PHYS 1220, Engineering Physics, Chapter 22 – Gauss's Law
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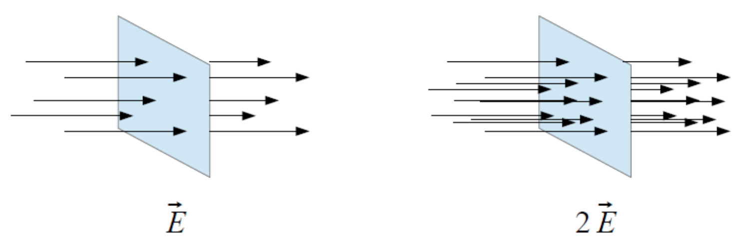
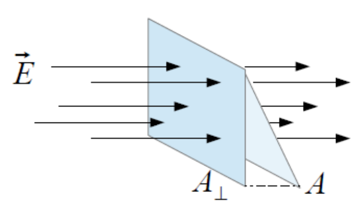
Goal of this chapter is to teach you what is Gauss's Law and how to use it to calculate Electric field

- Electric field lines vs Electric field
  - Electric field lines are continuous lines pointing out from positive charge and end at negative charge.
  - Electric field strength could be understood as the electric field line density. However, remember, electric field lines are conceptual lines. But you still can use the electric field lines to represent and even think how to calculate electric field.
  - For example, you can define the electric field strength, $\vec{E}$, as 5 electric field lines per unit area, $2\vec{E}$ would be 10 electric field lines passing through the same area.



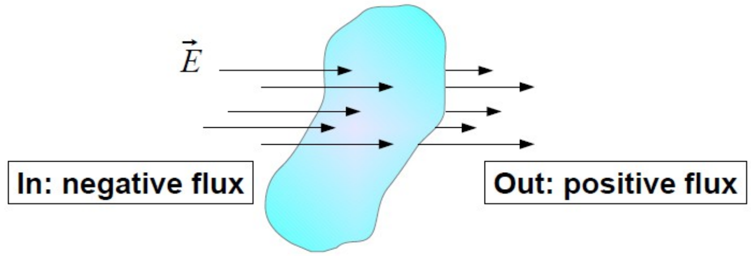
- Electric Flux: How much electric field is passing through a certain area.
  - Think electric field as electric line density: $|\vec{E}| \cdot A_{\perp}$.



  - So, the electric flux could be written as: $\Phi_E = \vec{E} \cdot \vec{A}$
  - But, what is the direction of the area vector: $\vec{A}$? $\vec{A} = A\hat{n}$ where $\hat{n}$ is a unit vector with direction pointing normal to the surface.

Do Example 22.1 (pg. 730)

- Electric flux of a closed surface.
  - What is interested is how much electric flux flowing "***out***" of the closed surface. In other words, if the electric field pointing **"*into*"** the closed surface, it is considered as **"*negative flux*"**; if the electric field pointing **"*out of*"** the closed surface, it is considered as **"positive *flux*"**.

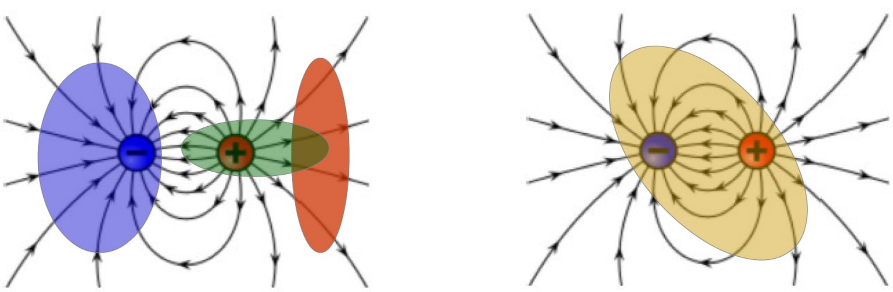


  - If you want to calculate the electric flux flowing out of the closed surface, mathematically, you can do it with the following equation. And a very important definition of the normal direction of the surface area vector, $\hat{n}$, to be pointing toward ***"outward"*** of the closed surface.

$$\Phi_E = \oint \vec{E} \cdot d\vec{A}$$

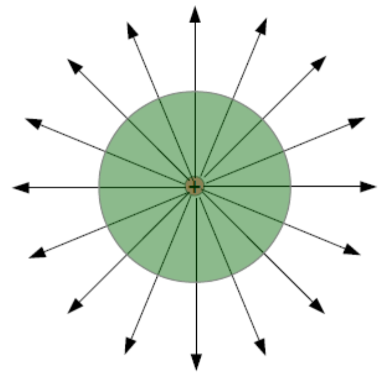
Do Example 22.2 (pg. 731)

- When do we have non-zero total outward electric flux for a particular closed surface?
  - Let's see four different closed surfaces:



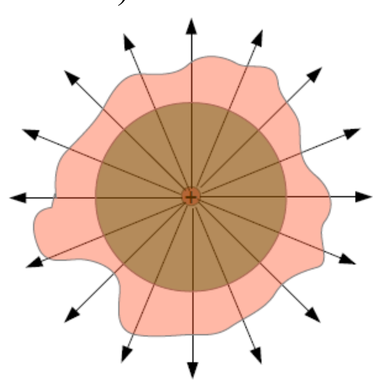
  - When the closed surface encloses a positive (negative) charge, the total flux is positive (negative); when the closed surface does not enclose any charge or enclose a zero net charge, the total flux is zero. (Note: the color on the texts are denoting the situation of the closed surfaces with that color in the figures above.)

- Let's try to calculate the electric flux in a simple case: point charge q, assume a imaginary sphere with radius R for the electric flux calculation.



- From Coulomb's Law, the electric field at a distance R from the charge q is: $\vec{E}=\frac{1}{4\pi\epsilon_0}\frac{q}{R^2}\hat{r}$
- From the electric flux equation, $\Phi_E=\oint\vec{E}\cdot d\vec{A}$, and the fact that, in this case, $\vec{E}$ and $d\vec{A}$ are parallel (both are pointing outward with charge q as center), we know that the electric flux, in this case, just be the product of $|\vec{E}|$ and total surface of the sphere $|\vec{A}|=4\pi R^2$. In other words,
$$\Phi_E=|\vec{E}|\cdot|\vec{A}|=\frac{1}{4\pi\epsilon_0}\frac{q}{R^2}4\pi R^2=\frac{q}{\epsilon_0}$$
- In short, the electric flux is only a function of the amount of the charge q.

- Even the shape of the closed surface is different from the sphere discussed above, the total electric flux is the same, as long as the closed surface also encloses the charge q. (You can count the number of electric field lines for the closed sphere and for the irregular closed surface to test this idea.)



- With these arguments, we can conclude that the total electric flux of a closed surface (regardless the shape, and it is up to your choice) is a function of the total charge enclosed in it. And this is Gauss's Law.

$$\Phi_E=\frac{Q_{encl}}{\epsilon_0}$$

Do Example 22.5 (pg. 737)

Do Example 22.6 (pg. 738)
Do Example 22.7 (pg. 739)
Do Example 22.9 (pg. 740)

- Think: could you construct a Gauss's Law for gravitational field by similar procedure? (This is out side of the teaching materials, so this will not be included in exams.)

- In Example 22.5, you already know that the electric field inside a charged conductor is zero. What is the electric field immediate out side of the charged conductor with general shape?

Math Preview for Chapter 23:

- Vector inner product
- unit vector
- Integral along a path in space
- Partial differentiate
- Gradient

Question to think:

- Do you know how to read map with contour lines (isoline, equal elevation lines)? If you do, then, think that if you stand in one particular point on the map, which direction (E, S, W, N) will be the steepest direction upward to the hill in height?