PHYS 1220, Engineering Physics, Chapter 23 – Electric Potential
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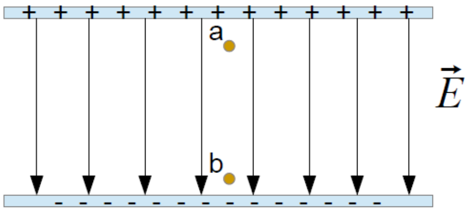
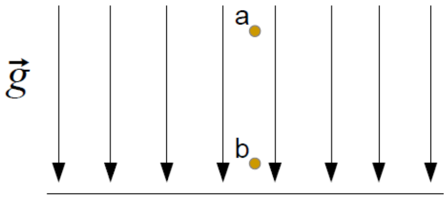
Goal of this chapter is to teach you what is Electric Potential and how to use it to calculate Electric field

- Electric potential energy change: defined as $\Delta U = -W$, where W is the work done by conservative force (here, electric force).

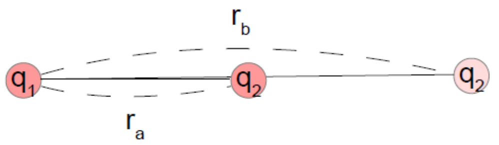
- In an **uniform** field

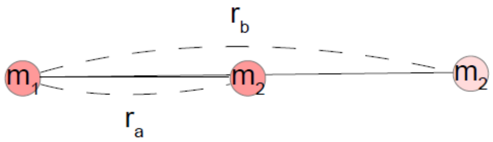




- Electric force for a charge q: $\vec{F} = q\vec{E}$. The work done by this electric field when the charge q moves from a to b is: $W_{ab} = \int_a^b \vec{F} \cdot d\vec{l} = q|\vec{E}|(y_a - y_b)$. So, the electric potential energy change is defined as: $\Delta U = -W_{ab} = q|\vec{E}|y_b - q|\vec{E}|y_a = U_b - U_a$. Thus, the electric potential energy: $U = q|\vec{E}|y$
- This is very similar to the same procedure to define **gravitational potential energy**: $U = m\vec{g}\,y$

- Electric potential energy of two point charge:





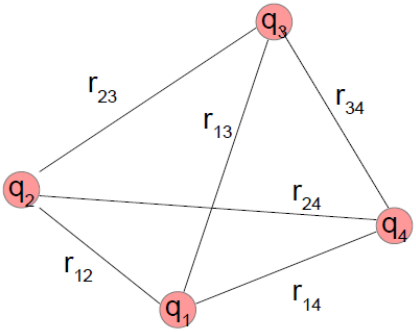
- Electric force between the two charges: $\vec{F} = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{r^2}\hat{r}$. The work done by this electric field when their separation changes from r_a to r_b is: $W_{ab} = \int_a^b \vec{F} \cdot d\vec{r} = \frac{q_1 q_2}{4\pi\epsilon_0}(\frac{1}{r_a} - \frac{1}{r_b})$. So, the electric potential energy change is defined as: $\Delta U = -W_{ab} = \frac{q_1 q_2}{4\pi\epsilon_0}(\frac{1}{r_b} - \frac{1}{r_a}) = U_b - U_a$. Thus, the electric potential energy: $U = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{r}$

- This is very similar to the same procedure to define **gravitational potential energy**: $U = -G\frac{m_1 m_2}{r}$

DO Example 23.2 (page 760)

- Electric potential energy with several point charges.

- Since U is a **scalar**, you can just **add** all contribution together.



- The total electric potential energy in above system is:

$$U = \frac{1}{4\pi\epsilon_0}\left(\frac{q_1 q_2}{r_{12}} + \frac{q_1 q_3}{r_{13}} + \frac{q_1 q_4}{r_{14}} + \frac{q_2 q_3}{r_{23}} + \frac{q_2 q_4}{r_{24}} + \frac{q_3 q_4}{r_{34}}\right) = \frac{1}{4\pi\epsilon_0}\sum_{i<j}\frac{q_i q_j}{r_{ij}}$$

- Again, in many cases, we want to know when we put one testing charge q in a system, what will that charge acts/feels. Similar to the reason we defined the electric field, now we define **Electric Potential** as: ***the potential energy per unit charge***.

$$V = \frac{U}{q_0}$$

- The unit of the electric potential is **volt**.

$$1\,V = 1\,volt = 1\,J/C = 1\ joule/coulomb$$

- Electric potential for various situations (remember that you are using a testing charge q_0 to figure out the electric potential.):

- Electric potential due to a point charge:

$$V = \frac{1}{4\pi\epsilon_0}\frac{q}{r}$$

- Electric potential due to a collection of point charges:

$$V = \frac{1}{4\pi\epsilon_0}\sum_i \frac{q_i}{r_i}$$

- Electric potential due to a continuous distribution of charges:

$$V = \frac{1}{4\pi\epsilon_0}\int\frac{dq}{r}$$

- What is the relationship among $\vec{F}$, $\vec{E}$, U , and V ?

$$\begin{tikzcd}
\vec{F}_e \arrow[r, shift left, "\Delta U=-W=-\int \vec{F}\cdot d\vec{l}"] \arrow[d, leftrightarrow, "\vec{E}=\frac{\vec{F}_e}{q}"'] & U_e \arrow[l, shift left, "\vec{F}=-\nabla U_e"] \arrow[d, leftrightarrow, "V=\frac{U_e}{q}"] \\
\vec{E} \arrow[r, shift left, "\Delta V=-\int \vec{E}\cdot d\vec{l}"] & V \arrow[l, shift left, "\vec{E}=-\nabla V"]
\end{tikzcd}$$

- **IMPORTANT:** Charge q could be either positive or negative. (1) The electric force is: $\vec{F}_e=q\vec{E}$. Then the electric force, $\vec{F}_e$, and the electric field, $\vec{E}$, will be the **same** (opposite) direction if ***q*** is **positive** (negative). (2) Same idea for the electric potential energy: $U_e=qV$. The electric potential energy, U_e , and the electric potential, V , will be the **same** (opposite) sign if ***q*** is **positive** (negative).
- **IMPORTANT #2**: The absolute number of the **potential energy** is not important, instead, the **DIFFERENCE** of the potential energy. Same for **electric potential energy** as well as **electric potential**.

- New unit for **ENERGY**: electron volt (eV)
1 eV = the electric potential energy gained by one electron with change of 1 V electric potential = (1.602×10^{-19} C) (1 V) = 1.602×10^{-19} J
  - Note: CV = J
  - Note #2: eV is an unit of **ENERGY**; while V is an unit of **electric potential**.

- Practice:
DO Example 23.8 (page 768)
DO Example 23.10 (page 769)

- Think: For charges on conductors, what determines them (the charges) to reach the equilibrium condition (all the charges stop moving, after the repelling from each other)?
  - The whole surface (even the whole volume) of the conductor reach the same electric potential.

- Electric field could be calculated from the electric potential (the derivation could be found on page 774 in the text book):

$$\vec{E}=-\vec{\nabla}V=-(\hat{i}\frac{\partial V}{\partial x}+\hat{j}\frac{\partial V}{\partial y}+\hat{k}\frac{\partial V}{\partial z})$$

Math Preview for Chapter 24:

- Nothing really special in Ch. 24

Question to think:

- Could we use the electric potential to move opposite signed charges to two different conductors, and use them later?