HW 1

Due: 11:00pm on Sunday, January 26, 2014

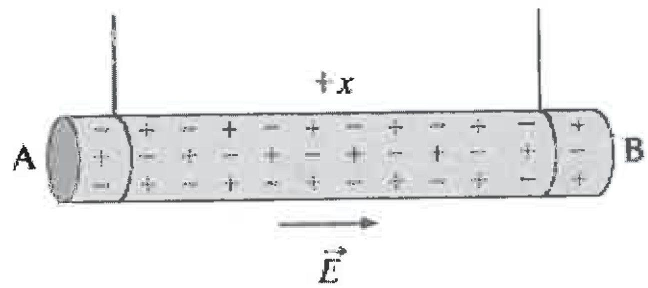
You will receive no credit for items you complete after the assignment is due. Grading Policy

The Electric Field inside a Conductor

Learning Goal:

To understand how the charges within a conductor respond to an externally applied electric field.

To illustrate the behavior of charge inside conductors, consider a long conducting rod that is suspended by insulating strings (see the figure). Assume that the rod is initially electrically neutral, and that it remains so for this discussion. The rod is positioned along the x axis, and an external electric field that points in the positive x direction (to the right) can be applied to the rod and the surrounding region. The atoms in the rod are composed of positive nuclei (indicated by plus signs) and negative electrons (indicated by minus signs). Before application of the electric field, these atoms were distributed evenly throughout the rod.



Part A

What is the force felt by the electrons and the nuclei in the rod when the external field described in the problem introduction is applied? (Ignore internal fields in the rod for the moment.)

You did not open hints for this part.

ANSWER: $\vec{F} = q\vec{E}$ — electron $-e$ ⇒ opposite to $\vec{E}$; — nuclei $+q$ ⇒ same to $\vec{E}$

- ○ Both electrons and nuclei experience a force to the right.
- ⊗ The nuclei experience a force to the right and the electrons experience a force to the left.
- ○ The electrons experience a force to the left but the nuclei experience no force.
- ○ The electrons experience no force but the nuclei experience a force to the right.

B

What is the motion of the negative electrons and positive atomic nuclei caused by the external field?

You did not open hints for this part.

ANSWER:

- ○ Both electrons and nuclei move to the right.
- ○ The nuclei move to the right and the electrons move to the left through equal distances.
- ⊗ The electrons move to the left and the nuclei are almost stationary.
- ○ The electrons are almost stationary and the nuclei move to the right.

* nuclei in material are usually stationary, while electrons in conductors can move freely

Part C

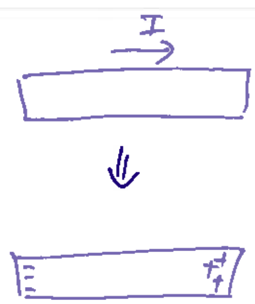
Imagine that the rightward current flows in the rod for a short time. As a result, what will the net charge on the right and left ends of the rod become?

Hint 1. How to approach this part

Remember that the rod as a whole must remain electrically neutral even if the charges are redistributed. This is because applying an electric field does not change the charge on the rod, only redistributes it.

ANSWER:

- ⊗ left end negative and right end positive
- ○ left end negative and right end negative
- ○ left end negative and right end nearly neutral
- ○ left end nearly neutral and right end positive
- ○ both ends nearly neutral



Part D

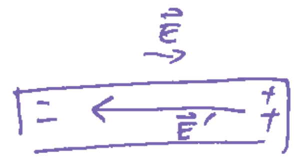
The charge imbalance that results from this movement of charge will generate an additional electric field in the region within the rod. In what direction will this field point?

Hint 1. Direction of the electric field

The electric field point away from positive charges and towards negative ones.

ANSWER:

- ○ It will point to the right and enhance the initial applied field.
- ⊗ It will point to the left and oppose the initial applied field.

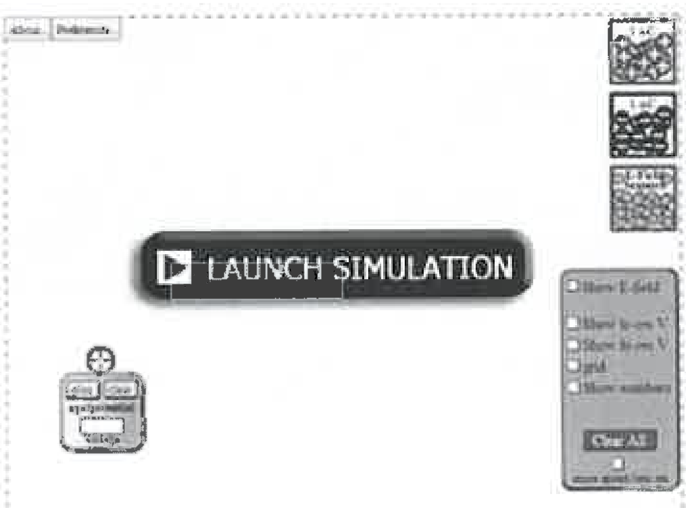


PhET Tutorial: Charges and Electric Fields

Learning Goal:

To understand the spatial distribution of the electric field for a variety of simple charge configurations.

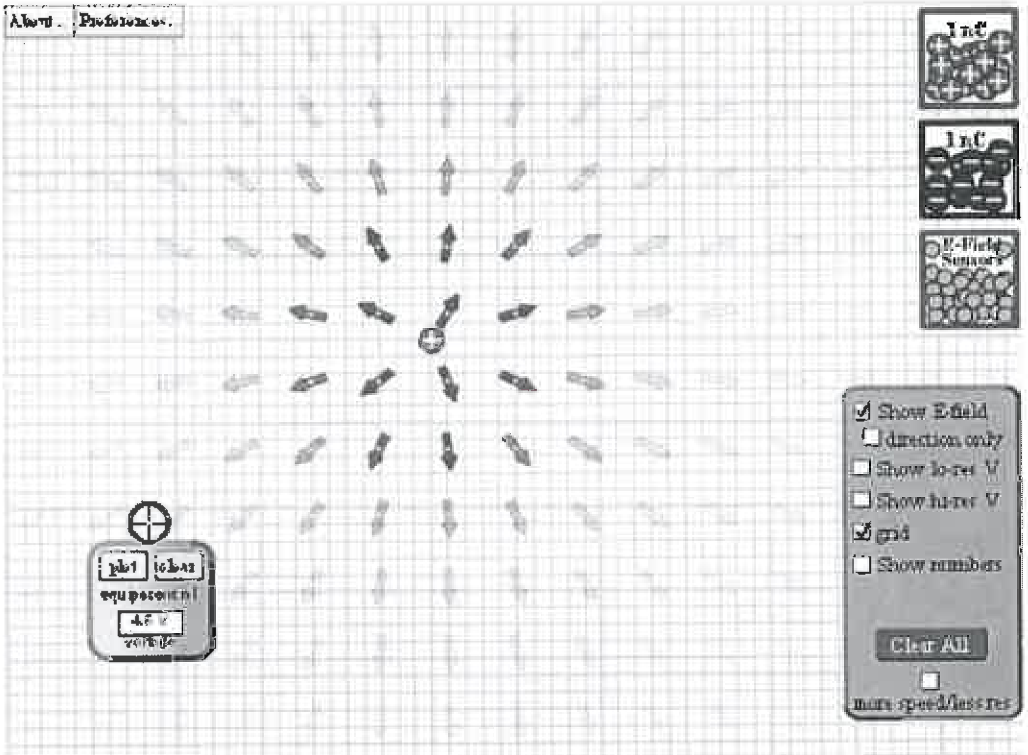
For this problem, use the PhET simulation *Charges and Fields*. This simulation allows you to place multiple positive and negative point-charges in any configuration and look at the resulting electric field.



Start the simulation. You can click and drag positive charges (red) or negative charges (blue) into the main screen. If you select **Show E-field** in the green menu, red arrows will appear, showing the direction of the electric field. Faint red arrows indicate that the electric field is weaker than at locations where the arrows are brighter (this simulation does not use arrow length as a measure of field magnitude).
Feel free to play around with the simulation. When you are done, click **Clear All** before beginning Part A.

Part A

Select **Show E-field** and **grid** in the green menu. Drag one positive charge and place it near the middle of the screen, right on top of two intersecting bold grid lines. You should see something similar to the figure below.



ANSWER:

The electric field produced by the positive charge

- [x] is directed radially away from the charge at all locations near the charge.
- [ ] wraps circularly around the positive charge.
- [ ] is directed radially toward the charge at all locations near the charge.

Part B

Now, let's look at how the distance from the charge affects the magnitude of the electric field. Select **Show numbers** on the green menu, and then click and drag one of the orange E-Field Sensors. You will see the magnitude of the electric field given in units of V/m (volts per meter, which is the same as newtons per coulomb).

Place the E-Field Sensor 1 m away from the positive charge (1 m is two bold grid lines away if going in a horizontal or vertical direction), and look at the resulting field strength.

Consider the locations to the right, left, above, and below the positive charge, all 1 m away.

ANSWER:

$\vec{E} = \frac{1}{4\pi\epsilon_0} \frac{q}{r^2} \hat{r}$, same $|\vec{r}|$, same magnitude

For these four locations, the magnitude of the electric field is

- [ ] greatest to the left of the charge.
- [ ] greatest below the charge.
- [x] the same.
- [ ] greatest to the right of the charge.
- [ ] greatest above the charge.

Part C

What is the magnitude of the electric field 1 m away from the positive charge compared to the magnitude of the electric field 2 m away?

Hint 1. How to approach the problem

Use an E-Field Sensor to determine the field strength both at 1 m away and at 2 m away from the charge. Then, take the ratio of the two field strengths.

ANSWER:

$|\vec{E}| = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}$ $|\vec{E_1}| = \frac{1}{4\pi\epsilon_0}\frac{q}{1^2} = 4|\vec{E_2}|$

The magnitude of the electric field 1 m away from the positive charge is

- ☒ four times
- ○ one-half
- ○ equal to
- ○ one-quarter
- ○ two times

$|\vec{E_2}| = \frac{1}{4\pi\epsilon_0}\frac{q}{2^2}$

the magnitude of the electric field 2 m away.

Part D

If the field strength is E = 9 $\mathrm{V/m}$ a distance of 1 m from the charge, what is the field strength E a distance of 3 m from the charge?

Hint 1. How to approach the problem

The magnitude of the electric field is inversely proportional to distance squared ($E \propto 1/r^2$). So if the distance is increased by a factor of three, the field strength must decrease by a factor of three squared. You could use the simulation to make a measurement (you might have to drag the charge away from the center so you have enough room to get 3 m away).

ANSWER: $|\vec{E}| = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}$ $9 = \frac{1}{4\pi\epsilon_0}\frac{q}{1^2} \Rightarrow \frac{q}{4\pi\epsilon_0} = 9$; $|\vec{E}| = \frac{q}{4\pi\epsilon_0}\frac{1}{3^2}$

E = 1 $\mathrm{V/m}$

$= \frac{9}{9} = 1$

Part E

Remove the positive charge by dragging it back to the basket, and drag a negative charge (blue) toward the middle of the screen. Determine how the electric field is different from that of the positive charge.
Which statement best describes the differences in the electric field due to a negative charge as compared to a positive charge?

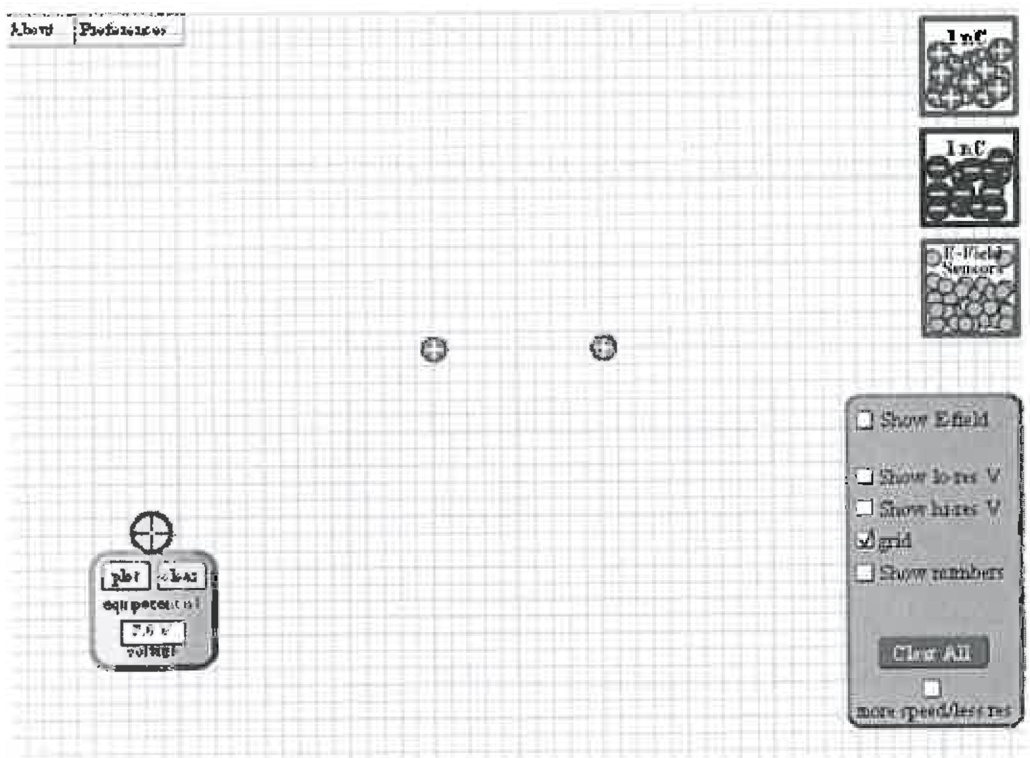
ANSWER:

- Nothing changes; the electric field remains directed radially outward, and the electric field strength doesnât change.
- The electric field changes direction (now points radially inward), but the electric field strength does not change.
- The electric field changes direction (now points radially inward), and the magnitude of the electric field decreases at all locations.

$\vec{E} = \frac{1}{4\pi\varepsilon_0} \frac{q}{r^2} \hat{r}$; q changed from "+" to "−"

Part F

Now, remove the negative charge, and drag two positive charges, placing them 1 m apart, as shown below.



Letâs look at the resulting electric field due to both charges. Recall that the electric field is a vector, so the net electric field is the vector sum of the electric fields due to each of the two charges.

Where is the magnitude of the electric field roughly equal to zero (other than very far away from the charges)?

ANSWER:

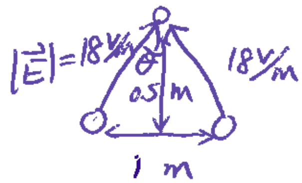
- The electric field is nonzero everywhere on the screen.
- The electric field is roughly zero near the midpoint of the two charges.
- The electric field is zero at any location along a vertical line going through the point directly between the two charges.

Part G

Consider a point 0.5 m above the midpoint of the two charges. As you can verify by removing one of the positive charges, the electric field due to only one of the positive charges is about 18 $\mathrm{V/m}$. What is the magnitude of the total electric field due to both charges at this location?

ANSWER:

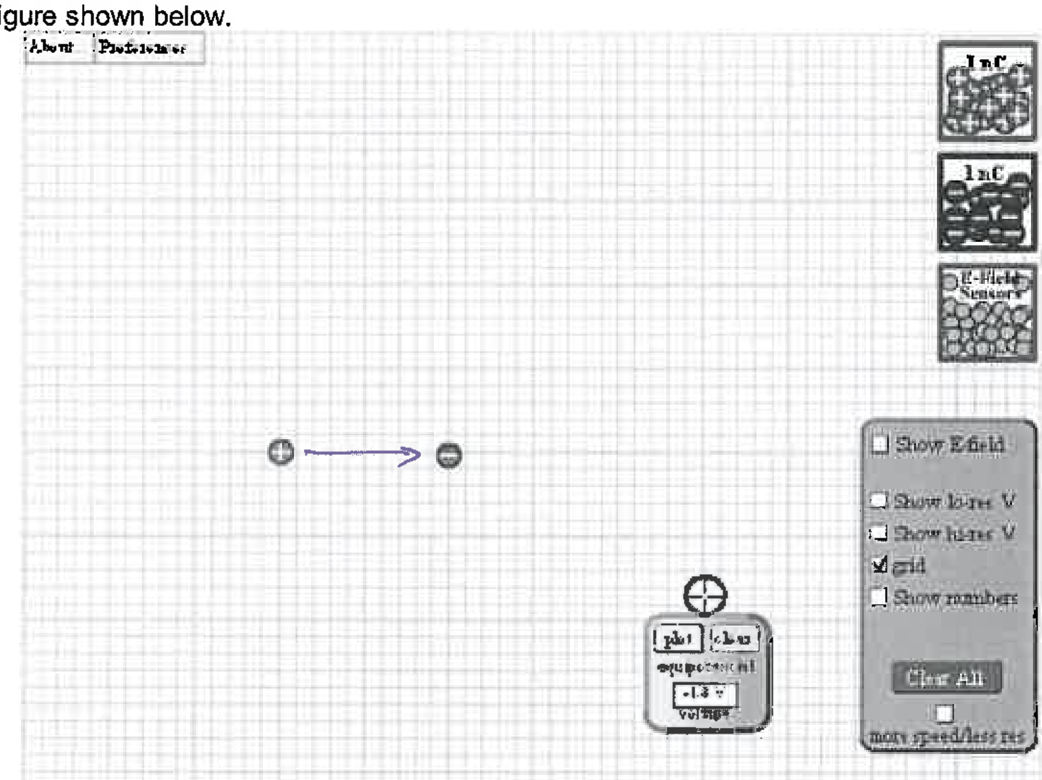
- zero
- 36 $\mathrm{V/m}$
- X 25 $\mathrm{V/m}$



$$|\vec{E}|_{total} = 18\cos\theta + 18\cos\theta$$
$$= 36\cos\theta$$
$$= 36 \times \frac{0.5}{\sqrt{(0.5)^2 + (0.5)^2}}$$
$$= 25.46 \ \mathrm{V/m}$$

Part H

Make an electric dipole by replacing one of the positive charges with a negative charge, so the final configuration looks like the figure shown below.



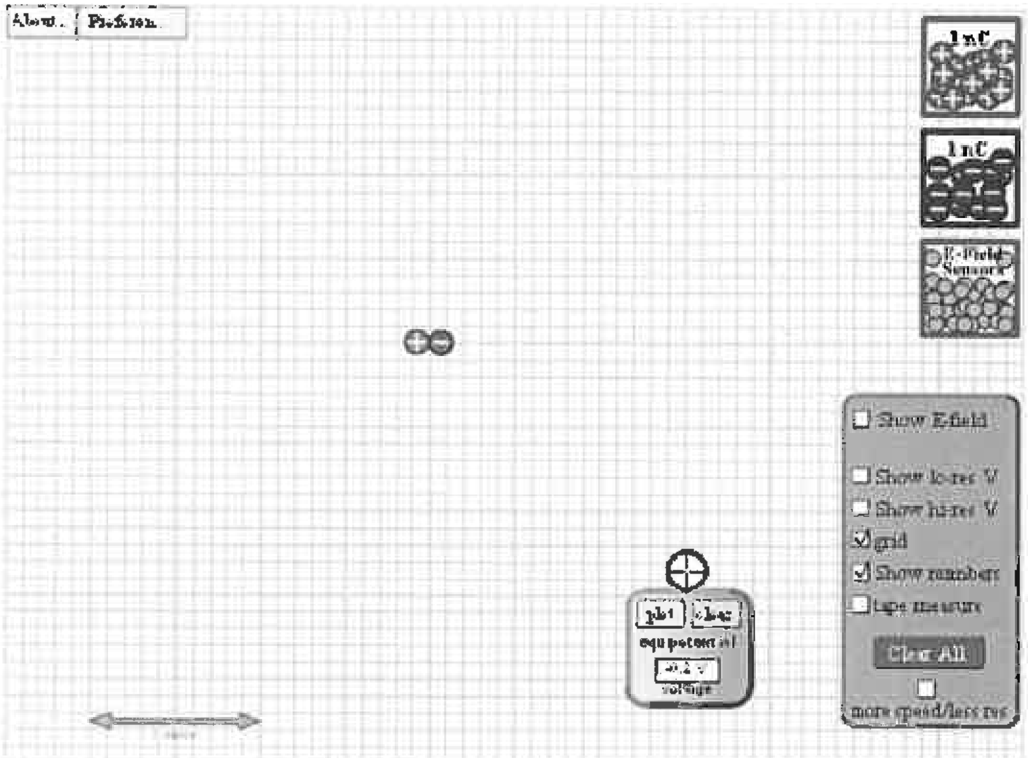
ANSWER:

The electric field at the midpoint is
- directed to the left.
- zero.
- ☒ directed to the right.

E - field starts @ "+"
ends @ "−"

Part I

Make a small dipole by bringing the two charges very close to each other, where they are barely touching. The midpoint of the two charges should still be on one of the grid point intersections (see figure below).



Measure the strength of the electric field 0.5 m directly above the midpoint as well as 1 m directly above. Does the strength of the electric field decrease as 1 over distance squared ($1/r^2$)?

Hint 1. How to approach the problem

If the strength of the field is decreasing as $1/r^2$, then the ratio of the magnitudes of the electric field measured at two distances, say 0.5 m away and 1 m away, would be

$$E_{r=0.5}/E_{r=1} = (1/0.5)^2/(1/1)^2 = 4.$$

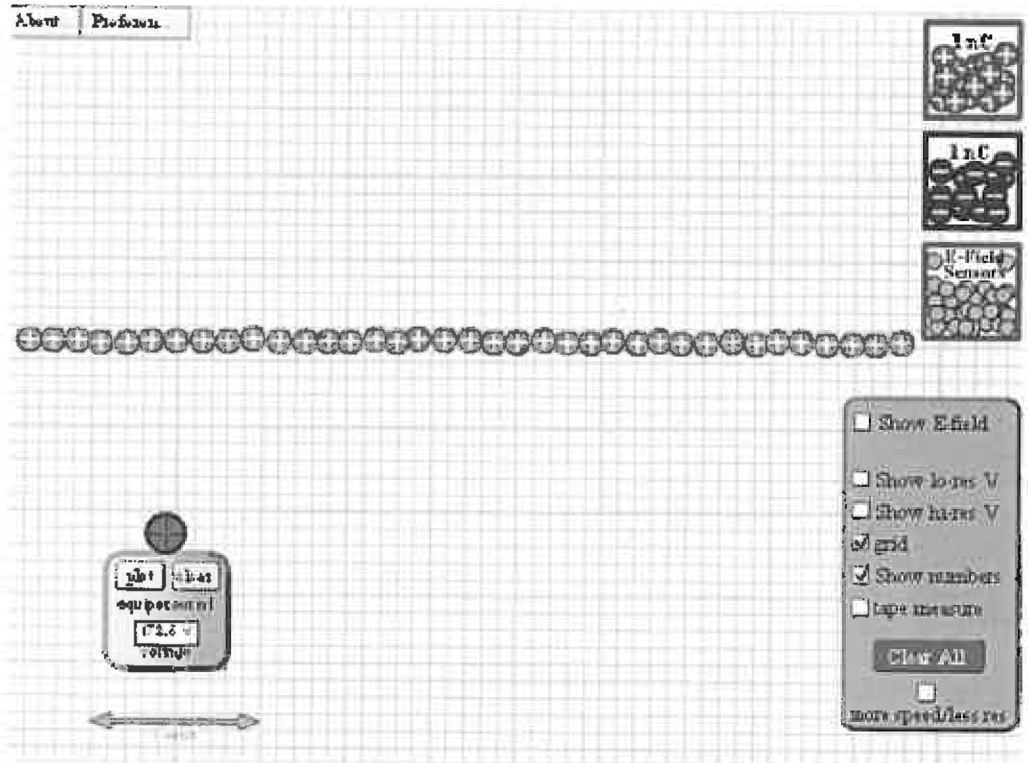
Compare this value to the value you measure with an E-Field Sensor.

ANSWER:

- ○ Yes, it does.
- ⊗ No, it decreases more quickly with distance.
- ○ No, it decreases less quickly with distance.

Part J

Make a long line of positive charges, similar to that shown in the figure below. Try to place all of the charges centered along a horizontal grid line. Feel free to look at the electric field, as it is interesting.



Measure the strength of the electric field 1 m directly above the middle as well as 2 m directly above. Does the strength of the electric field decrease as 1 over distance squared ($1/r^2$)?

ANSWER:

- ○ No, it decreases more quickly with distance.
- ○ Yes, it does.
- ⊗ No, it decreases less quickly with distance.

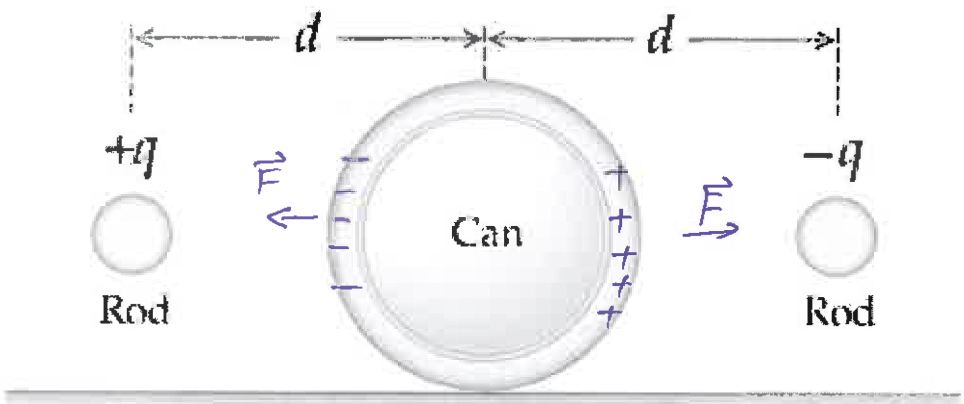
PhET Interactive Simulations
University of Colorado
http://phet.colorado.edu

Video Tutor: Charged Rod and Aluminum Can

First, launch the video below. You will be asked to use your knowledge of physics to predict the outcome of an experiment. Then, close the video window and answer the questions at right. You can watch the video again at any point.

Part A

Consider the situation in the figure below, where two charged rods are placed a distance d on either side of an aluminum can. What does the can do?



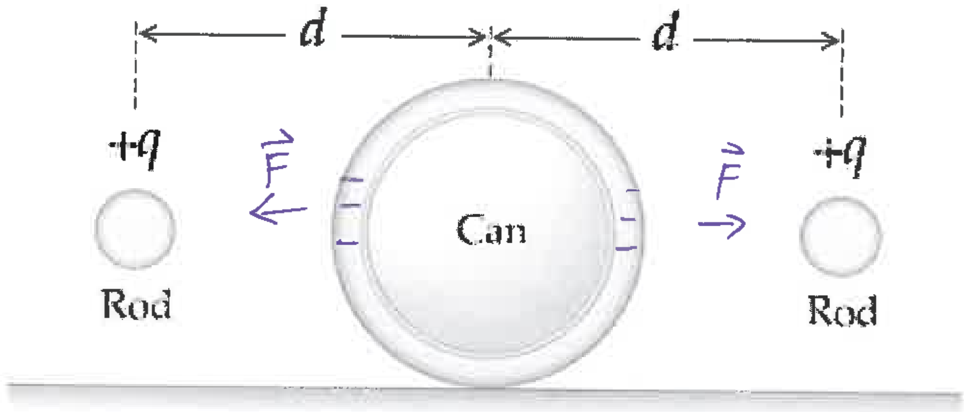
You did not open hints for this part.

ANSWER:

- ○ Rolls to the right
- ○ Rolls to the left
- ⊗ Stays still

Part B

Now, consider the situation shown in the figure below. What does the can do?



ANSWER:

- Rolls to the right
- Stays still
- Rolls to the left

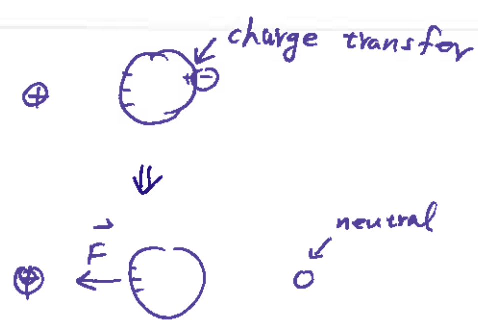
Part C

Using the setup from the *first* question, imagine that you briefly touch the *negatively* charged rod to the can. You then hold the two rods at equal distances on either side of the can. What does the can do?

You did not open hints for this part.

ANSWER:

- Rolls toward the positively charged rod
- Does not move
- Rolls away from the positively charged rod



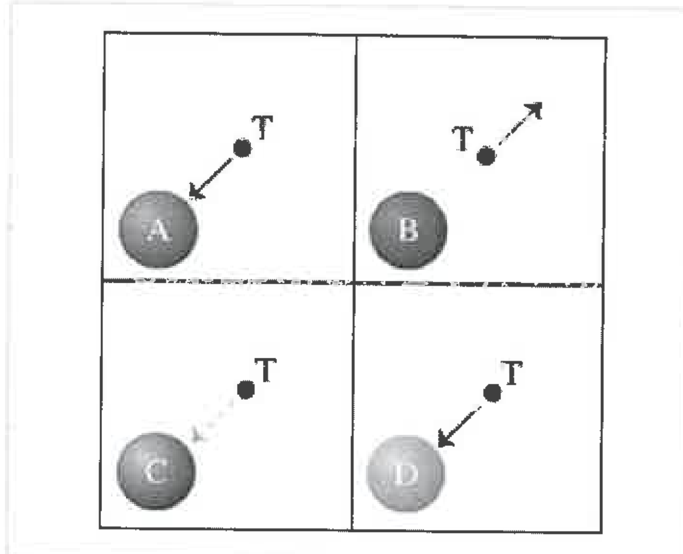
A Test Charge Determines Charge on Insulating and Conducting Balls

Learning Goal:

To understand the electric force between charged and uncharged conductors and insulators.

When a test charge is brought near a charged object, we know from Coulomb's law that it will experience a net force (either attractive or repulsive, depending on the nature of the object's charge). A test charge may also experience an electric force when brought near a *neutral* object. Any attraction of a neutral insulator or neutral conductor to a test charge must occur through induced polarization. In an insulator, the electrons are bound to their molecules. Though they cannot move freely throughout the insulator, they can shift slightly, creating a rather weak net attraction to a test charge that is brought close to the insulator's surface. In a conductor, free electrons will accumulate on the surface of the conductor nearest the positive test charge. This will create a strong attractive force if the test charge is placed very close to the conductor's surface.

Consider three plastic balls (A, B, and C), each carrying a uniformly distributed charge equal to either $+Q$, $-Q$ or zero, and an uncharged copper ball (D). A positive test charge (T) experiences the forces shown in the figure when brought very near to the individual balls. The test charge T is strongly attracted to A, strongly repelled from B, *weakly* attracted to C, and strongly attracted to D.



Assume throughout this problem that the balls are brought very close together.

Part A

What is the nature of the force between balls A and B?

You did not open hints for this part.

ANSWER:

- ☒ strongly attractive
- ○ strongly repulsive
- ○ weakly attractive
- ○ neither attractive nor repulsive

$+Q$ $-Q$

A $\rightarrow$ $\vec{F}$ $\leftarrow$ B

Part B

What is the nature of the force between balls A and C?

Hint 1. What is the charge on ball C?

Recall that ball C is composed of insulating material, which means that it can be polarized, but the charges inside are otherwise not free to move around inside the ball. Since the test charge experiences only a weak force due to ball C, if we compare to ball A we conclude that the charge on ball C must be

ANSWER:

- ○ +Q
- ○ -Q
- ○ zero

ANSWER:

- ○ strongly attractive
- ○ strongly repulsive
- ⊗ weakly attractive
- ○ neither attractive nor repulsive

Part C

What is the nature of the force between balls A and D?

Hint 1. What are the surface charges on ball D?

Recall that copper is a conductor, in which charges can freely flow. When ball D is brought close to ball A, what will be the nature of the surface charge density on the side of ball D that is closest to ball A?

ANSWER:

- ○ positive
- ○ negative
- ○ zero

ANSWER:

- ⊗ attractive
- ○ repulsive
- ○ neither attractive nor repulsive

Part D

What is the nature of the force between balls D and C?

ANSWER:

- ○ attractive
- ○ repulsive
- ⊗ neither attractive nor repulsive

both are uncharged

Coulomb's Law Tutorial

Learning Goal:

To understand how to calculate forces between charged particles, particularly the dependence on the sign of the charges and the distance between them.

Coulomb's law describes the force that two charged particles exert on each other (by Newton's third law, those two forces must be equal and opposite). The force $\vec{F}_{21}$ exerted *by* particle 2 (with charge q_2) *on* particle 1 (with charge q_1) is proportional to the charge of each particle and inversely proportional to the square of the distance r between them:

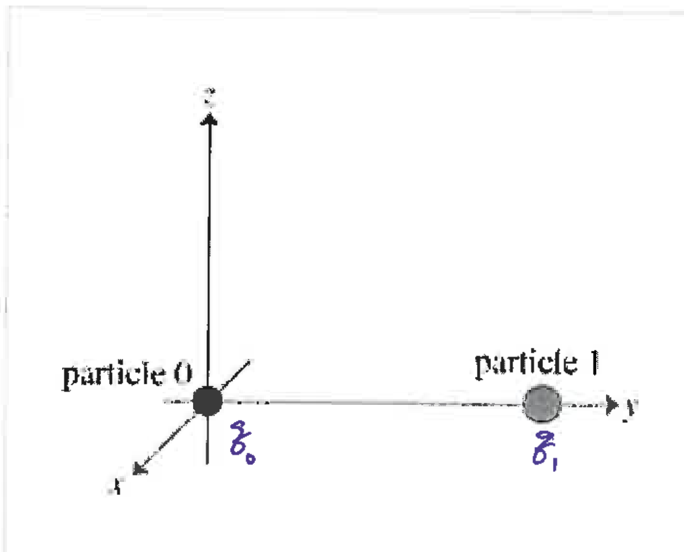
$$\vec{F}_{21} = \frac{k q_2 q_1}{r^2} \hat{r}_{21},$$

where $k = \frac{1}{4\pi\epsilon_0}$ and $\hat{r}_{21}$ is the unit vector pointing *from* particle 2 *to* particle 1. The force vector will be parallel or antiparallel to the direction of $\hat{r}_{21}$, parallel if the product $q_1 q_2 > 0$ and antiparallel if $q_1 q_2 < 0$; the force is *attractive* if the charges are of opposite sign and *repulsive* if the charges are of the same sign.

Part A

Consider two positively charged particles, one of charge q_0 (particle 0) fixed at the origin, and another of charge q_1 (particle 1) fixed on the *y*-axis at $(0, d_1, 0)$. What is the net force $\vec{F}$ *on* particle 0 *due to* particle 1?

Express your answer (a vector) using any or all of k, q_0, q_1, d_1, $\hat{i}$, $\hat{j}$, and $\hat{k}$.



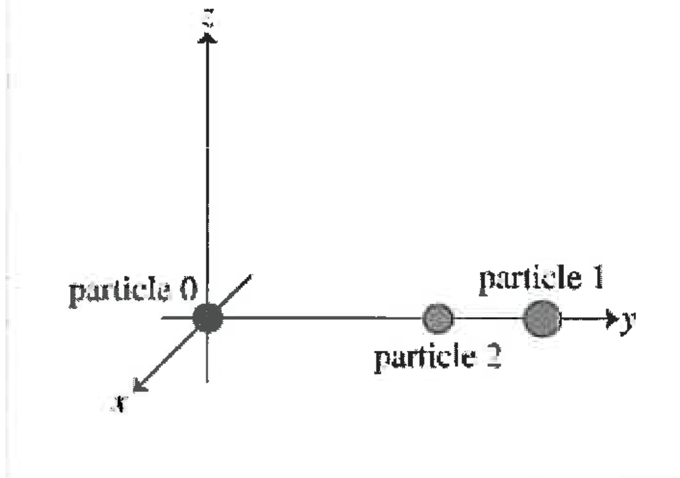
ANSWER: ★ the negative sign is because you know q_1 will repell q_0 to left, which is $-\hat{j}$ direction

$\vec{F} = -k \cdot \frac{q_0 q_1}{d_1^2} \hat{j}$

Part B

Now add a third, negatively charged, particle, whose charge is $-q_2$ (particle 2). Particle 2 fixed on the *y*-axis at position $(0,\, d_2,\, 0)$. What is the new net force *on* particle 0, *from* particle 1 and particle 2?

Express your answer (a vector) using any or all of k, q_0, q_1, q_2, d_1, d_2, $\hat{i}$, $\hat{j}$, and $\hat{k}$.



ANSWER:

$\vec{F} = \left(-k\dfrac{q_0 q_1}{d_1^2} + k\dfrac{q_0 q_2}{d_2^2}\right)\hat{j}$

Part C

Particle 0 experiences a repulsion *from* particle 1 and an attraction *toward* particle 2. For certain values of d_1 and d_2, the repulsion and attraction should balance each other, resulting in no net force. For what ratio d_1/d_2 is there no net force on particle 0?

Express your answer in terms of any or all of the following variables: k, q_0, q_1, q_2.

ANSWER:

$\vec{F} = 0 \Rightarrow -k\dfrac{q_0 q_1}{d_1^2} + k\dfrac{q_0 q_2}{d_2^2} = 0$

$d_1/d_2 = \sqrt{\dfrac{q_1}{q_2}}$

$\Rightarrow \dfrac{q_1}{d_1^2} = \dfrac{q_2}{d_2^2}$

$\Rightarrow \dfrac{d_1}{d_2} = \sqrt{\dfrac{q_1}{q_2}}$

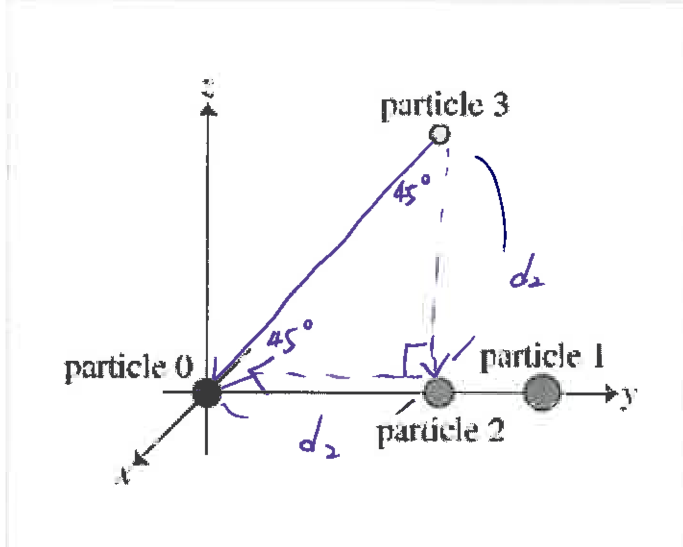
Part D

Now add a fourth charged particle, particle 3, with positive charge q_3, fixed in the *yz*-plane at $(0,\, d_2,\, d_2)$. What is the net force $\vec{F}$ on particle 0 due *solely* to this charge?

Express your answer (a vector) using k, q_0, q_3, d_2, $\hat{i}$, $\hat{j}$, and $\hat{k}$. Include only the force caused by particle 3.

$$|\vec{F}| = k\frac{q_0 q_3}{r^2} = k\frac{q_0 q_3}{2d_2^2}$$

$$\vec{F} = -k\frac{q_0 q_3}{2d_2^2}\cdot\frac{1}{\sqrt{2}}\hat{j} - k\frac{q_0 q_3}{2d_2^2}\frac{1}{\sqrt{2}}\hat{k}$$



You did not open hints for this part.

ANSWER:

$\vec{F}$ = $-k\frac{q_0 q_3}{2\sqrt{2}d_2^2}\hat{j} - k\frac{q_0 q_3}{2\sqrt{2}d_2^2}\hat{k}$

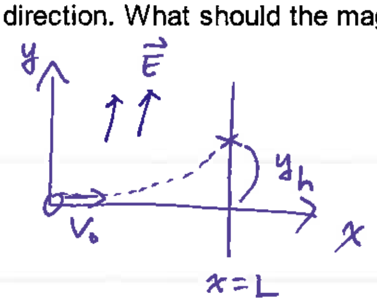
The Trajectory of a Charge in an Electric Field

An charge with mass m and charge q is emitted from the origin, $(x,y) = (0,0)$. A large, flat screen is located at $x = L$. There is a target on the screen at y position y_h, where $y_h > 0$. In this problem, you will examine two different ways that the charge might hit the target. Ignore gravity in this problem.

Part A

Assume that the charge is emitted with velocity v_0 in the positive x direction. Between the origin and the screen, the charge travels through a constant electric field pointing in the positive y direction. What should the magnitude E of the electric field be if the charge is to hit the target on the screen?

Express your answer in terms of m, q, y_h, v_0, and L.



You did not open hints for this part.

$$y_h = \frac{1}{2}\frac{q}{m}|\vec{E}|\frac{L^2}{V_0^2}$$

$$|\vec{E}| = \frac{2mV_0^2 y_h}{qL^2}$$

ANSWER:

E = $\frac{2mV_0^2 y_h}{qL^2}$

x: constant speed (V_0) motion

$L = V_0 \cdot t \Rightarrow t = \frac{L}{V_0}$

y: constant acceleration motion

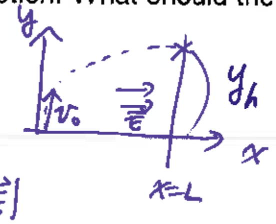
$\vec{F} = q\vec{E} = m\vec{a} \Rightarrow |\vec{a}| = \frac{q}{m}|\vec{E}|$

$y_h = 0 + \frac{1}{2}|\vec{a}|t^2$ $(s = V_0 t + \frac{1}{2}at^2)$

Part B

Now assume that the charge is emitted with velocity v_0 in the positive *y* direction. Between the origin and the screen, the charge travels through a constant electric field pointing in the positive *x* direction. What should the magnitude E of the electric field be if the charge is to hit the target on the screen?

Express your answer in terms of m, q, y_h, v_0, and L.



You did not open hints for this part.

x: constant acceleration motion

$\vec F = q\vec E = m\vec a \Rightarrow |\vec a| = \frac{q}{m}|\vec E|$

$(S = v_0 t + \frac{1}{2}at^2)$

$L = 0 + \frac{1}{2}\frac{q}{m}|\vec E|t^2 \Rightarrow t = \sqrt{\frac{2mL}{q|\vec E|}}$

y: constant speed motion

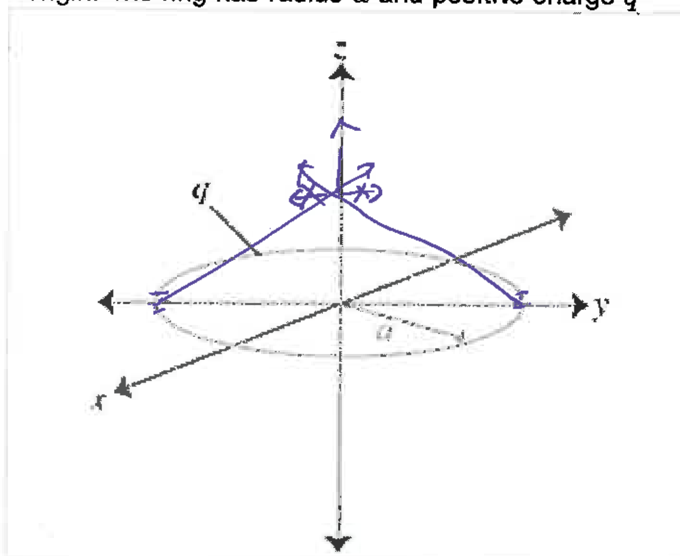
$y_h = v_0 \cdot t = v_0\sqrt{\frac{2mL}{q|\vec E|}} \Rightarrow |\vec E| = \frac{2mLv_0^2}{q y_h^2}$

ANSWER:

$E = \frac{2mLv_0^2}{q y_h^2}$

Charged Ring

Consider a uniformly charged ring in the *xy* plane, centered at the origin. The ring has radius a and positive charge q distributed evenly along its circumference.



Part A

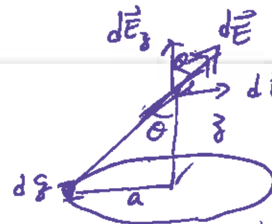
What is the direction of the electric field at any point on the z axis?

You did not open hints for this part.

ANSWER:

- ○ parallel to the *x* axis
- ○ parallel to the *y* axis
- ⊗ parallel to the *z* axis
- ○ in a circle parallel to the *xy* plane

Part B



What is the magnitude of the electric field along the positive *z* axis?

Use k in your answer, where $k = \frac{1}{4\pi\epsilon_0}$.

$(\vec{E} = k\frac{q}{r^2}\hat{r})$

$$dE_z = k\frac{dq}{a^2+z^2}\cdot\cos\theta \qquad \cos\theta = \frac{z}{\sqrt{a^2+z^2}}$$

$$= k\frac{dq}{a^2+z^2}\cdot\frac{z}{\sqrt{a^2+z^2}} = k\frac{dq\cdot z}{(a^2+z^2)^{3/2}}$$

You did not open hints for this part.

ANSWER:

$E(z)$ = $k\frac{q\,z}{(a^2+z^2)^{3/2}}$

$$E_z = \int dE_z = \int k\frac{dq\cdot z}{(a^2+z^2)^{3/2}}$$

$$= k\frac{q\cdot z}{(a^2+z^2)^{3/2}}$$

Part C

Imagine a small metal ball of mass m and negative charge $-q_0$. The ball is released from rest at the point $(0, 0, d)$ and constrained to move along the z axis, with no damping. If $0 < d \ll a$, what will be the ball's subsequent trajectory?

ANSWER:

- ○ repelled from the origin
- ○ attracted toward the origin and coming to rest
- ⊗ oscillating along the z axis between $z = d$ and $z = -d$
- ○ circling around the z axis at $z = d$

Part D

The ball will oscillate along the z axis between $z = d$ and $z = -d$ in simple harmonic motion. What will be the angular frequency ω of these oscillations? Use the approximation $d \ll a$ to simplify your calculation; that is, assume that $d^2 + a^2 \approx a^2$.

Express your answer in terms of given charges, dimensions, and constants.

Hint 1. Simple harmonic motion

Recall the nature of simple harmonic motion of an object attached to a spring. Newton's second law for the system states that

$$F_x = m\frac{d^2x}{dt^2} = -k'x,$$ leading to oscillation at a frequency of $\omega = \sqrt{\frac{k'}{m}}$

(here, the prime on the symbol representing the spring constant is to distinguish it from $k = \frac{1}{4\pi\epsilon_0}$). The solution to this differential equation is a sinusoidal function of time with angular frequency ω. Write an analogous equation for the ball near the charged ring in order to find the ω term.

Hint 2. Find the force on the charge

What is F_z, the z component of the force on the ball on the ball at the point $(0, 0, d)$? Use the approximation $d^2 + a^2 \approx a^2$.

Express your answer in terms of q_0, k, q, d, and a.

Hint 1. A formula for the force on a charge in an electric field

The formula for the force $\vec{F}$ on a charge q in an electric field $\vec{E}$ is

$$\vec{F} = q\vec{E}.$$

Therfore, in particular,

$$F_z = qE_z.$$

You have already found $E_z(z)$ in Part B. Use that expression in the equation above to find an expression for the z component of the force F_z on the ball at the point $(0, 0, d)$. Don't forget to use the approximation given.

ANSWER:

$F_z =$

use answer in Part B

$\vec{F} = q\vec{E} = -q_0 \cdot k \frac{qz}{(a^2+z^2)^{3/2}}$; $z = d$

$= -k\frac{qq_0 d}{(a^2+d^2)^{3/2}} = -k\frac{qq_0 d}{(a^2)^{3/2}}$ $(a^2+d^2 \simeq a^2)$

$= -k\frac{qq_0}{a^3}d$

$\vec{F}(z) = -k\frac{qq_0}{a^3}z = m|\vec{a}_z| = m\frac{d^2z}{dt^2}$ $(\vec{F} = m\vec{a})$

SHM: $z(t) = A\sin(wt)$

$\ddot{z} = -Aw^2\sin(wt) = -w^2 z$

$\frac{d^2z}{dt^2} = -\frac{kqq_0}{ma^3}z$

$\Rightarrow w = \sqrt{\frac{kqq_0}{ma^3}}$

ANSWER:

$\omega = \sqrt{\frac{kqq_0}{ma^3}}$

The Electric Field Produced by a Finite Charged Wire

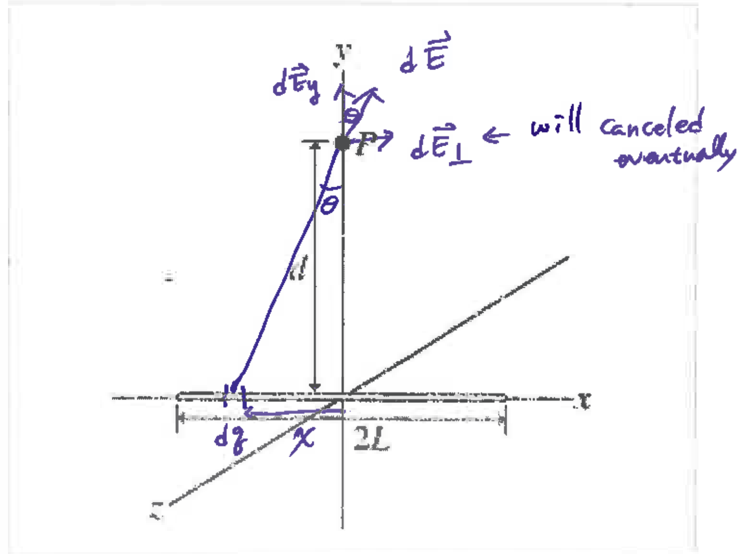
A charged wire of negligible thickness has length $2L$ units and has a linear charge density λ. Consider the electric field $\vec{E}$ at the point P, a distance d above the midpoint of the wire.

$$d\vec{E}_y = \underbrace{k\frac{dq}{d^2+x^2}}_{k\frac{dq}{r^2}} \cdot \underbrace{\frac{d}{\sqrt{d^2+x^2}}}_{\cos\theta}$$

$$\star\ dq = \lambda \cdot dx$$

$$d\vec{E}_y = k\,\frac{\lambda d}{(d^2+x^2)^{3/2}}\,dx$$

$$E_y = \int dE_y = \int_{-L}^{L} k\,\frac{\lambda d}{(d^2+x^2)^{3/2}}\,dx$$



$$E_y = k\lambda d \int_{-L}^{L} \frac{1}{(d^2+x^2)^{3/2}}\,dx$$

use integral eq. two pages later

$$E_y = k\lambda d \cdot \frac{x}{d^2\sqrt{d^2+x^2}}\Bigg|_{-L}^{L}$$

$$= k\lambda d\left(\frac{L}{d^2\sqrt{d^2+L^2}} - \frac{-L}{d^2\sqrt{d^2+L^2}}\right)$$

$$= \frac{k\lambda d \cdot 2L}{d^2\sqrt{d^2+L^2}}$$

Part A

The field $\vec{E}$ points along one of the primary axes. Which one?

You did not open hints for this part.

ANSWER:

- ○ $\hat{i}$
- ⊗ $\hat{j}$
- ○ $\hat{k}$

Part B

What is the magnitude E of the electric field at point P? Throughout this part, express your answers in terms of the constant k, defined by $k = \frac{1}{4\pi\epsilon_0}$.

Express your answer in terms of L, λ, d, and k.

Hint 1. How to approach the problem

To compute the field at point P, divide the rod into infinitesimal segments and find the electric field produced by each. Then integrate to find the total electric field. The electric field due to an arbitrary infinitesimal segment of wire has both *x* and *y* components. You could integrate these two components separately, but because of symmetry, you already know that the total electric field points only in the *y* direction. So you need to work only with the *y* component of the field.

Hint 2. Find the field due to an infinitesimal segment

Find $dE_y(x)$, the y component of the electric field produced at point P by the infinitesimal segment of the wire between x and $x+dx$.

Express your answer in terms of x, d, dx, λ, and k.

Hint 1. Find the total electric field

Find the magnitude dE of the electric field at point P produced by the infinitesimal wire segment between x and $x+dx$.

Express your answer in terms of λ, d, dx, x, and the constant k.

Hint 1. Formula for electric field due to a point charge

The electric field $\overrightarrow{dE}$ at position $\vec{r}$ away from an infinitesimal charge element dq is given by $\overrightarrow{dE} = k\frac{dq}{r^2}\hat{r}$.

Hint 2. Find $|\vec{r}|$

What is the distance from an infinitesimal segment of wire at position x to the point P?

Express your answer in terms of x and d.

ANSWER:

$|\vec{r}|$ =

Hint 3. Find an expression for dq

The wire has charge per unit length λ, and the length of an infinitesimal charge element is dx. Write an expression for dq, the total charge on the infinitesimal element dx.

Express your answer in terms of λ and dx.

ANSWER:

dq =

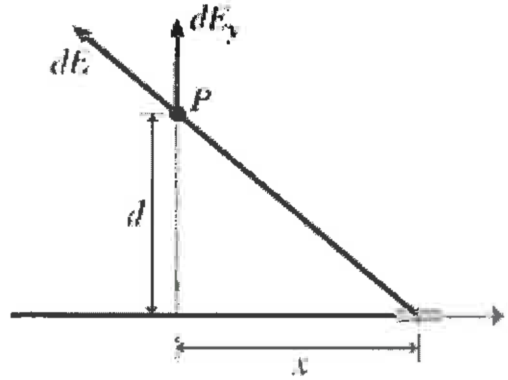
ANSWER:

dE =

Hint 2. Find the *y* component of dE

What is dE_y, the *y* component of the electric field at point P due to an infinitesimal segment of wire located at position x? Assume that the magnitude of the electric field at point P due to the infinitesimal wire segment is dE.

Express your answer in terms of dE, x, and d.



ANSWER:

dE_y =

ANSWER:

$dE_y(x)$ =

Hint 3. A necessary integral

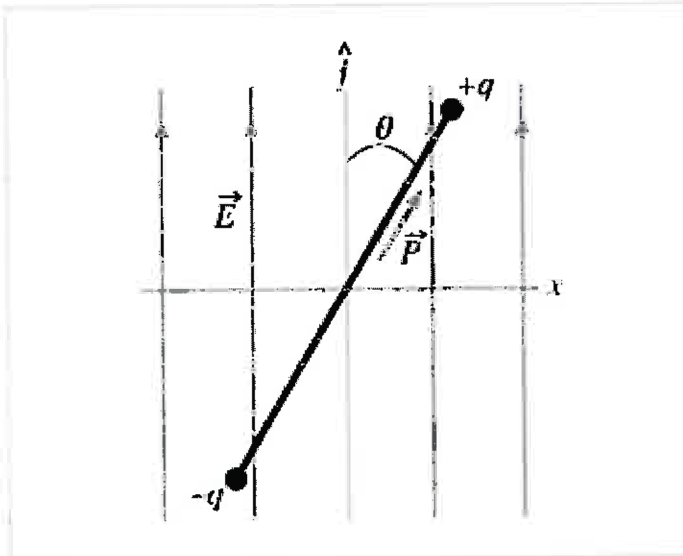
$$\int \frac{1}{\sqrt{(a^2+x^2)^3}}\, dx = \frac{x}{a^2\sqrt{a^2+x^2}}$$

ANSWER:

$E = \frac{2k\lambda L}{d\sqrt{d^2+L^2}}$ (see calculations two page above)

Dipole Motion in a Uniform Field

Consider an electric dipole located in a region with an electric field of magnitude E pointing in the positive y direction. The positive and negative ends of the dipole have charges $+q$ and $-q$, respectively, and the two charges are a distance D apart. The dipole has moment of inertia I about its center of mass. The dipole is released from angle $\theta = \theta_0$, and it is allowed to rotate freely.



Part A

What is ω_{max}, the magnitude of the dipole's angular velocity when it is pointing along the y axis?

Express your answer in terms of quantities given in the problem introduction.

use energy concept

You did not open hints for this part.

$\cancel{\vec{\tau} = \vec{p}\times\vec{E} = q\vec{d}\times\vec{E}}$

$\cancel{|\vec{\tau}| = |q||\vec{d}||\vec{E}|\sin\theta}$

$E_i = E_f$

$U_i + K_i = U_f + K_f$

$U = -\vec{p}\cdot\vec{E}$

$-|\vec{p}|\cdot|\vec{E}|\cos\theta_i + 0 = -|\vec{p}||\vec{E}| + \frac{1}{2}I\omega_{max}^2$

ANSWER:

$$\omega_{max} = \sqrt{\frac{2qd|\vec{E}|(1-\cos\theta_i)}{I}}$$

$$\Rightarrow \omega_{max}^2 = \frac{2q\cdot d\cdot|\vec{E}|(1-\cos\theta_i)}{I}$$

Part B

If θ_0 is small, the dipole will exhibit simple harmonic motion after it is released. What is the period T of the dipole's oscillations in this case?

Express your answer in terms of π and quantities given in the problem introduction.

You did not open hints for this part.

$\vec{\tau} = \vec{p}\times\vec{E}$

$|\vec{\tau}| = -|\vec{p}|\cdot|\vec{E}|\cdot\sin\theta = I\cdot\alpha = I\cdot\frac{d^2\theta}{dt^2}$

ANSWER:

when θ_0 is small (sin$\theta \approx \theta$)

$-|\vec{p}||\vec{E}|\cdot\theta = I\cdot\frac{d^2\theta}{dt^2} \Rightarrow \frac{d^2\theta}{dt^2} = -\frac{q\cdot d\cdot|\vec{E}|}{I}\theta$

SHM: $\theta(t) = A\sin(wt)$

$\ddot{\theta} = -Aw^2\sin(wt) = -w^2\theta$

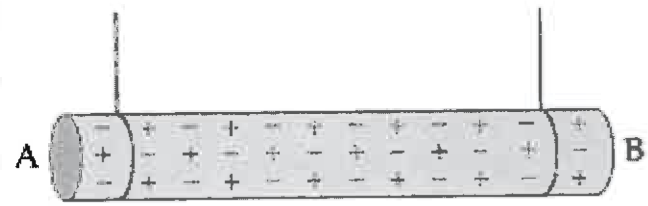
$\Rightarrow w^2 = \frac{qd|\vec{E}|}{I}$

$w = \sqrt{\frac{qd|\vec{E}|}{I}} = \frac{2\pi}{T}$ ⇒ next page

$T = 2\pi\sqrt{\frac{I}{qd|\vec{E}|}}$

Charging an Insulator

This problem explores the behavior of charge on realistic (i.e. non-ideal) insulators. We take as an example a long insulating rod suspended by insulating wires. Assume that the rod is initially electrically neutral. For convenience, we will refer to the left end of the rod as end A, and the right end of the rod as end B . In the answer options for this problem, "weakly attracted/repelled" means "attracted/repelled with a force of magnitude similar to that which would exist between two balls, one of which is charged, and the other acquires a small induced charge". An attractive/repulsive force greater than this should be classified as "strongly attracted/repelled".



Part A

A small metal ball is given a negative charge, then brought near (i.e., within a few millimeters) to end A of the rod. What happens to end A of the rod when the ball approaches it closely this first time?

Select the expected behavior.

You did not open hints for this part.

ANSWER:

- ○ strongly repelled
- ○ strongly attracted
- ○ weakly attracted
- ○ weakly repelled
- ○ neither attracted nor repelled

Now consider what happens when the small metal ball is *repeatedly* given a negative charge and then brought *into contact* with end A of the rod

Part B

After several contacts with the charged ball, how is the charge on the rod arranged?

Select the best description.

You did not open hints for this part.

ANSWER:

- ○ positive charge on end B and negative charge on end A
- ○ negative charge spread evenly on both ends
- ☒ negative charge on end A with end B remaining almost neutral
- ○ positive charge on end A with end B remaining almost neutral
- ○ none of the above

Part C

How does end A of the rod react when the ball approaches it after it has already made several contacts with the rod, such that a fairly large charge has been deposited at end A?

Select the expected behavior.

ANSWER:

- ☒ strongly repelled
- ○ strongly attracted
- ○ weakly attracted
- ○ weakly repelled
- ○ neither attracted nor repelled

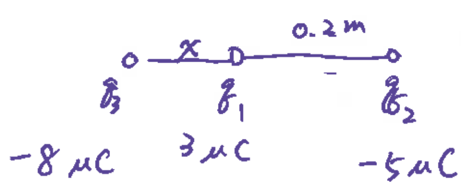
Exercise 21.19

Three point charges are arranged along the x-axis. Charge q_1 = +3.00 μC is at the origin, and charge q_2 = -5.00 μC is at x = 0.200 m. Charge q_3 = -8.00 μC.

Part A

Where is q_3 located if the net force on q_1 is 7.00 N in the $-x$ direction?

ANSWER:



$-8\,\mu C$

$\vec{F} = k\frac{q_1 q_2}{(0.2)^2} - k\frac{q_1 q_3}{x^2}$

(next page)

$k \approx 9\times10^{9}\ Nm^2 C^{-2}$

$\vec{F} = k\frac{(3\mu)(5\mu)}{(0.2)^2} - k\frac{(3\mu)(8\mu)}{x^2}$
$= -7\ (N)$
$\Rightarrow x = 0.144\ m$

x_3 = 0.144 m

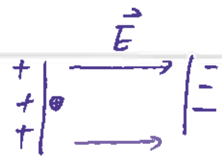
Exercise 21.38

$q = e = 1.6\times10^{-19}\ C$
$// \ m_p = 1.67\times10^{-27}\ kg$

A uniform electric field exists in the region between two oppositely charged plane parallel plates. A proton is released from rest at the surface of the positively charged plate and strikes the surface of the opposite plate, 1.52cm distant from the first, in a time interval of 1.57×10^{-6}s .

Part A

Find the magnitude of the electric field.



$\vec{F} = q\vec{E} = m_p\vec{a}$
$\vec{a} = \frac{q}{m_p}\vec{E}$
$s = v_0 t + \frac{1}{2}at^2$
$1.52\times10^{-2} = 0 + \frac{1}{2}\frac{q}{m_p}|\vec{E}|(1.57\times10^{-6})^2$
$= \frac{1}{2}\frac{1.6\times10^{-19}}{1.67\times10^{-27}}|\vec{E}| \times(1.57\times10^{-6})^2$

ANSWER:

E = 1.31×10^{2} N/C

$1.52\times10^{-2} = 1.18\times10^{-4}\ |\vec{E}|$

Part B

Find the speed of the proton when it strikes the negatively charged plate.

$V = V_0 + at$
$= 0 + \frac{q}{m_p}|\vec{E}|\cdot t$
$= \frac{1.6\times10^{-19}}{1.67\times10^{-27}}\cdot 1.31\times10^{2}\cdot 1.57\times10^{-6}$

ANSWER:

v = 1.97×10^{4} m/s

Exercise 21.53

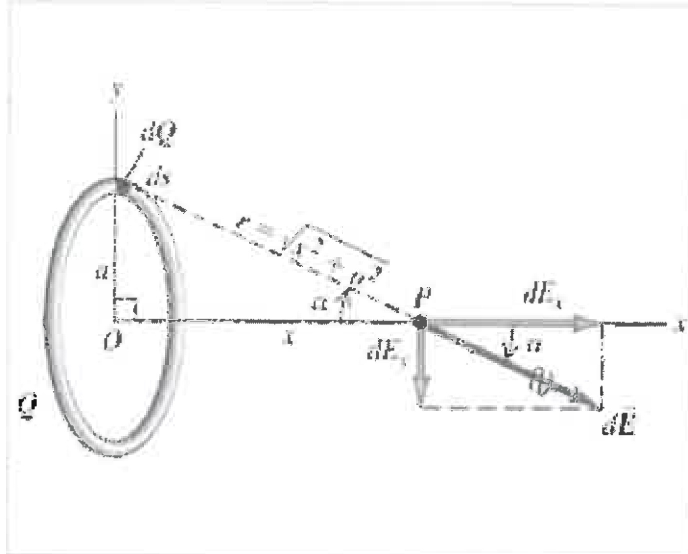
A ring-shaped conductor with radius a = 2.80cm has a total positive charge Q = 0.123nC uniformly distributed around it.

see page 18

$$E_x = k \frac{q x}{(a^2+x^2)^{3/2}}$$

$$= 9\times10^9 \frac{0.123\times10^{-9} \cdot 39\times10^{-2}}{((2.8\times10^{-2})^2 + (39\times10^{-2})^2)^{3/2}}$$

$$= \frac{43.173\times10^{-2}}{(0.1529)^{3/2}} = 7.22$$



Part A

What is the magnitude of the electric field at point P, which is on the positive x-axis at x = 39.0cm ?

ANSWER:

E = 7.22 N/C

Part B

What is the direction of the electric field at point P?

ANSWER:

- ☒ +x-direction
- ○ -x-direction

Part C

A particle with a charge of $-2.90\mu C$ is placed at the point P described in part A. What is the magnitude of the force exerted *by* the particle *on* the ring?

$F = q\vec{E} = -2.9\,\mu C \cdot 7.22$
$= 2.9\times10^{-6} \times 7.22$

ANSWER:

F = 2.09×10^{-5} N

Part D

What is the direction of the force exerted *by* the particle *on* the ring?

ANSWER:

- ☒ +*x*-direction
- ○ -*x*-direction

Exercise 21.57

Point charges $q_1 = -4.20\mathrm{nC}$ and $q_2 = +4.20\mathrm{nC}$ are separated by a distance of 3.20mm, forming an electric dipole.

Part A

Find the magnitude of the electric dipole moment.

$|\vec{P}| = q \cdot d = 4.2\times10^{-9} \times 3.2\times10^{-3}$

$= 13.44\times10^{-12}$

ANSWER:

$p = 1.344\times10^{-11}$ $\mathrm{C\cdot m}$

Part B

Find the direction of the electric dipole moment?

for e-dipole $\vec{P}$: from $-$ to $+$

ANSWER:

- ☒ from q_1 to q_2
- ○ from q_2 to q_1

Part C

The charges are in a uniform electric field whose direction makes an angle of 37.0° with the line connecting the charges. What is the magnitude of this field if the torque exerted on the dipole has magnitude $7.70\times10^{-9}\mathrm{N\cdot m}$?

ANSWER:

$E = 9.5\times10^{2}$ $\mathrm{N/C}$

$\vec{\tau} = \vec{P}\times\vec{E}$

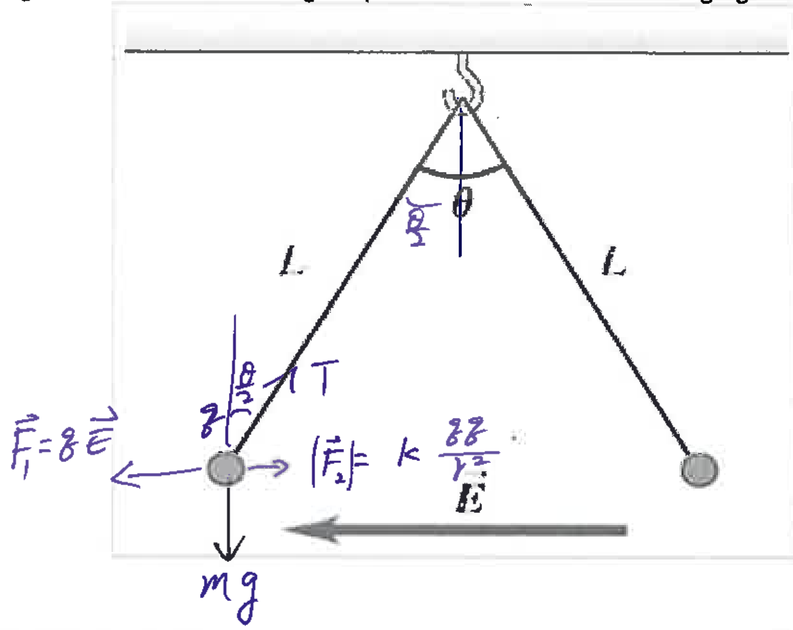
$|\vec{\tau}| = |\vec{P}|\cdot|\vec{E}|\sin\theta$

$7.7\times10^{-9} = 1.344\times10^{-11}\,|\vec{E}|\cdot\sin 37°$

$|\vec{E}| = 9.5\times10^{2}$

Problem 21.82

Two tiny spheres of mass m = 5.90mg carry charges of equal magnitude, 72.0 nC, but opposite sign. They are tied to the same ceiling hook by light strings of length 0.530 m. When a horizontal uniform electric field E that is directed to the left is turned on, the spheres hang at rest with the angle θ between the strings equal to 50.0° in the following figure.



Part A

Which ball (the one on the right or the one on the left) has positive charge?

ANSWER:

- [x] The one on the left
- [ ] The one on the right

y: $mg = T\cos\frac{\theta}{2} \Rightarrow T = \frac{mg}{\cos\frac{\theta}{2}}$

x: $T\sin\frac{\theta}{2} + F_2 = F_1$

$mg\tan\frac{\theta}{2} + k\frac{q^2}{r^2} = q|\vec E|$

$|\vec E| = \frac{mg}{q}\tan\frac{\theta}{2} + k\frac{q}{r^2}$ | $r = 2L\sin\frac{\theta}{2}$

Part B

What is the magnitude E of the field?

Express your answer with the appropriate units.

ANSWER:

E = 3.6×10^3

$= \frac{5.9\times10^{-6}\times9.8}{72\times10^{-9}}\tan25^\circ + 9\times10^9\cdot\frac{72\times10^{-9}}{(2\cdot0.53\sin25^\circ)^2}$

$= 0.37\times10^3 + 3.23\times10^3$

$= 3.6\times10^3$

Score Summary:

Your score on this assignment is 0.0%.
You received 0 out of a possible total of 15 points.