. Calculate the kinetic energy of a proton whose velocity is $0.8c$. *Ans.* 625.5 MeV

8.32. Calculate the momentum of a proton whose kinetic energy is 200 MeV. *Ans.* 644.5 MeV/c

8.33. Calculate the kinetic energy of a neutron whose momentum is 200 MeV/c *Ans.* 21.0 MeV

$\hookrightarrow \quad \frac{d\vec{p}}{dv} = \frac{m_0}{\left(1-\frac{v^2}{c^2}\right)^{3/2}}$

$\rightarrow \quad dp = \text{"} \cdot dv$

$$K = \int \frac{dp}{dt}\, dx = \int dp\, \frac{dx}{dt} = \int v\, dp$$

$$= \int_0^{v_1} v \left(\frac{m_0}{\left(1-\frac{v^2}{c^2}\right)^{3/2}}\right) dv$$

$$= \left(m_0 \frac{c^2}{\left(1-\frac{v^2}{c^2}\right)^{1/2}} \right|_0^{v_1}$$

$$= \frac{m_0 c^2}{\sqrt{1-\frac{v_1^2}{c^2}}} - \frac{m_0 c^2}{\sqrt{1-0}}$$

$$\equiv \gamma m_0 c^2 - m_0 c^2 \equiv \underbrace{mc^2}_{E} - \underbrace{m_0 c^2}_{\text{rest } E_0} \equiv K$$

- at any speed: $K = \Delta m c^2$

for $v \ll c$, $\gamma \approx 1 \rightarrow m \approx m_0$ and $K \cong \frac{1}{2} m_0 v^2$

Schaum 8.23

- find max. speed for $\frac{1}{2} m_0 v^2$ w/ error $\leq 0.5\%$

condition at max speed $\quad \frac{K - \frac{1}{2} m_0 v^2}{K} = 0.005$

or $\quad K = \frac{\frac{1}{2} m_0 v^2}{0.995}$

$$K = E - E_0 = \gamma m_0 c^2 - m_0 c^2 = m_0 c^2 \cdot \left[\left(1 - \frac{v^2}{c^2}\right)^{-\frac{1}{2}} - 1 \right]$$

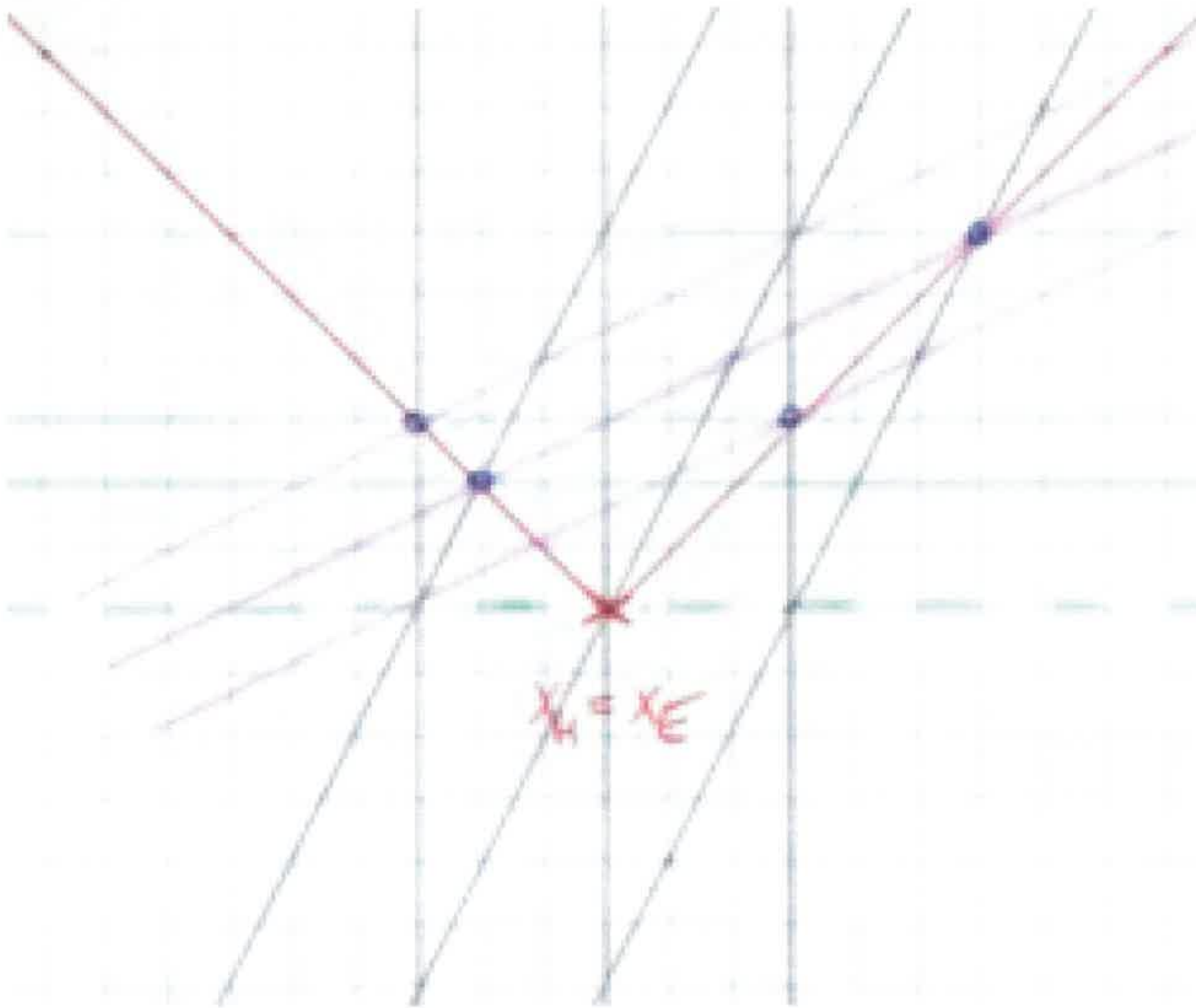
bin. exp.

$$= m_0 c^2 \left[\left(1 + \frac{1}{2}\frac{v^2}{c^2} + \frac{3}{8}\frac{v^4}{c^4} + \dots \right) - 1 \right]$$

$$= \frac{1}{2} m_0 v^2 + \frac{3}{8} m_0 v^2 \left(\frac{v^2}{c^2}\right) + \dots$$

$$\hookrightarrow \frac{\frac{1}{2} m_0 v^2}{0.995} \approx \frac{1}{2} m_0 v^2 + \frac{3}{8} m_0 v^2 \left(\frac{v^2}{c^2}\right) \Rightarrow v \approx 0.082c$$

Spare copy of the drawing in problem 6:



all signals. Even if the "sig-
hock wave) travels along the
ıat signal has 20 feet to travel
eet, before reaching the back
ıuld be a dead heat if v were
pole *more* than just gets in.
whose length was as little as
ɛt in motion: the garage front
feet against the shock wave's
e ratio 17.3 to 20, i.e., .865

ɔral to this story: whatever
ıing in any one frame, must
e true when viewed from any
ıysical laws we are using are
be an explanation of the re-
ɔugh it may be quite a differ-
he first frame. Recall Ein-
experienced by an electric
magnetic field is equivalent
est frame of the charge.
e, a "rigid" rod of rest length
eter $\frac{1}{2}L$ on a smooth table.

Indeed, "rigidity" is an impossible requirement in relativity. A rod pushed at one end cannot start to move at the other end at once, since that would allow us to send a "signal" at infinite speed. A body being acted on by various forces at various points simultaneously will yield to *each* force initially as though all the others were absent; for at each point it takes a finite time for the effects of the other forces to arrive. Hence, in relativity, a body has infinitely many degrees of freedom. Again, a body which appears rigid in one inertial frame need not appear rigid in another: the rod falling into the hole may keep its precise shape in the frame of the table (at least, until it hits), while in its own original rest frame it bends. For reasons similar to those preventing the existence of rigid bodies in relativity, incompressible fluids are equally impossible.

⇒ rigidity impos[illegible] rod pushed, can[illegible] start all at once [illegible] [illegible] → info [illegible]

[illegible] a body has ∞ DoF

similar w/ incompress. fluids

Time Dilation 34

Let us again consider two inertial frames S and S' in standard configuration. Let a standard clock be fixed in S' and consider two events at that clock at which it indicates times t_1' and t_2' differing by T_0. We inquire what time inter-

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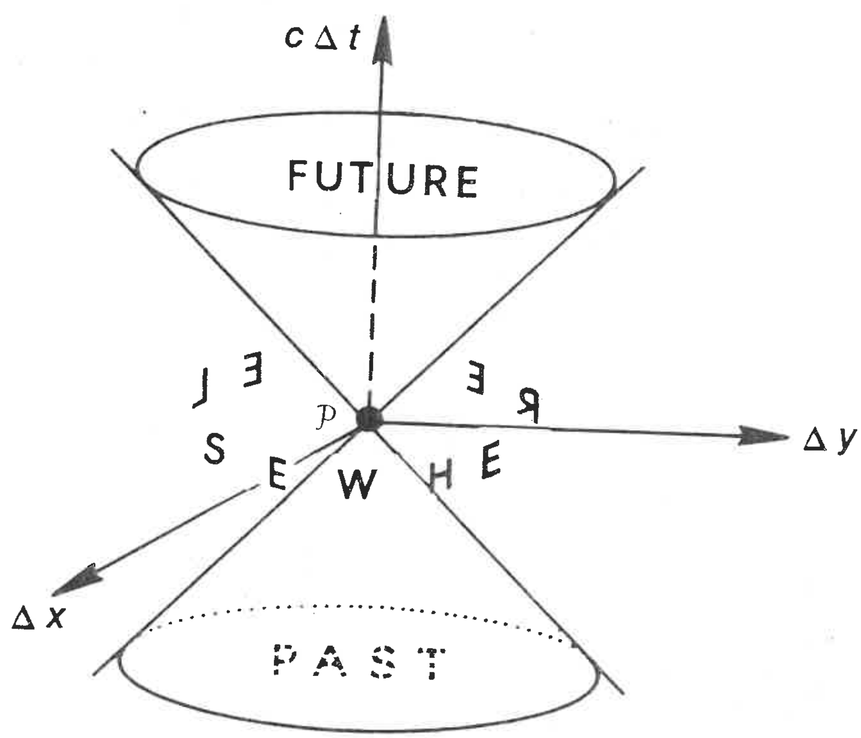


FIGURE 10.

$\mathcal{P}$ have positive squared norm lie inside the cone, i.e., where the time axis is, whereas those with negative squared norm lie outside

where it can only be discovered by ...
cause of length contraction resides in the preferred ether frame.

The relativistic length contraction is no "illusion": it is real in every way. Though no direct experimental verification has yet been attempted, there is no question that in principle it could be done. Consider the admittedly unrealistic situation of a man carrying horizontally a 20-foot pole and wanting to get it into a 10-foot garage. He will run at speed $v = .866c$ to make $\gamma = 2$, so that his pole contracts to 10 feet. It will be well to insist on having a sufficiently massive block of concrete at the back of the garage, so that there is no question of whether the pole finally stops in the inertial frame of the garage, or vice versa. Thus the man runs with his (now contracted) pole into the garage and a friend quickly closes the door. In principle we do not doubt the feasibility of this experiment, i.e., the reality of length contraction. When the pole stops in the rest frame of the garage, it is, in fact, being "rotated in spacetime" and will tend to assume, if it can, its original length relative to the garage. Thus, if it survived the impact, it must now either bend, or burst the door.

At this point a "paradox" might occur to the reader: what about the symmetry of the phenomenon? Relative to the runner, won't the garage be only 5 feet long? Yes, indeed. Then how can the 20-foot pole get into the 5-foot garage? Very well, let us consider what happens in the rest frame of the pole. The open garage now comes towards the stationary pole. Because of the concrete wall, it keeps on going even after the impact, taking the front end of the pole with it. But the back end of the pole is still at rest: it cannot yet "know" that the front end has been struck, because of the finite speed of propagation of *all* signals. Even if the "signal" (in this case the elastic shock wave) travels along the pole with the speed of light, that signal has 20 feet to travel against the garage front's 15 feet, before reaching the back end of the pole. This race would be a dead heat if v were $.75c$. But v is $.866c$! So the pole *more* than just gets in. (It could even get into a garage whose length was as little as 5.4 feet at rest and thus 2.7 feet in motion: the garage front would then have to travel 17.3 feet against the shock wave's 20 feet, requiring speeds in the ratio 17.3 to 20, i.e., .865 to 1 for a dead heat.)

There is one important moral to this story: whatever result we get by correct reasoning in any one frame, must be true; in particular, it must be true when viewed from any other frame. As long as the physical laws we are using are Lorentz-invariant, there *must* be an explanation of the result in every other frame, although it may be quite a different explanation from that in the first frame. Recall Einstein's "hunch" that the force experienced by an electric charge when moving through a magnetic field is equivalent to a simple electric force in the rest frame of the charge.

Consider, as another example, a "rigid" rod of rest length L sliding over a hole of diameter $\frac{1}{2}L$ on a smooth table. When its Lorentz factor is 10, the length of the rod is $\frac{1}{5}$ of the diameter of the hole, and in passing over the hole, it will fall into it under the action of gravity* (at least slightly: enough to be stopped). This *must* be true also in the frame of the rod—in which however, the diameter of the hole is only $\frac{1}{20}L$! The only way in which this can happen is that the front of the "rigid" rod *bends* into the hole. Moreover, even after the front end strikes the far edge of the hole, the back end keeps coming in (not yet "knowing" that the front end has been stopped), as it must, since it does so in the first description.

*We are here violating our resolve to work in *strict* inertial frames only! The conscientious reader may replace the force of gravity acting down the hole by a sandblast from the top—the result will be the same. For a full discussion of this paradox, see W. Rindler, *Am. J. Phys.* **29**, 365 (1961).

S17 HW1 sol

1) a) Schaum 3.2, 3.6 ✓

orig! $v = 0.8c$, $x = 50\,m$, $t = 2\cdot10^{-7}\,s$; t'?

$$t' = \frac{t - \frac{vx}{c^2}}{\sqrt{1-0.8^2}} = \frac{2\cdot10^{-7} - \frac{0.8\cdot50}{3\cdot10^8}}{\sqrt{0.36}} = 1.1\cdot10^{-7}$$

$$\text{hw1} \cong \frac{1/6.6E\text{-}8}{\sqrt{1-0.85^2}} \cong \cancel{1.266}\ 1.07 \cdot 10^{-7}$$

$$\cong \frac{1/}{\sqrt{1-0.96^2}} \cong 2.\cancel{381}\ 1.43 \cdot 10^{-7}$$

$$\cong \frac{1/}{\sqrt{1-0.9999^2}} \cong \cancel{47.14}\ 23.6 \cdot 10^{-7}$$

(A) time and space mix when seen from another inertial obs pov

our t' scales w/ γ, has $\gamma \cong \cancel{1.77}\ 1.9 < 70$

|| 12.5 < 7.5, 5

b) c, c

Speed of light is always c

12.5

2) Schaum 5.1, 5.6 $v = 0.8c, 0.92c, 0.99c$

$$\gamma \Delta t_0 = \frac{\Delta t}{\gamma} ; \frac{2\cdot10^{-6}s}{\sqrt{1-0.8^2}} \simeq \frac{2\cdot10^{-6}}{0.6} = 3.3\cdot10^{-6}, \quad \frac{2\cdot10^{-6}}{\sqrt{1-0.92^2}} \cong 5.1\cdot10^{-6} \ \}\sim2.55, \quad \frac{2\cdot10^{-6}}{\sqrt{1-0.99^2}} \sim 1.42\cdot10^{-5} \ \}\sim7.09$$

$$d = v\Delta t = 0.8c\cdot3.3\cdot10^{-6} \sim \cancel{1211}\ 792\ m$$

$$= 0.92c\cdot5.1\cdot10^{-6} \sim 1407.6\,m$$

$$= 0.99c\cdot14.2\cdot10^{-6} \sim 4217.4$$

(A) In moving frames half-life is longer, acc. to γ, the particles travel farther b/c the distance appears larger compared to them / b/c they live longer

15

3) normal wave fronts 30

spectral change 45

explained 25 sound/travel

4) 30 a) $L = \frac{L_0}{\gamma} = \frac{L_0}{2} \Rightarrow \gamma = 2$, $\frac{1}{\sqrt{1-\frac{v^2}{c^2}}} = 2$ or $v = 0.866c$

30 b) paradox is pole rest frame → garage is even smaller ($\gamma=2$) → 5 ft long!

→ pole must bend when it comes to rest

30 c) new clock look:

backhand pole doesn't know of collision at front end until that info has been transmitted to it, at most w/ c!

dead head for $v = 0.75c$

10 d) under a) pole fits in car [illegible]

$$L = \frac{L_0}{\sqrt{1-0.866^2}} = \cancel{X} \_ 10\,ft$$

$v = 0.75^2 \Rightarrow \sim 1.511 \cdot 10\,ft = 15.1\,ft$ to suffice

better

$$m = \frac{m_0}{\sqrt{1-(u^2/c^2)}}$$

when m_0, the *rest mass*, is the mass of the body measured when it is at rest with respect to the observe Problem 8.1.

8.3 NEWTON'S SECOND LAW IN RELATIVITY

The classical expression of Newton's second law is that the net force on a body is equal to the r change of the body's momentum. To include relativistic effects, allowance must be made for the fa the mass of a body varies with its velocity. Thus the relativistic generalization of Newton's second

$$\mathbf{F} = \frac{d\mathbf{p}}{dt} = \frac{d}{dt}\left[\frac{m_0\mathbf{u}}{\sqrt{1-(u^2/c^2)}}\right] = \frac{d}{dt}(m\mathbf{u})$$

8.4 MASS AND ENERGY RELATIONSHIP: $E = mc^2$

In relativistic mechanics, as in classical mechanics, the kinetic energy, K, of a body is equal work done by an external force in increasing the speed of the body from zero to some value u, i.e

$$K = \int_{u=0}^{u=u} \mathbf{F} \cdot d\mathbf{s}$$

Using Newton's second law, $\mathbf{F} = d(m\mathbf{u})/dt$, one finds (Problem 8.21) that this expression reduces

$$K = mc^2 - m_0c^2$$

The kinetic energy, K, represents the difference between the *total energy*, E, of the moving partic the *rest energy*, E_0, of the particle when at rest, so that

$$E - E_0 = mc^2 - m_0c^2$$

If the rest energy is chosen so that $E_0 = m_0c^2$, we obtain Einstein's famous relation

$$E = mc^2$$

which shows the equivalence of mass and energy. Thus, even when a body is at rest it still has an e content given by $E_0 = m_0c^2$, so that in principle a massive body can be completely converted into an more familiar, form of energy.

8.5 MOMENTUM AND ENERGY RELATIONSHIP

Since momentum is conserved, but not velocity, it is often useful to express the energy of a b terms of its momentum rather than its velocity. To this end, if the expression

$$m = \frac{m_0}{\sqrt{1-(u^2/c^2)}}$$

is squared and both sides are multiplied by $c^4[1-(u^2/c^2)]$, one obtains

$$m^2c^4 - m^2u^2c^2 = m_0^{\,2}c^4$$

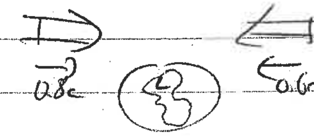
readings: p. 149–154 chapter started; 167–169

S131 video ~~7~~6 ≈ 52% Harry Doolittle Dodge ~41

D7 — solving a 3 party vel. problem

Schaum 7.3

1 rocket A travels to +x w/ 0.8c
B to −x' w/ 0.6c
rel. to earth along x−x' axis



Q What is v_A meas. by B?

Let S be earth, S' be B, particle be A

$$\rightarrow \quad u_x' = \frac{u_x - v}{1 - \frac{v u_x}{c^2}} = \frac{0.8c - (-0.6c)}{\left(1 - \frac{(-0.6c)\,0.8c}{c^2}\right)} = \frac{1.4c}{1+0.48} \approx 0.946c$$

Q whiteboard task: solve for S ≙ A, S' ≙ B, P ≙ earth

$$(A)\quad 0.6c = u_x' = \frac{-0.8c - v}{1 - \frac{v\,(-0.8c)}{c^2}} \implies v = -0.946c$$

which agrees (note signs accordingly)

wk 7 ≈ ~~48~~ 52% carry over wk8 ≤ wk6

S131 wk8 ≈ wk6 ... HtA twin paradox

Schaum 7.5 — P at 0.8c angle 30° to x as det. by S
vel. P as seen by S' : −0.6c along x−x'

S: $u_x = 0.8c \cdot \cos 30° \cong 0.693c$

$u_y = 0.8c \cdot \sin 30° = 0.4c$

$$S': \quad u_x' = \frac{0.693c - (-0.6c)}{1 - \frac{(-0.6c)}{c^2}\cdot 0.693c} \approx 0.913c$$

$$u_y' = \frac{0.4c \cdot \sqrt{1-0.6^2}}{1 - \frac{(-0.6c)\cdot 0.693c}{c^2}} \approx 0.226c$$

$$u' = \sqrt{u_x'^2 + u_y'^2} \approx 0.941c$$

• proper length + proper time : as meas. in rest frame : l_0
same clock, same location

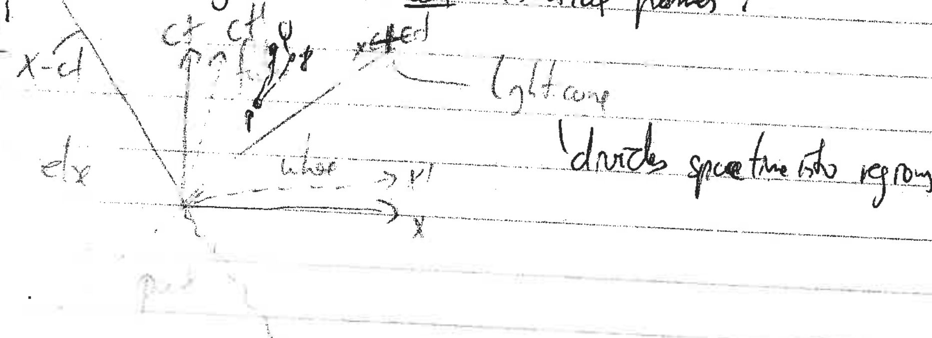
the problem of meas. time simultaneously (p 107 – 109, won't expand in lecture)
also 111 – 117 'mode strictly'

– space time intervals

$$s^2 = (ct')^2 - (x')^2 - (y')^2 - (z')^2$$ is an invariant under LT

i.e. agreed on in all inertial frames!

(1-dim)



$(\Delta s)^2$, interval $\sqrt{(\Delta s)^2}$ b/w 2 events

>0 dep. on $c\Delta t > \Delta x$

<0 $\qquad <$

$=0$ $\qquad =$

Light line connects point events b/w which $\sqrt{(\Delta s)^2} = 0$

if >0 can transform to ct' axis $\parallel \overline{P\text{-}Q}$
meaning P and Q occur at the same place in S'
sep. by time only = time-like interval

if <0 can transform P, Q to simult. events at diff. places
'space-like'
to connect to such events by space-like interval req. $v > c$
(causality!)

terms higher than the first order are ignored. But the [illegible] result has a special symmetry that the previous result lacks.

The most dramatic manifestation of this form of the Doppler effect, for relative motion of source and observer along the line joining them, is the famous *red shift* of distant galaxies. The spectrum of a complete galaxy, being a synthesis from all the different radiating objects in it, is close to being a continuous smear. But astrophysicists are able to distinguish a few very prominent dark lines—i.e., narrow gaps in the otherwise continuous spectrum—produced as the escaping radiation passes through cooler gases or vapors and undergoes selective absorption before leaving the galaxy. Two such lines in particular, the so-called H and K absorption lines of ionized calcium, can be distinguished even when all other characteristic features have been lost. (Ionized calcium atoms present an extraordinarily high cross section for light of these particular wavelengths.) They lie near the extreme violet end of the spectrum for a stationary source, but have been observed drastically shifted toward the region of longer wavelengths for certain very distant galaxies. A selection of galactic spectra with progressively increasing Doppler shifts is shown on pages 140 and 141. In each photograph the spectrum of the galaxy appears as a rather ill-defined horizontal streak, interrupted by the H and K absorption gaps. A line spectrum from a laboratory source is recorded above and below each galactic spectrum for purposes of comparison. In the last photograph, for example, the H and K absorption lines are found to be shifted to a wavelength of about 4750 Å, as compared to about 3940 Å for a stationary source. This is a very large increase of wavelength—nearly 25%. Using Eq. (5–14) we have

$$\lambda' = c\tau' = \left(\frac{1+\beta}{1-\beta}\right)^{1/2} c\tau = \left(\frac{1+\beta}{1-\beta}\right)^{1/2} \lambda$$

Therefore,

$$\beta = \frac{(\lambda'/\lambda)^2 - 1}{(\lambda'/\lambda)^2 + 1} \tag{5–16}$$

Putting $\lambda'/\lambda \approx 4750/3940 = 1.21$, we find

$$\beta = \frac{0.46}{2.46} \approx 0.2$$

Therefore,

$$v = 0.2c \approx 6 \times 10^7 \text{ m/sec}$$

Edwin Hubble, who did so much to advance the study of the depths of space outside our own galaxy, established the existence of a linear relation between the velocity of recession and the distance for remote galaxies. Part (B) of the illustration on pages 140 and 141 shows the data of Part (A) plotted so as to exhibit this spectacular relationship, which is known as *Hubble's law*. The determination of the galactic distances is much less direct and definite than the measurement of the Doppler shifts and ultimately involves such profound questions as whether space on the grand scale is describable by Euclidean geometry. But this is beyond the scope of our immediate topic, and if you want further details you should hunt them up for yourself in a book on astronomy.[1] It must suffice here to lay the chief emphasis on the Doppler shifts themselves.

MORE ABOUT DOPPLER EFFECTS

As the first Sputnik sped around the earth it emitted a radio-frequency signal that was picked up by many tracking stations. Figure 5–6 shows one example of such observations.[2] When the satellite is very far away, approaching or receding, it gives maximum or minimum Doppler frequency-shifts corresponding to the one-dimensional problem we have been discussing. But the switch from augmented to diminished frequency is not instantaneous, as it would be if the moving object passed right through the position of the observer. Instead, it follows a smooth curve that can yield information about the altitude as well as the speed of the moving source. Let us analyze a situation of this kind.

In Fig. 5–7 we show the path of a satellite passing at a height h above an observation point O. We shall regard the path as being an approximation to a horizontal straight line, so that the satellite's position can be described by the following equations:

$$x = vt \qquad y = h$$

The time $t = 0$ marks the instant when the satellite is directly overhead.

We suppose the satellite to have a transmitter that sends out

[1] See, for example, F. Hoyle, *Frontiers of Astronomy*, Harper, New York, 1955, or his beautiful, more recent book, *Astronomy*, Doubleday, New York, 1962.

[2] R. R. Brown et al. (M.I.T. Lincoln Lab.), *Proc. IRE* **45**, 1552 (1957).

– Relativistic Kinematics (ch. 5)

b/w frames only one uniquely diff direction involved

↳ that of the rel. motion of the in. frames, $S, S' \to v$

any displacement → compon. ∥, ⊥ to it

Relation b/w comp. of vel meas. in S and in S', w/ v b/w frames?

1-d LT:
$$x = \gamma(x' + vt') \qquad t = \gamma\left(t' + \frac{vx'}{c^2}\right)$$
$$y = y'$$

$\Rightarrow$ ~~$dx = \gamma($~~ suppose meas. u_x', u_y' vel. in S'

$$dx = \gamma(u_x' + v)\,dt'$$
$$dy = u_y'\,dt'$$
$$dt = \gamma\left(1 + \frac{vu_x'}{c^2}\right)dt'$$

then
$$u_x = \frac{dx}{dt} = \frac{u_x' + v}{1 + \frac{vu_x'}{c^2}} \qquad u_x' = \frac{u_x - v}{1 - \frac{vu_x}{c^2}}$$
$$u_y = \frac{dy}{dt} = \frac{u_y'/\gamma}{1 + \frac{vu_x'}{c^2}} \qquad u_y' = \frac{u_y/\gamma}{1 - \frac{vu_x}{c^2}}$$

if at least one of the velocities $\approx c$

→ result diff from Galilean kinematics of vector add.

e.g. $u_x' = c$, $v = c$: $\dfrac{c + c}{1 + 1} = \dfrac{2c}{2} = c$

$u_x' = 0.9c$, $v = 0.8c$: $\dfrac{1.7c}{1 + 0.72} = \dfrac{1.7}{1.72}c = 0.988c$

added before 'fam'

accelerated motion

This is a misconception. We can meaningfully discuss a displacement and all its time derivatives within the context of the Lorentz transformations.

Just as with the velocity transformations, it is very advantageous to distinguish between longitudinal and transverse accelerations with respect to the direction of relative motion of two inertial frames. We have

$$u_x = \frac{u_x' + v}{1 + vu_x'/c^2} \tag{5–2}$$

$$u_y = \frac{u_y'/\gamma}{1 + vu_x'/c^2} \tag{5–3}$$

LT

$$t = \gamma(t' + vx'/c^2) \tag{5–1}$$

Therefore,

$$du_x = \frac{du_x'}{1 + vu_x'/c^2} - \left[\frac{u_x' + v}{(1 + vu_x'/c^2)^2}\,\frac{v\,du_x'}{c^2}\right]$$

(Remember that v = constant for the purpose of this calculation.) Collecting the terms together, we have

$$du_x = \frac{(1 - v^2/c^2)\,du_x'}{(1 + vu_x'/c^2)^2} = \frac{du_x'}{\gamma^2(1 + vu_x'/c^2)^2}$$

Also, from Eq. (5–1),

$$dt = \gamma(dt' + v\,dx'/c^2) = \gamma(1 + vu_x'/c^2)\,dt'$$

Therefore,

$$a_x = \frac{du_x}{dt} = \frac{du_x'/dt'}{\gamma^3(1 + vu_x'/c^2)^3}$$

i.e.,

$$a_x = \frac{a_x'}{\gamma^3(1 + vu_x'/c^2)^3} \tag{5–24}$$

Similarly, from Eq. (5–3) we have

$$du_y = \frac{du_y'}{\gamma(1 + vu_x'/c^2)} - \frac{u_y'}{\gamma(1 + vu_x'/c^2)^2}\,\frac{v\,du_x'}{c^2}$$

$$a_y = \frac{du_y}{dt} = \frac{du_y'/dt'}{\gamma^2\left(1+\frac{vu_x'}{c^2}\right)^2} - \frac{u_y'}{\gamma^4\left(1+\frac{vu_x'}{c^2}\right)^3} \cdot \frac{v\,\frac{du_x'}{dt'}}{c^2}$$

$$a_y = \frac{a_y'}{\gamma^2(1 + vu_x'/c^2)^2} - \frac{(vu_y'/c^2)a_x'}{\gamma^2(1 + vu_x'/c^2)^3} \qquad (5\text{–}25)$$

Only if $u_y' = 0$ or $a_x' = 0$ (or both) does the expression for a_y become relatively simple. But for these cases we have

Special case ($u_y' = 0$ or $a_x' = 0$):

$$a_y = \frac{a_y'}{\gamma^2(1 + vu_x'/c^2)^2} \qquad (5\text{–}26)$$

It may be noted that if a body is instantaneously at rest in S' ($u_x' = u_y' = 0$), its acceleration components as measured in S are diminished by the factors γ^3 for the x direction and γ^2 for the y direction, as compared with the accelerations measured in the instantaneous rest frame S'.

The main lesson to be learned from the above calculations is that acceleration is a quantity of limited and questionable value in special relativity. Not only is it not an invariant, but the expressions for it are in general cumbersome, and moreover its different components transform in different ways. Certainly the proud position that it holds in Newtonian dynamics has no counterpart here.

THE TWINS

Of all the supposed paradoxes engendered by relativity theory, the *twin paradox* (or clock paradox) is the most famous and has been the most controversial. It asserts that if one clock remains at rest in an inertial frame, and another, initially agreeing with it, is taken off on any sort of path and finally brought back to its starting point, the second clock will have lost time as compared with the first. In today's parlance, the astronaut will end up by becoming younger than his twin brother. This result, which was stated by Einstein in his very first relativity paper (1905), became the subject of a raging controversy in the physics literature during the years 1957–1959, after preliminary skirmishes dating back

expres
ess Kinematic state as sgl 4-vector (x, y, z, ict)
of length $s\sqrt{-1}$ in any frame!

$$-s^2 i = x^2 + y^2 + z^2 + (ict)^2$$

$i = \sqrt{-1}$, so $i^2 = -1$

n dynamic state as sgl 4-vector $(\underbrace{p_x, p_y, p_z}_{\text{kin } E}, \frac{iE}{c})$

of length $\frac{iE_0}{c}$ in any frame

forces
ces in Relativity : in relativist accel is not an invariant
$\vec{F}_0$ in general meas. + well defined ; in rest frame F_{0x} applied parallel to v causes a_{0x}
is force that similar : rest frame $- \vec{F}_0$

$$\vec{F} = \frac{d\vec{p}}{dt} = \frac{d}{dt}(m\vec{v}), \text{ where } m = \gamma m_0$$

$$F_x = \frac{dp_x}{dt} = \overbrace{\gamma m_0 a_x}^{m_0 \frac{dv}{dt}\gamma + m_0 v \frac{d}{dt}\gamma} \neq m_0 a_{0x} = \underbrace{F_{0x}}_{\text{rest force}}$$

collects terms

i.e. however: $a_x = \frac{1}{\gamma^3} a_{0x}$

$F_y = \gamma m_0 a_y$ (force appl. perp. to $m\vec{v}$)
$\rightarrow a_y = \frac{1}{\gamma^2} a_{0y}$ and $F_y = \frac{1}{\gamma} F_{0y}$ sle F_x loses parallel $\rightarrow$ no chg of magn. just directn

$$\frac{F_y}{F_x} = \frac{m_0 a_y \gamma}{\gamma^3 m_0 a_x} = \frac{1}{\gamma^2}\frac{a_y}{a_x}$$ in moving frame

$$F_x = \gamma^3 m_0 a_x = \gamma^3 m_0 \frac{a_{0x}}{\gamma^3} = m_0 a_{0x} = F_{0x}$$

spite change of meas. of m and a in two frames, F_x remains the same

is gen $\vec{F} \nparallel \vec{a}$
only rest frame, $\gamma = 1$, $F \parallel a$ guar.

atoms: emission & abs. of photon

Absorption:

Stationary particle struck by a photon of energy Q (completely absorbed)

→ combined sys: M' recoils w/ v

Conservation of energy:

$$E = M_0c^2 + Q = M'c^2 \qquad (1)$$

Conservation of linear momentum:

$$p = Q/c = M'v \qquad (2)$$

Therefore,

$$M' = M_0 + Q/c^2$$

$$\frac{(2)}{(1)} \qquad \beta = v/c = Q/(M_0c^2 + Q) \tag{6–21}$$

Note that, for $Q \ll M_0c^2$, we have simply $\beta \approx Q/M_0c^2$ —corresponding to a Newtonian type of calculation in which a body of invariable mass M_0 is given an impulse of magnitude Q/c by the photon.

Emission

Consider a stationary atom of mass M_0 that emits a photon of energy Q. This is already more complicated than the previous example, because the emitting atom undergoes a recoil. Let the recoiling atom have mass M' (and rest mass M_0') and velocity v. Then

$$E = M_0c^2 = M'c^2 + Q = E' + Q$$

$$p = 0 = M'v - Q/c = p' - Q/c$$

i.e.,

$$E' = M_0c^2 - Q$$

$$cp' = Q$$

We will solve these equations for Q by taking advantage of the relation between E' and p' for the recoiling atom. Using Eq. (6–19), we have

$$\begin{aligned}(M_0'c^2)^2 &= (E')^2 - (cp')^2 \\ &= (M_0c^2 - Q)^2 - (Q)^2\end{aligned}$$

i.e.,

$$(M_0'c^2)^2 = (M_0c^2)^2 - 2M_0c^2Q \tag{6–22}$$

Therefore,

$$(M_0'c^2)^2 = (M_0c^2)^2 - 2M_0c^2Q_0 + Q_0^2 \tag{6–24}$$

Combining Eqs. (6–22) and (6–24), we get

$$Q = Q_0\left(1 - \frac{Q_0}{2M_0c^2}\right) \tag{6–25}$$

Since the photon energy is proportional to the frequency, the corresponding frequency is lowered and the wavelength increased. Only if the emitting atom could somehow be prevented from recoiling would the total energy release Q_0 be conferred on the photon.

These results have important physical implications, because they place restrictions on the ability of atoms and nuclei to reabsorb their own characteristic radiations. Any element when suitably stimulated (as in an electric-discharge tube) emits a characteristic line spectrum—for example, the Balmer series of hydrogen. These lines are very sharp; that is to say, each line represents an extremely small spread of wavelengths about some average. This sharpness is an expression of the fact that the emitting atoms themselves cannot exist in states with any arbitrary energy but are limited to a series of sharp energy levels. The emission of a photon corresponds to a certain decrease of energy (or mass) of an atom, as described by Eq. (6–23), when the atom falls from a state A to a state B. The photon, however, is cheated out of a small fraction of this energy by the atomic recoil. Thus, if such a photon encounters another similar atom which is in its lower state B and at rest, there is not enough energy to raise the atom back to state A (and the situation is exacerbated by the fact that the absorption process in turn involves a recoil). If atomic energy states were *perfectly* sharp, and if emitting and absorbing atoms were both initially stationary, a vapor would thus be transparent to its own radiation. Of course, the situation we have described is unrealistic on two counts. Atomic energy-levels are not perfectly sharp, and the atoms of a vapor have thermal motions that can, if the velocities are right, nullify the effects of recoil. It turns out, in fact, that the thermal motions completely mask the effect in the case of visible light. But with the much more energetic photons that are ejected from nuclei as γ rays the recoil effect is relatively much greater [note that ac-

p. 184

Creation of particles
Cons. of E = mc² — when E of mc² is about particle can 'condense'
In practice more E is needed
1) fundamental particle cons. laws beyond E, p: charge, spin, etc

exp. chg conserv

process to be discovered was the creation of an electron-positron pair from the energy of a γ-ray photon[1]:

$$\gamma \rightarrow e^- + e^+$$

Although, on energetic grounds alone, a γ ray of 0.51 MeV would suffice to provide the rest-mass energy of one electron, the only type of process that nature allows requires at least twice this amount.

Actually, although charge conservation applies invariably in these transmutations, it is by no means the only restriction. For example, one could envisage the creation of the constituents of a neutral hydrogen atom—one proton and one electron—using the energy of a single photon ($\geq$ 938 MeV). But this is not an observed process. It appears that many types of particles (including electrons, protons, and neutrons) cannot be created without calling into existence their so-called antiparticles—particles of the same rest mass, but with electric charge, magnetic moment, etc., of the opposite sign. The creation of a neutron, even though it is uncharged, does not occur without the simultaneous creation of an antineutron (differing from it in the sign of the magnetic moment).

2. The other reason that may step up the energy requirements for particle creation is a purely practical one. It arises from the fact that the creation process normally is made to take place by causing energetic collisions between preexisting particles. Thus, for example, positively charged π mesons (pions) can be made by bombarding a hydrogen target with high-energy protons:

$$P_1 + P_2 \rightarrow P + N + \pi^+$$

The colliding protons, P_1 and P_2, give rise to a proton, a neutron, and a pion, as indicated. (The π meson happens to be a particle that can be created singly, without an associated antiparticle.) Since a neutron and a proton have almost equal rest masses, the only new rest energy needed is that represented by the pion, about 140 MeV. But if the target proton P_2 is initially at rest and P_1 has a large momentum, a good deal of kinetic energy is locked up in motion of the system as a whole, and is unavailable for conversion into the rest mass of new particles.

It is clear that (2) is not a fundamental limitation in prin-

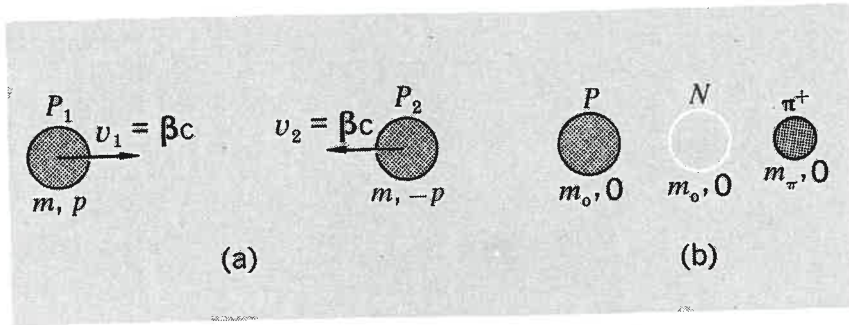


Fig. 6–5 (a) Two protons colliding with equal and opposite velocities in the zero-momentum frame. (b) The final state in this frame at the threshold for pion production, yielding a proton, a neutron, and a π^+ meson at rest.

ciple. If particles P_1 and P_2 could be made to collide with equal and opposite momenta, the amount of energy associated with the general motion of the system would be zero. All the kinetic energy of collision would then be available for particle creation. To produce colliding beams of particles traveling in opposite directions is technically a great deal harder than to have one beam striking a stationary target, but the payoff can be great, as we shall see.

Let us now consider in more detail some of these creation processes.

Pion production

Whether or not we have colliding beams, we can always *imagine* ourselves to be in a frame of reference where the total momentum is zero. Suppose we do this for two colliding protons, so that they have equal and opposite momenta, $\pm p$, and a total energy $2mc^2$ [Fig. 6–5(a)]. It is conceivable that in this zero-momentum frame we have a final state, as represented by Fig. 6–5(b), in which all particles are at rest. This will represent the most economical condition for particle creation, since nothing is wasted on kinetic energy, and will give us

$$E = 2mc^2 = 2m_0c^2 + m_\pi c^2$$

where m_0 is the rest mass of a nucleon—i.e., of either a proton or a neutron, disregarding the slight mass difference between them—and m_π is the rest mass of a charged pion.[1] Thus we have

$$\frac{m}{m_0} = 1 + \frac{m}{2m_0}$$

[1]Taking the electron mass as a unit, the proton mass is 1836.1 and the neu-

With $m_\pi = 273m_e$, $m_0 = 1837m_e$, this gives $m/m_0 = 1.074$, or $m_0/m = 0.93$. We can use this value of m_0/m to fix the speed (β) of each proton in the zero-momentum frame, for we have

$$m/m_0 = \gamma = (1 - \beta^2)^{-1/2} \tag{6–27}$$

$$\beta^2 = 1 - (m_0/m)^2 = 0.135$$

$$\beta \approx 0.37$$

Now if proton P_2 is actually at rest in the laboratory frame, the zero-momentum frame must have the speed β relative to the laboratory. Thus the proton P_1, which has the speed β in the zero-momentum frame, has a velocity β_1 in the laboratory frame given by

$$\beta_1 = \frac{\beta + \beta}{1 + \beta^2} = \frac{2\beta}{1 + \beta^2} \approx 0.65$$

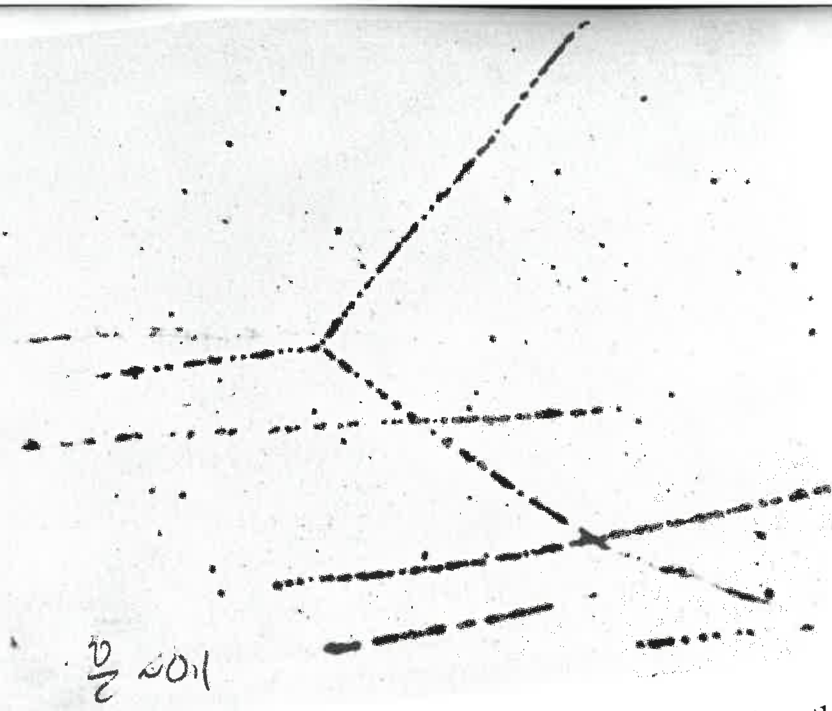
according to the relativistic velocity-addition theorem [cf. Eq. (6–6)]. From this we have

$$\gamma_1 = (1 - \beta_1{}^2)^{-1/2} \approx 1.31$$

This means that the bombarding proton must have a *kinetic* energy of $(\gamma_1 - 1)m_0c^2$, or $0.31m_0c^2$. The rest energy of a nucleon is 938 MeV, so the kinetic energy required is about 290 MeV, or rather more than twice the rest energy of the created pion. It would have been precisely a factor 2 if we could have ignored the relativistic increase of mass with velocity for the protons. (Satisfy yourself that this is so.)

The bombarding energy as calculated here is what is called the *threshold energy* for the process. We know that anything less than this is insufficient, and in practice the bombardment is carried out at energies appreciably above threshold, because this enhances the efficiency of the process—i.e., the probability that in a proton-proton collision a pion will in fact be created. But this last statement raises questions beyond our present discussion, which is limited strictly to the collision dynamics and the calculation of threshold energies. Figure 6–6 is a bubble-chamber photograph showing the kind of evidence from which the occurrence of particle-creation events like these can be inferred.

Fig. 6–8 Elastic scattering of an incident proton of about 5 MeV by an initially stationary proton in a photographic emulsion. The collision is "non-relativistic" ($K/m_0c^2 \ll 1$) with a 90° angle between the tracks of the protons after collision. (From C. F. Powell and G. P. S. Occhialini, Nuclear Physics in Photographs, Oxford Univ. Press, New York.)



$\frac{v}{c} \approx 0.1$

Elastic scatt of relat particles!

classically angle b/w traject after collision is 90

b/c of incr. of inertial mass w/ v; tho more true is rel. eqn of E

→ squeezing forward ⇒ angle < 90

For simplicity we shall limit ourselves to considering the special case in which, after collision, the two particles (as observed in the laboratory frame S) travel symmetrically at equal angles to the direction of the incident particle. Let the incident particle have total energy E_1 and momentum $\mathbf{p}_1$, and let the momenta of the particles after collision be of magnitude p_2 at angles $\pm\theta/2$ to $\mathbf{p}_1$, as shown in Figure 6–9. Then by conservation of energy and momentum we have

$$E_1 + E_0 = 2E_2 \tag{6–28}$$

$$p_1 = 2p_2 \cos\frac{\theta}{2} \tag{6–29}$$

Also we have

$$c^2p_1{}^2 = E_1{}^2 - E_0{}^2 \qquad c^2p_2{}^2 = E_2{}^2 - E_0{}^2 \tag{6–30}$$

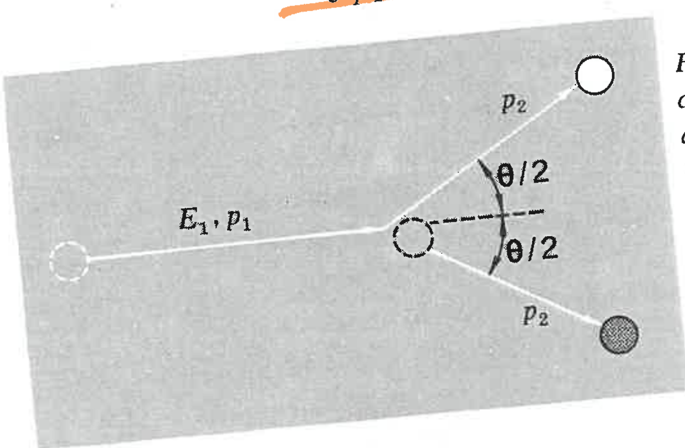


Fig. 6–9 Relativistic elastic collision of a particle with a similar particle initially at rest. The final state is assumed to be a symmetrical one in which the particles have equal speeds and hence make equal angles with the initial direction of particle 1.

It proves convenient to introduce the kinetic energy K_1 of the incident particle, so that we put

$$E_1 = E_0 + K_1$$

Using Eqs. (6–28) and (6–30) we then find

$$c^2 p_1{}^2 = (E_0 + K_1)^2 - E_0{}^2 = K_1(2E_0 + K_1)$$
$$c^2 p_2{}^2 = (E_0 + K_1/2)^2 - E_0{}^2 = K_1(E_0 + K_1/4)$$

Substituting these in Eq. (6–29) gives us half K after collision [illegible]

$$\cos^2\frac{\theta}{2} = \frac{2E_0 + K_1}{4E_0 + K_1} \quad ✓ \qquad p_1 = 2p_2\cos\frac{\theta}{2}$$

$$\frac{p_1}{2p_2} = \cos\frac{\theta}{2}$$

$$\frac{p_1^2}{4p_2^2} = \cos^2\frac{\theta}{2}$$

$$\frac{\cancel{K_1}(2E_0+K_1)/\cancel{c^2}}{4\cancel{K_1}/\cancel{c^2}\,(E_0+\frac{K_1}{4})} = \cos^2\frac{\theta}{2}$$

Putting

$$\cos\theta = 2\cos^2\frac{\theta}{2} - 1 \qquad 2\,\frac{2E_0+K_1}{4E_0+K_1} - 1$$

we find

$$\frac{4E_0+2K_1}{4E_0+K_1} - 1$$

$$\cos\theta = \frac{K_1}{4E_0 + K_1} \tag{6–31}$$

The change in the appearance of the collision as we go from low to high energies is nicely displayed in Eq. (6–31). For $K_1 \ll E_0$ we have $\cos\theta \to 0$, $\theta \to \pi/2$. For $K_1 \gg E_0$, we have $\cos\theta \to 1$, $\theta \to 0$. This relativistic compression of the scattering angles was first experimentally verified by F. C. Champion in 1932 for fast electrons (β particles).[1] Using a cloud chamber, he studied the elastic collisions of these electrons with the electrons of the atoms of the air in the chamber. Since that time the effect has become a commonplace in high-energy particle physics. Figure 6–10 shows a bubble-chamber photograph of a proton-proton

[1]F. C. Champion, *Proc. Roy. Soc.* (London), **A 136,** 630 (1932).

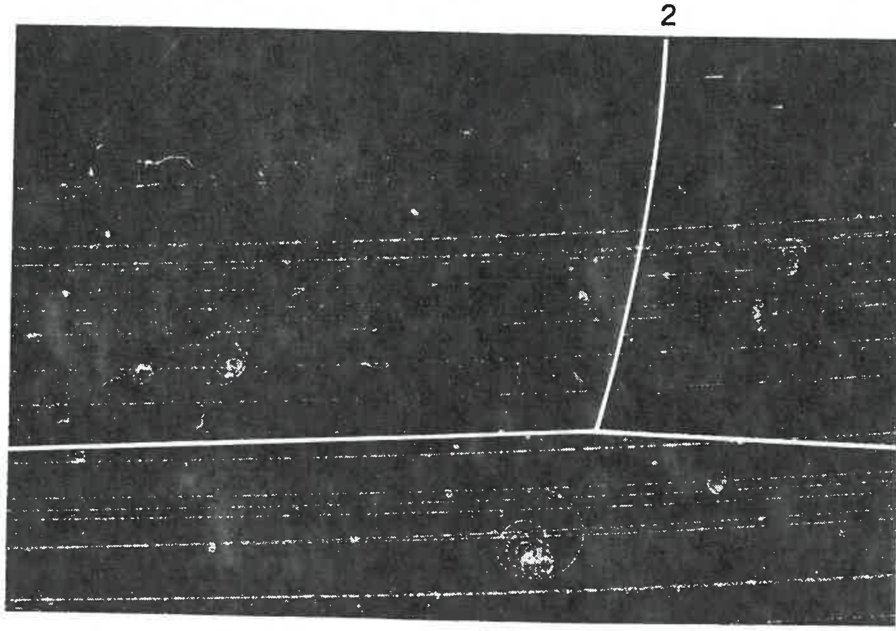


Fig. 6–10 Elastic proton-proton collision in a liquid-hydrogen bubble chamber, using incident protons of about 3 Gev. The incident proton enters at 1, and the two recoiling protons leave at 2 and 3. One cannot tell which of the latter was the incident proton. Relevant tracks emphasized. (*Brookhaven National Laboratory.*)

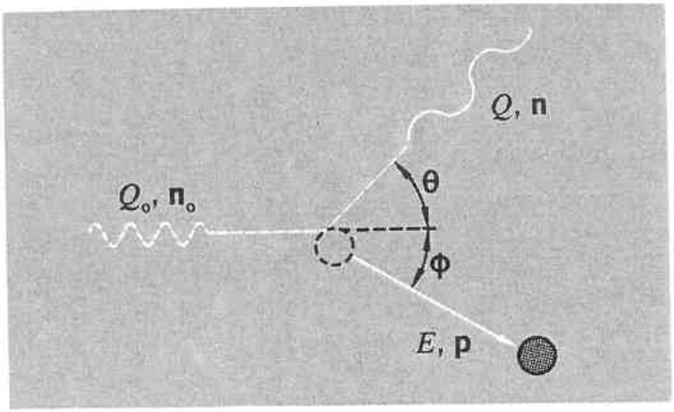


Fig. 6–11 Compton effect. An incident photon is scattered and degraded in energy as the result of an elastic collision with an initially stationary electron.

collision of this type, at an incident proton energy equivalent to several proton rest-masses.

The Compton effect

Of all the phenomena pointing to the corpuscular properties of photons, the Compton effect is perhaps the most direct and convincing. It is the collision of a photon with a free electron—which in practice means an electron loosely bound to an atom, so that it is effectively free. The collision is elastic, in the sense that no energy is siphoned off from kinetic energy into other forms, but because the electron recoils, the scattered photon has a lower energy, and hence a longer wavelength, than the incident photon. The systematic study of this phenomenon during the years 1919–1923 by A. H. Compton,[1] using X-ray photons, brought him a Nobel prize in 1927.

The Compton scattering process is an essentially relativistic collision, and can be described as follows. A photon of energy Q_0 strikes a stationary electron, which recoils in the direction φ (Fig. 6–11). The photon is scattered in the direction θ with energy Q. Conservation of energy and momentum give us the following:

$$Q_0 + m_0c^2 = E + Q \tag{6–32}$$

$$\mathbf{n}_0 Q_0/c = \mathbf{n}Q/c + \mathbf{p} \tag{6–33}$$

where E and $\mathbf{p}$ are the energy and momentum of the recoiling electron. If we are interested in the scattered photon and not in the electron, we can proceed as follows:

$$(Q_0 - Q) + m_0c^2 = E$$

$$(\mathbf{n}_0 Q_0 - \mathbf{n}Q) = c\mathbf{p}$$

[1] A. H. Compton, *Phys. Rev.*, **22**, 409 (1923).

where $\mathbf{n}_0$ and $\mathbf{n}$ are unit vectors in the initial and final photon directions, as shown. Square each of the above (i.e., form the scalar product of each side with itself in the second case):

$$(Q_0 - Q)^2 + 2(Q_0 - Q)m_0c^2 + (m_0c^2)^2 = E^2 \tag{6–34}$$

$$Q_0{}^2 - 2Q_0Q\cos\theta + Q^2 = c^2p^2 \tag{6–35}$$

Subtracting Eq. (6–34) from (6–35),

$$2Q_0Q(1 - \cos\theta) - 2(Q_0 - Q)m_0c^2 = 0$$

Therefore,

$$\frac{1}{Q} - \frac{1}{Q_0} = \frac{1}{m_0c^2}(1 - \cos\theta)$$

If the quantum energy is Q, the wavelength is given by

$$Q = h\nu = \frac{hc}{\lambda}$$

Thus in terms of wavelength the Compton effect is described by the following equation:

$$\lambda - \lambda_0 = \frac{h}{m_0c}(1 - \cos\theta) \tag{6–36}$$

For electrons, $h/m_0c = 0.02426$ Å, or 2.4×10^{-10} m. What Compton did was to establish that the scattered X-ray wavelength conformed to Eq. (6–36), both in its angular dependence and in the absolute size of the shift.[1] Figure 6–12 is a graph constructed from Compton's published data. It remained a matter of great interest, however, to demonstrate the ballistic nature of the collision by showing that the recoiling electron appeared simultaneously with the photon, and in a direction φ uniquely defined by the dynamics. The latter feature was convincingly demonstrated by Cross and Ramsey in 1950, using incident photons (γ rays, in this instance) with a sharply defined energy of 2.6 MeV. The experiment confirmed that the angle between photon and electron after scattering had the theoretical value within narrow limits (see Fig. 6–13). The coincidence in time between the particles in a Compton scattering process has

[1]The latter point is important, because even on a classical wave picture of radiation one can picture a free electron as being given a velocity under the action of radiation pressure. Radiation scattered from it would then be Doppler-shifted with the same angular variation as that given by Eq. (6–36). But the size of the shift would not be sharply defined, because the electron velocity would increase continuously from zero.

shall now discuss a classic experiment that exploited the Doppler effect to provide convincing quantitative evidence of the time-dilation phenomenon.

DOPPLER EFFECT AND TIME DILATION

3

Back in 1907 Einstein had suggested that a measurement might be made of the apparent wavelength of light emitted at right angles to their direction of motion by rapidly moving atoms. According to Eq. (5–17), the radiation traveling at an angle θ to the direction of a moving source has an observed frequency given by

$$\nu'(\theta) = \nu \frac{(1 - \beta^2)^{1/2}}{1 - \beta \cos \theta}$$

This defines an apparent wavelength given by

$$\lambda'(\theta) = \lambda \frac{1 - \beta \cos \theta}{(1 - \beta^2)^{1/2}} = \gamma\lambda(1 - \beta \cos \theta) \tag{5–19}$$

The angle θ is the direction as measured by the observer. If we set $\theta = \pi/2$, the apparent wavelength is larger than λ by just the factor γ. Now if a proton accelerated through about 5 kV picks up an electron, it forms a hydrogen atom moving at a speed of about 10^6 m/sec, so that $\beta \approx 1/300$ and $\gamma - 1 \approx 5 \times 10^{-6}$. This value of $\gamma - 1$ represents the fractional change of measured wavelength for any light emitted sideways by the moving atom and for a line in the visible spectrum at 5000 Å would mean an absolute wavelength shift of about 0.025 Å. This is extremely small but might in principle be measurable. There is, however, a very serious practical difficulty. If one is to establish the existence of this transverse, or second-order, Doppler effect (as it is variously called), one must be sure that the angle θ is precisely $\pi/2$. A deviation from it by the amount β radians (equal to about 0.2° in this example) would cause the first-order Doppler factor (that

$$t_2 - t_1 = \frac{\ldots}{c-v}$$

$$x_2 - x_1 = v\,\frac{cn\tau}{c-v}$$

$$\text{So}\quad \tau' \quad t_2' - t_1' = \gamma\left[(t_2 - t_1) - \frac{v(x_2 - x_1)}{c^2}\right] \quad L$$

$$= \gamma\left(\frac{cn\tau}{c-v} - \frac{v^2}{c^2}\,\frac{cn\tau}{c-v}\right)$$

for n periods $\Rightarrow$ for 1 period divide by n

$$\tau' = \frac{\gamma c\tau}{c-v}\left(1 - \frac{v^2}{c^2}\right) \qquad \text{let } \beta = \frac{v}{c}$$

$$= \gamma\tau\frac{(1-\beta^2)}{(1-\beta)} \qquad \text{b/c}\quad \frac{c}{c-v} = \frac{1}{1-\ldots}$$

$$= \gamma\tau(1+\beta) \qquad \text{Simonul}$$

$$\text{now, } \gamma = (1-\beta^2)^{-\frac{1}{2}}$$

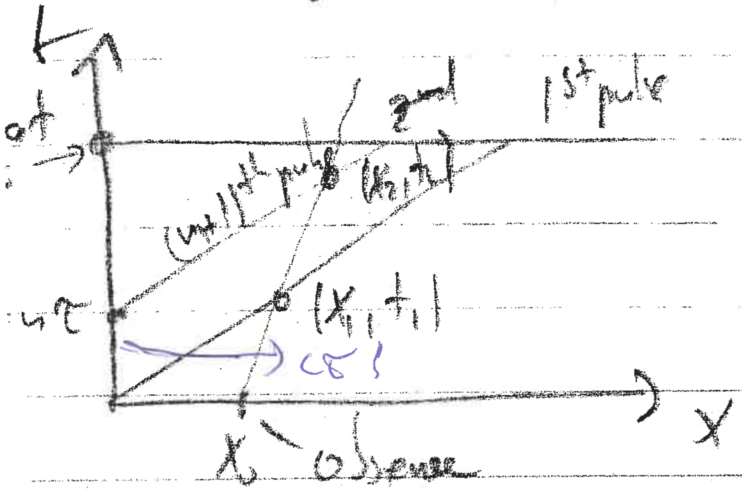
$$\text{b/c}\quad \frac{1+\beta}{\sqrt{1-\beta^2}} = \frac{1+\ldots}{\sqrt{(\ldots}}$$

$$\Rightarrow \tau' = \left(\frac{1+\beta}{1-\beta}\right)^{\frac{1}{2}}\tau$$

$$\text{and}\quad \nu' = \left(\frac{1-\beta}{1+\beta}\right)^{\frac{1}{2}}\nu \qquad \text{b/c}\quad \nu = \frac{1}{\tau}$$

$$\cancel{\sqrt{\frac{1-\frac{v}{c}}{1+\frac{v}{c}}} = \sqrt{\frac{c}{c}\,\frac{c-v}{c+v}}}$$

~~Space time diagram of rel. Doppler Effect~~



src starts at origin, S
obs moves w/ v rel to S
his rest frame S'
each pulse travels w/ c
$(n+1)^{st}$ pulse sent at $t = n\tau$
covers n periods of vibn

$\Rightarrow \nu = \frac{1}{\tau}$

$$\cancel{t_2 - t_1} = \frac{cn\tau}{c-v}$$

$$x_2 - x_1 = v\,\frac{cn\tau}{c-v}$$

$$\cancel{t_2' - t_1'} = \gamma\left[(t_2 - t_1) - \frac{v(x_2 - x_1)}{c^2} \right] \quad LT$$

$$= \gamma\left(\frac{cn\tau}{c-v} - \frac{v^2}{c^2}\frac{cn\tau}{c-v} \right)$$

for n periods $\Rightarrow$ ~~for 1 period~~ divide by n

$$\cancel{\tau'} = \cancel{\frac{\gamma c\tau}{c-v}\left(1 - \frac{v^2}{c^2}\right)} \quad \cancel{\text{let } \beta = \frac{v}{c}}$$

$$= \ldots (1-\beta^2) \quad \ldots \quad \frac{c}{\ldots} \ldots \frac{1}{\ldots}$$

$\hookrightarrow \quad \frac{dp}{dv} = \frac{m_0}{\left(1-\frac{v^2}{c^2}\right)^{3/2}}$

$\rightarrow \quad dp = \text{''} \; dv$

$K = \int \frac{dp}{dt}\, dx = \int dp\, \frac{dx}{dt} = \int v\, dp$

$= \int_0^{v_1} v \left(\frac{m_0}{\left(1-\frac{v^2}{c^2}\right)^{3/2}}\right) dv$

$= \left(m_0 \frac{c^2}{\left(1-\frac{v^2}{c^2}\right)^{1/2}} \right|_0^{v_1}$

$= \frac{m_0 c^2}{\sqrt{1-\frac{v_1^2}{c^2}}} - \frac{m_0 c^2}{\sqrt{1-0}}$

$\equiv \gamma m_0 c^2 - m_0 c^2 \equiv \underbrace{mc^2}_{E} - \underbrace{m_0 c^2}_{\text{rest } E_0} \equiv K$

- at any speed: $K = \Delta m c^2$

for $v \ll c$, $\gamma \approx 1 \rightarrow m \approx m_0$ and $K \cong \frac{1}{2} m v^2$

Schaum 8.25

- find max. speed for $\frac{1}{2} m_0 v^2$ w/ error $\leq 0.5\%$

condition at max speed $\quad \frac{K - \frac{1}{2} m_0 v^2}{K} = 0.005$

or $\quad K = \frac{\frac{1}{2} m_0 v^2}{0.995}$

$K = E - E_0 = \gamma m_0 c^2 - m_0 c^2 = m_0 c^2 \cdot \left[\left(1-\frac{v^2}{c^2}\right)^{-\frac{1}{2}} - 1 \right]$

bin. exp.

$= m_0 c^2 \left[\left(1 + \frac{1}{2}\frac{v^2}{c^2} + \frac{3}{8}\frac{v^4}{c^4} + \dots \right) - 1 \right]$

$= \frac{1}{2} m_0 v^2 + \frac{3}{8} m_0 v^2 \left(\frac{v^2}{c^2}\right) + \dots$

$\hookrightarrow \quad \frac{\frac{1}{2} m_0 v^2}{0.995} \approx \frac{1}{2} m_0 v^2 + \frac{3}{8} m_0 v^2 \left(\frac{v^2}{c^2}\right) \Rightarrow v \approx 0.082c$

to work w/ N: relativity $m_0 = \frac{E_0}{c^2} \to \frac{1}{2}mv^2$

m_0 'rest mass'

$E_0 = \frac{m_0}{c^2}$ 'rest energy'

e.g. e^-: $m_0 = 9.11 \cdot 10^{-31}$ kg

$E_0 = 8.2 \cdot 10^{-14}$ J $= 0.511$ MeV

then, $\boxed{m(v) = \frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}}}$ mass is Rel.

n/b $K = E - E_0$ not by subbing $m(v)$ in $\frac{1}{2}mv^2$

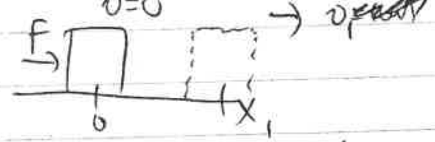
$$\boxed{\vec{p} = m(v)\,\vec{v}, \qquad E = m(v)\cdot c^2}$$

R9 → video #20: $E=mc^2$ demo

pull everywhere & is inelastic relativistic collision of m_0 w/ m_0 (at rest) the resulting rest mass of the composite particle is

1- one can't tell
2- $< 2m_0$
3- $> 2m_0$
4- $= 2m_0$

$\vec{F} = \frac{d\vec{p}}{dt}$



$W = \int_{x=0}^{x_1} F\,dx \to K_2 - K_1 = W = K_2 - 0 \equiv K$

$\vec{p} = m\vec{v} = \gamma m_0 \vec{v} = \frac{m_0 \vec{v}}{\sqrt{1-\frac{v^2}{c^2}}}$

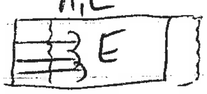
- Relativistic Dynamics

- the inertia of energy (ch.1, p.16)

"$\frac{p}{v} = \frac{mv}{v}$"

<u>Einstein's box</u>

M, L



photons emitted → rad. carries mom. $\frac{E}{c}$

mom. cons. : recoil box, mom $-\frac{E}{c}$, $v = -\frac{E}{Mc}$

after $\Delta t \approx \frac{L}{c}$ $(v \ll c)$ → box comes to rest

+ moved by $\Delta x = v\Delta t = -\frac{E}{Mc}\frac{L}{c}$

the rad. carried eq. mass, m, so that CM unmoved: $mL + M\Delta x = 0$

together, $mL - \frac{M}{M}\frac{E}{c^2}L = 0$ or $m = \frac{E}{c^2}$

<u>for photons</u> $E = cp$, $m = \frac{E}{c^2}$ $\Rightarrow$ $m_{ph} = \frac{p}{c}$

Newt. mech: $m = \frac{p}{v}$ w/ $m = \frac{E}{c^2}$

↳ Relat: $\cancel{m = p}$ $E = \frac{pc^2}{v}$

Next: $dE = F\,dx = \frac{dp}{dt}dx = v\,dp$

combine w/ $\frac{pc^2}{v}$:

$$E\,dE = c^2 p\,dp \text{, integrate}$$

$$\boxed{E^2 = c^2p^2 + \underbrace{E_0^2}_{\text{int. const}}}$$ the relativ. relation b/w E and p

substitute $E = \frac{pc^2}{v}$ as $cp = \frac{Ev}{c}$, for $v \ll c$

use bin. expansion: $(1 - \frac{v^2}{c^2})^{-\frac{1}{2}} \simeq 1 + \frac{1}{2}\frac{v^2}{c^2} + \ldots$

↳ $E(v) = E_0 + \frac{1}{2}\left(\frac{E_0}{c^2}\right)v^2 + \ldots$

$$k \approx 1 - \frac{1}{n^2} = 1 - \frac{1}{(4/3)^2} = 0.438$$

which agrees with Fizeau's experimental result.

7.10. Evaluate the Doppler equation to first order in v/c when the source and observ each other.

Ans.

$$\nu = \nu_0 \sqrt{\frac{1 - \dfrac{v}{c}}{1 + \dfrac{v}{c}}} \times \frac{\sqrt{1 + \dfrac{v}{c}}}{\sqrt{1 + \dfrac{v}{c}}} = \nu_0 \frac{\sqrt{1 - \dfrac{v^2}{c^2}}}{1 + \dfrac{v}{c}} \approx \nu_0 \frac{c}{c + v}$$

which is the classical expression for the Doppler effect when the receiver is stat the medium.

7.11. A car is approaching a radar speed trap at 80 mi/hr. If the radar set works 20 × 10⁹ Hz, what frequency shift is observed by the patrolman at the radar s

Ans. To first order in v/c, the frequency received by the car is

$$\frac{1}{\sqrt{1 - \frac{v}{c}}} \approx \sqrt{(1-\frac{v}{c})^{-1}}$$

bm. eq's $\quad \sqrt{1 - \frac{v}{c}} \approx 1 - \left(-\frac{v}{c}\right)$ small $\quad 1 + \frac{v}{c}$

$$\nu' = \nu_0 \sqrt{\frac{1 + \dfrac{v}{c}}{1 - \dfrac{v}{c}}} \approx \nu_0 \sqrt{\left(1 + \frac{v}{c}\right)\left(1 + \frac{v}{c}\right)} = \nu_0 \left(1 + \frac{v}{c}\right)$$

The car then acts as a moving source with this frequency. The frequency received is

$$\nu'' \approx \nu' \left(1 + \frac{v}{c}\right) \approx \nu_0 \left(1 + \frac{v}{c}\right)^2 \approx \nu_0 \left(1 + \frac{2v}{c}\right)$$

bm $\left(1 + 2\frac{v}{c}\right)$

from which (80 mi/hr = 35 m/s)

$$\nu'' - \nu_0 \approx 2\frac{v}{c}\nu_0 = \frac{2 \times 35\ \text{m/s}}{3 \times 10^8\ \text{m/s}} \times 20 \times 10^9\ \text{Hz} = 4.67 \times 10^3\ \text{H}$$

7.12. A star is receding from the earth at a speed of $5 \times 10^{-3}c$. What is the wavele sodium D_2 line (5890 Å)?

as measured in the frame of reference attached to the ground.
Thus we have

$$\tau' = \gamma\tau(1 - v\cos\theta/c)$$

But $1/\tau'$ represents the received frequency ν' of the signals under these conditions. Hence

$$\nu' = \frac{\nu}{\gamma(1 - \beta\cos\theta)}$$

i.e.,

$$\nu' = \nu\frac{(1-\beta^2)^{1/2}}{1 - \beta\cos\theta} \tag{5-17}$$

If we wanted to proceed to construct (or analyze) the graph of observed frequency versus time (Fig. 5–6), we would make use of the relationship

$$\cos\theta = \frac{-vt}{(h^2 + v^2t^2)^{1/2}} \tag{5-18}$$

We did not really need special relativity to discuss the Doppler effect of Sputnik I, because the measurements involved were not sensitive to the differences of the order of β^2—i.e., a few parts in 10^{10}—between relativistic and nonrelativistic behavior in this case. It is true that, with the atomic clocks now available as frequency standards, such subtle changes are by no means beyond the reach of detection. But we shall not pursue this topic. The satellite problem simply provided a nice framework within which to develop the theory of Doppler effects for a source moving in an arbitrary direction. The really important applications of the Doppler formula as expressed by Eq. (5–17) are in the analysis of radiation from swiftly moving atoms, nuclei, or other subatomic particles. And as one example of this, we

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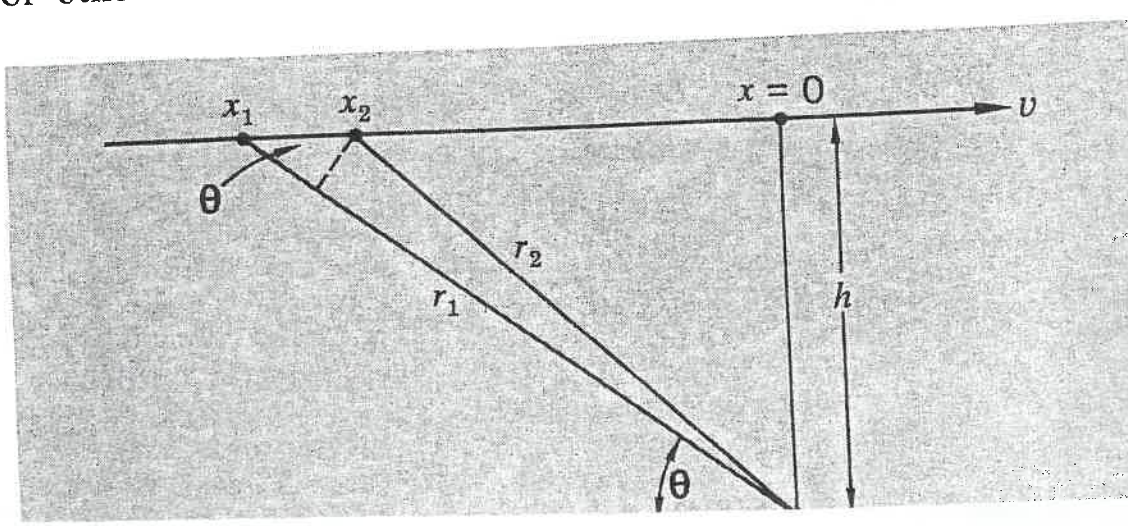
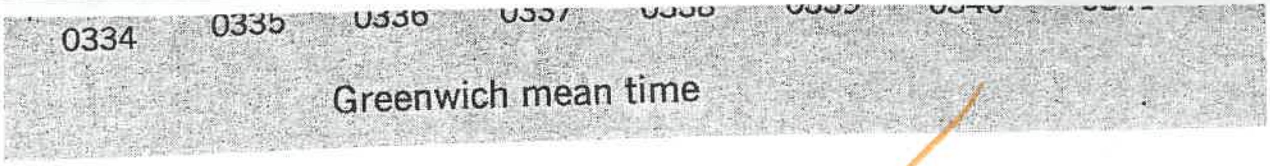


Fig. 5–7 Diagram for consideration of Doppler effect with signals emitted at angle θ to line of



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pulses at a frequency ν in its own rest system. Consider two successive pulses that are emitted from the positions x_1 and x_2 as shown, at times we can denote t_1 and t_2. The time interval τ between the pulses is $1/\nu$ in the inertial frame of the satellite but is greater than this by the time-dilation factor γ in the observer's frame. Thus we have

* go to page 6 [illegible]

3a)

$$t_2 - t_1 = \gamma\tau = \gamma/\nu$$

The pulses take times r_1/c and r_2/c respectively to reach O, so that the measured time separation τ' between them is given by

$$\begin{aligned} \tau' &= t_2 + r_2/c - t_1 - r_1/c \\ &= \gamma\tau - (r_1 - r_2)/c \end{aligned}$$

Now if the distance $x_2 - x_1$ is very much less than r_1 (i.e., if the satellite travels a very small distance during one cycle of its transmitter signals), we can with good accuracy put

$$\begin{aligned} r_1 - r_2 &\approx (x_2 - x_1)\cos\theta \\ &= v\gamma\tau\cos\theta \end{aligned}$$

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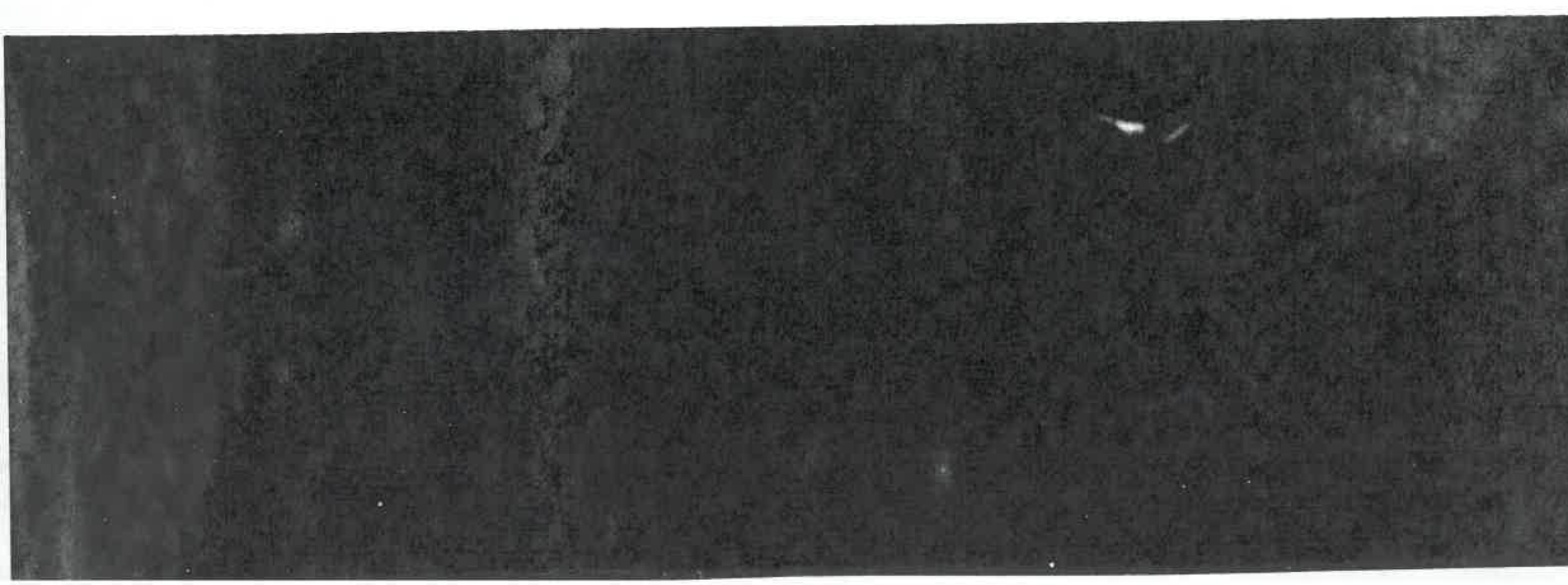

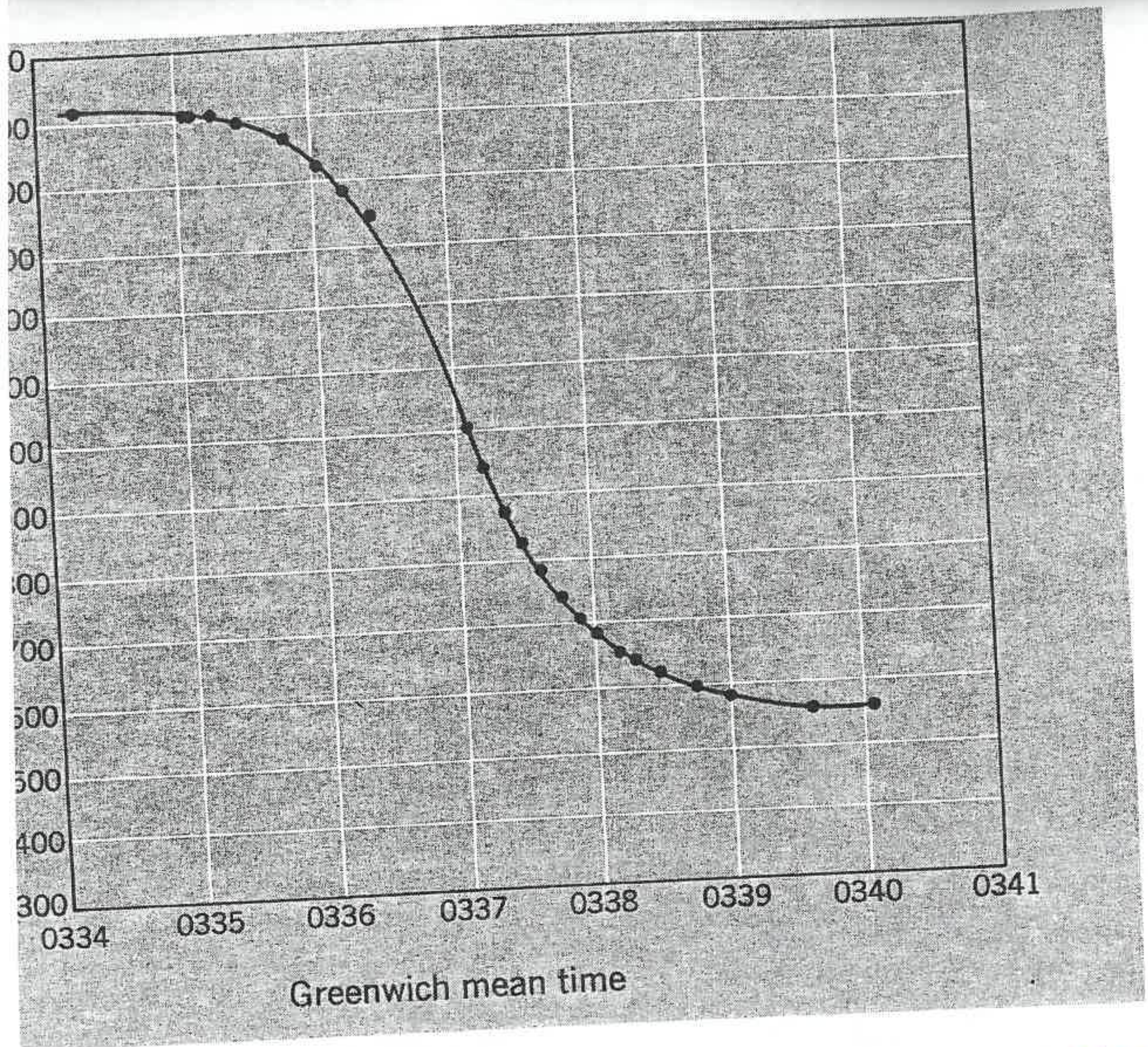

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$$t_2 - t_1 = \gamma\tau = \gamma/\nu \qquad \text{3a}$$

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↙ p208 ff

$$\boxed{\begin{aligned} p_x' &= \gamma_v\left(p_x - \frac{vE}{c^2}\right) \\ p_y' &= p_y\,,\quad p_z' = p_z \\ E' &= \gamma_v\left(E - v p_x\right) \end{aligned}} \qquad \boxed{\begin{aligned} p_x &= \gamma_v\left(p_x' + \frac{vE'}{c^2}\right) \\ &\text{''} \\ E &= \gamma_v\left(E' + v p_x'\right) \end{aligned}}$$

where $\gamma_v = \left(1 - \frac{v^2}{c^2}\right)^{-\frac{1}{2}}$

a new invariant $E^2 - (cp)^2 = E_0^2$ in all inertial frames

and v is the vel. of S' as meas. by S

important eqn: read p. 176–180 ; absorption and emission

later work: p184 ff creation of particles

p189 pair prod. in pairs

p191–194 scatt. of part.

p.194 ff Compton eff. in part

putting it all together

– use 4-vectors (p.214 ff)

the 3 comp. of $\vec{p}$ transform like the 3 comp. of $\vec{r}$

tot E, a scalar, " " t

↳ quantity $E^2 - (cp)^2 = E_0^2$ is an invariant – same in all inertial frames

- Newtonian thinking: p, E represent different prop. of a body
- Relativistic " : the distinctions are blurred b/c p and E mix (like x and t mix under LT)

(there is a third such mixing: $\vec{E}, \vec{H}$)

this mixing is the deeper reason for giving up the idea of space and being separate prop. : think of single 4 dim space-time structure (already used in space-time diagrams) cons. of mass (dep. on obs. frame! choose the close one)

$P_{i,sys} = P_{f,sys}$

$$\frac{m_0 u_A'}{\sqrt{1-\frac{u_A'^2}{c^2}}} + \frac{m_0 u_B'}{\sqrt{1-\frac{u_B'^2}{c^2}}} \overset{!}{=} \frac{M_0 u_C'}{\sqrt{1-\frac{u_C'^2}{c^2}}}$$

E cons. $E_{sys,i} = E_{sys,f}$

$$\frac{m_0 c^2}{\sqrt{1-\frac{u_A'^2}{c^2}}} + \underbrace{\frac{m_0 c^2}{\sqrt{1-\frac{u_B'^2}{c^2}}}}_{\text{n.b. what happens here, if } u_B'=0\,?} \overset{!}{=} \frac{M_0 c^2}{\sqrt{1-\frac{u_C'^2}{c^2}}}$$

→ 2 eqn for 2 unknowns (M_0, u_C')

- ~~Lorentz Tr for E and p~~

particle vel. in S : u

" S' : u'

$$u_x' = \frac{u_x - v}{1-\frac{v u_x}{c^2}} \qquad u_y' = \frac{\frac{u_y}{\gamma}}{1-\frac{v u_x}{c^2}}$$

~~in S~~

$$\left.\begin{aligned} E &= \gamma_u m_0 c^2 \\ p_x &= \gamma_u m_0 u_x \\ p_y &= \gamma_u m_0 u_y \end{aligned}\right\} \quad \text{where } \gamma_u = \frac{1}{\sqrt{1-\frac{u^2}{v^2}}}$$

~~in S'~~ same, but with u' and $\gamma_{u'} = \frac{1}{\sqrt{1-\frac{u'^2}{v^2}}}$

to relate the expressions in S and S', express $\gamma_{u'}$ in terms of meas. in S

(and vice versa)

The energy and momentum of the particle in the two frames are as follows:

In S:

$$\left.\begin{aligned} E &= \gamma(u)m_0c^2 \\ p_x &= \gamma(u)m_0u_x \\ p_y &= \gamma(u)m_0u_y \end{aligned}\right\} \quad \text{where } \gamma(u) = (1 - u^2/c^2)^{-1/2} \tag{7–3}$$

In S':

$$\left.\begin{aligned} E' &= \gamma(u')m_0c^2 \\ p_x' &= \gamma(u')m_0u_x' \\ p_y' &= \gamma(u')m_0u_y' \end{aligned}\right\} \quad \text{where } \gamma(u') = (1 - u'^2/c^2)^{-1/2} \tag{7–4}$$

The one big step in relating these two sets of dynamical quantities is to express $\gamma(u)$ in terms of quantities measured in S', or $\gamma(u')$ in terms of quantities measured in S. Let us take the latter. We have

$$\begin{aligned} \gamma(u') &= [1 - (u')^2/c^2]^{-1/2} \\ &= [1 - (u_x')^2/c^2 - (u_y')^2/c^2]^{-1/2} \end{aligned} \tag{7–5}$$

We shall treat this by easy stages. First, consider the following:

$$\begin{aligned} 1 - (u_x')^2/c^2 &= 1 - \frac{(u_x - v)^2}{c^2(1 - vu_x/c^2)^2} \\ &= \frac{(1 - vu_x/c^2)^2 - (u_x - v)^2/c^2}{(1 - vu_x/c^2)^2} \\ &= \frac{1 - u_x^2/c^2 - v^2/c^2 + (vu_x/c^2)^2}{(1 - vu_x/c^2)^2} \end{aligned}$$

Therefore,

$$1 - (u_x')^2/c^2 = \frac{(1 - u_x^2/c^2)(1 - v^2/c^2)}{(1 - vu_x/c^2)^2} \tag{7–6a}$$

Next, note that, from equations (7–2), we have ≡ LT

$$(u_y')^2/c^2 = \frac{(u_y^2/c^2)(1 - v^2/c^2)}{(1 - vu_x/c^2)^2} \tag{7–6b}$$

Subtracting Eq. (7–6b) from (7–6a), we get

$$1 - (u')^2/c^2 = \frac{(1 - u^2/c^2)(1 - v^2/c^2)}{(1 - vu_x/c^2)^2}$$

in which we recognize the squares of the reciprocals of $\gamma(u')$, $\gamma(u)$, and $\gamma(v)$.

We have, in fact,

$$\gamma(u') = \gamma(v)\gamma(u)(1 - vu_x/c^2) \tag{7-7}$$

Now taking this result in conjunction with the first of equations (7–4), we have

$$E' = \gamma(v)[\gamma(u)m_0c^2 - v\gamma(u)m_0u_x]$$

which, by reference to equations (7–3), can be expressed as follows:

$$E' = \gamma(v)(E - vp_x) \tag{7-8}$$

Again, taking the equation for p_x', we have

$$p_x' = \gamma(v)\gamma(u)m_0(u_x - v)$$

i.e.,

$$p_x' = \gamma(v)(p_x - vE/c^2) \tag{7-9}$$

Finally, taking the equation for p_y', we find

$$p_y' = \gamma(u)m_0u_y$$

Therefore,

$$p_y' = p_y \tag{7-10}$$

Let us collect together the transformations from S to S' expressed by Eqs. (7–8), (7–9), and (7–10), plus the corresponding transformations from S' to S:

LORENTZ TRANSFORMATIONS
FOR MOMENTUM AND ENERGY

$$\begin{aligned} p_x' &= \gamma(p_x - vE/c^2) & \qquad p_x &= \gamma(p_x' + vE'/c^2) \\ p_y' &= p_y & p_y &= p_y' \\ p_z' &= p_z & p_z &= p_z' \\ E' &= \gamma(E - vp_x) & E &= \gamma(E' + vp_x') \end{aligned}$$

with $\gamma = (1 - v^2/c^2)^{-1/2}$, where v is the velocity of S' as measured in S

(7–11)

One striking feature of equations (7–11) is that the momen-