HW 3

Due: 11:59pm on Sunday, February 9, 2014

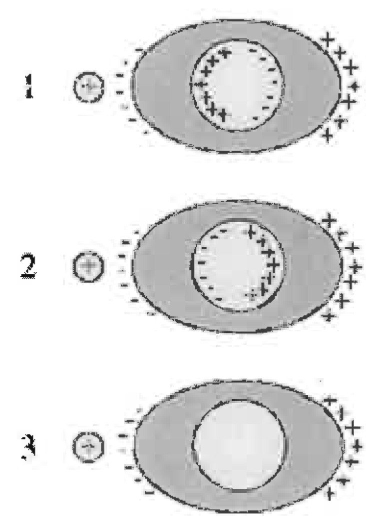
You will receive no credit for items you complete after the assignment is due. Grading Policy

Charge Distribution on a Conductor with a Cavity

A positive charge is brought close to a fixed neutral conductor that has a cavity. The cavity is neutral; that is, there is no *net* charge inside the cavity.

Part A

Which of the figures best represents the charge distribution on the inner and outer walls of the conductor?



You did not open hints for this part.

ANSWER:

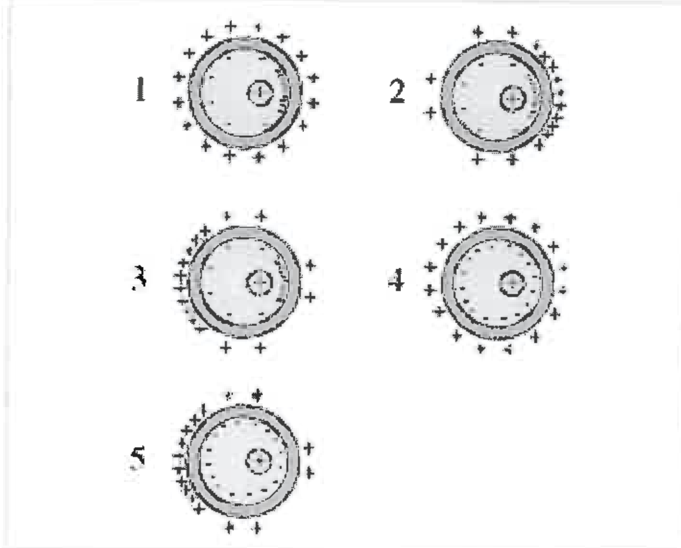
- 1
- 2
- 3

Charge Distribution on a Conducting Shell - 2

A positive charge is kept (fixed) off-center inside a fixed spherical conducting shell that is electrically neutral, and the charges in the shell are allowed to reach electrostatic equilibrium.

Part A

The large positive charge inside the shell is roughly 16 times that of the smaller charges shown on the inner and outer surfaces of the spherical shell. Which of the following figures best represents the charge distribution on the inner and outer walls of the shell?



You did not open hints for this part.

ANSWER:

- [x] 1
- [ ] 2
- [ ] 3
- [ ] 4
- [ ] 5

Exercise 22.15

Two very long uniform lines of charge are parallel and are separated by $0.350\mathrm{m}$. Each line of charge has charge per unit length $4.90\mu\mathrm{C}/\mathrm{m}$.

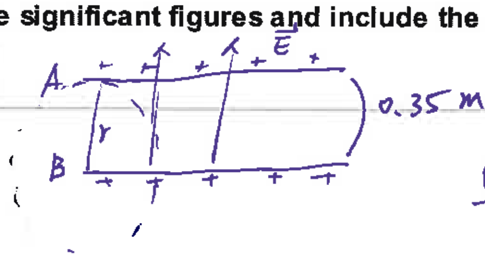
Part A

What magnitude of force does one line of charge exert on a $5.90\times10^{-2}\mathrm{m}$ section of the other line of charge?

Express your answer to three significant figures and include the appropriate units.

ANSWER:

$F = 7.2\times10^{-2}$



* First, use Gauss law to find $\vec{E}$ due to B line @ A position

$\Phi_E = \oint \vec{E}\cdot d\vec{A} = |\vec{E}|\cdot 2\pi r\cdot L = \frac{L\cdot\lambda}{\epsilon_0}$

$|\vec{E}| = \frac{1}{2\pi\epsilon_0}\frac{\lambda}{r} = \frac{1}{2\pi\times 8.85\times10^{-12}}\frac{4.9\times10^{-6}}{0.35}$

$= 0.25\times10^{6} = 2.5\times10^{5}$

$\vec{F} = Q\vec{E} = \lambda\cdot l\cdot|\vec{E}|$

$= 4.9\times10^{-6}\times 5.9\times10^{-2}\times 2.5\times10^{5}$

$= 72\times10^{-3}$

Electric Potential Energy versus Electric Potential

Learning Goal:

To understand the relationship and differences between electric potential and electric potential energy.

In this problem we will learn about the relationships between electric force $\vec{F}$, electric field $\vec{E}$, potential energy U, and electric potential V. To understand these concepts, we will first study a system with which you are already familiar: the uniform gravitational field.

Gravitational Force and Potential Energy

First we review the force and potential energy of an object of mass m in a uniform gravitational field that points downward (in the $-\hat{k}$ direction), like the gravitational field near the earth's surface.

Part A

Find the force $\vec{F}(z)$ on an object of mass m in the uniform gravitational field when it is at height $z = 0$.

Express $\vec{F}(z)$ in terms of m, z, $\hat{k}$, and g.

ANSWER:

$\vec{F}(z) = -mg\hat{k}$

Part B

Now find the gravitational potential energy $U(z)$ of the object when it is at an arbitrary height z. Take zero potential to be at position $z = 0$. Keep in mind that the potential energy is a scalar, not a vector.

Express $U(z)$ in terms of m, z, and g.

ANSWER:

$U(z) = mgz$

Part C

In what direction does the object accelerate when released with initial velocity upward?

ANSWER:

- ○ upward
- ⊗ downward
- ○ upward or downward depending on its mass
- ○ downward only if the ratio of g to initial velocity is large enough

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Electric Force and Potential Energy

Now consider the analogous case of a particle with charge q placed in a uniform electric field of strength E, pointing downward (in the $-\hat{k}$ direction)

Part D

Find $\vec{F}(z)$, the electric force on the charged particle at height z.

Express $\vec{F}(z)$ in terms of q, E, z, and $\hat{k}$.

You did not open hints for this part.

ANSWER:

$\vec{F}(z) = -qE\hat{k}$

Part E

Now find the potential energy $U(z)$ of this charged particle when it is at height z. Take zero potential to be at position $z = 0$.

Express $U(z)$ in terms of q, E, and z.

ANSWER:

$U(z) = qEz$

Part F

In what direction does the charged particle accelerate when released with upward initial velocity?

ANSWER:

- ○ upward
- ○ downward
- ⊗ upward or downward depending on its charge
- ○ downward only if the ratio of qE to initial velocity is large enough

Electric Field and Electric Potential

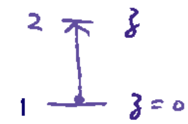
The electric potential V is defined by the relationship $U = qV$, where U is the electric potential energy of a particle with charge q.

Part G

Find the electric potential V of the uniform electric field $\vec{E} = E\hat{k}$. Note that this field is not pointing in the same direction as the field in the previous section of this problem. Take zero potential to be at position $z = 0$.

Express V in terms of q, E, and z.

$$\Delta V = -\int_1^2 \vec{E}\cdot d\vec{\ell} = -\int_1^2 |\vec{E}|\,dz = -(Ez - 0)$$



ANSWER:

$V = -Ez$

Part H

The electric field can be derived from the electric potential, just as the electrostatic force can be determined from the electric potential energy. The relationship between electric field and electric potential is $\vec{E} = -\vec{\nabla}V$, where $\vec{\nabla}$ is the gradient operator:

$$\vec{\nabla}V = \frac{\partial V}{\partial x}\hat{i} + \frac{\partial V}{\partial y}\hat{j} + \frac{\partial V}{\partial z}\hat{k}.$$

The partial derivative $\frac{\partial V}{\partial x}$ means the derivative of V with respect to x, holding all other variables constant.

Consider again the electric potential $V = -Ez$ corresponding to the field $\vec{E} = E\hat{k}$. This potential depends on the z coordinate only, so $\frac{\partial V}{\partial x} = \frac{\partial V}{\partial y} = 0$ and $\frac{\partial V}{\partial z} = \frac{dV}{dz}$.

Find an expression for the electric field $\vec{E}$ in terms of the derivative of V.

Express your answer as a vector in terms of the unit vectors $\hat{i}$, $\hat{j}$, and/or $\hat{k}$. Use dV/dz for the derivative of V with respect to z.

ANSWER:

$$\vec{E} = -\vec{\nabla}V = -\left(\frac{\partial V}{\partial x}\hat{i} + \frac{\partial V}{\partial y}\hat{j} + \frac{\partial V}{\partial z}\hat{k}\right)$$

* $V = -Ez$ (in Part G)

$$\vec{E} = -\left(0\hat{i} + 0\hat{j} \underbrace{-E}_{dV/dz}\hat{k}\right) = E\hat{k} = -\frac{dV}{dz}\hat{k}$$

$\vec{E} = -\frac{dV}{dz}\hat{k}$

Part I

A positive test charge will accelerate toward regions of ________ electric potential and ________ electric potential energy.

Choose the appropriate answer combination to fill in the blanks correctly.

You did not open hints for this part.

ANSWER:

- ○ higher; higher
- ○ higher; lower
- ○ lower; higher
- ⊗ lower; lower

Part J

A negative test charge will accelerate toward regions of ________ electric potential and ________ electric potential energy.

Choose the appropriate answer combination to fill in the blanks correctly.

You did not open hints for this part.

ANSWER:

- ○ higher; higher
- ⊗ higher; lower
- ○ lower; higher
- ○ lower; lower

Video Tutor: Charged Conductor with Teardrop Shape

First, launch the video below. You will be asked to use your knowledge of physics to predict the outcome of an experiment. Then, close the video window and answer the question at right. You can watch the video again at any point.

Part A

Two conducting spheres are each given a charge Q. The radius of the larger sphere is three times greater than that of the smaller sphere. If the electric field just outside of the smaller sphere is E_0, then the electric field just outside of the larger sphere is

You did not open hints for this part.

$\vec{E} = k\frac{Q}{r^2}\hat{r} = \vec{E}_0$

$\vec{E} = k\frac{Q}{(3r)^2}\hat{r} = k\frac{Q}{9r^2}\hat{r} = \frac{\vec{E}_0}{9}$

ANSWER:

- 1/9 E_0
- 9 E_0
- 1/3 E_0
- E_0
- 3 E_0

Bouncing Electrons

Two electrons, each with mass m and charge q, are released from positions very far from each other. With respect to a certain reference frame, electron A has initial nonzero speed v toward electron B in the positive x direction, and electron B has initial speed $3v$ toward electron A in the negative x direction. The electrons move directly toward each other along the x axis (very hard to do with real electrons). As the electrons approach each other, they slow due to their electric repulsion. This repulsion eventually pushes them away from each other.

Part A

Which of the following statements about the motion of the electrons in the given reference frame will be true at the instant the two electrons reach their minimum separation?

ANSWER: ☆ think of relative velocity at any moment

$\vec{V}_A$ A $\quad$ $\vec{V}_B$ B $\quad$ $\vec{V}_{AB} = \vec{V}_A - \vec{V}_B$

the minimum distance happen when

$\vec{V}_{AB} = 0 \Rightarrow \vec{V}_A = \vec{V}_B$ (same speed, same direction)

- ○ Electron A is moving faster than electron B.
- ○ Electron B is moving faster than electron A.
- ○ Both electrons are moving at the same (nonzero) speed in opposite directions.
- ☒ Both electrons are moving at the same (nonzero) speed in the same direction.
- ○ Both electrons are momentarily stationary.

Part B

What is the minimum separation r_{min} that the electrons reach?

Express your answer in term of q, m, v, and k (where $k = \frac{1}{4\pi\epsilon_0}$).

Hint 1. How to approach the problem

Since no external or nonconservative forces act on the system of the two electrons, both momentum and total energy (kinetic plus potential) are conserved. Find one expression for the energy when the electrons are far apart, and another when they reach their minimum separation r_{min}. This will give you an equation in which the only unknown is the speed of the electrons at the moment of their minimum separation. Apply conservation of momentum, using the same initial and final states, to obtain a second equation involving the speed of the electrons. Solve the simultaneous energy and momentum equations to obtain r_{min}.

Hint 2. Find the initial energy

What is the total energy E_{initial} of the two electrons when they are initially released? Assume that the electrons are so far apart that their potential energy is zero.

Express your answer in terms of m and v.

ANSWER:

E_{initial} =

Hint 3. Find the final energy

What is the total energy E_{final} of the electrons when they reach their minimum separation r_{min}? Assume that the (identical) speed of the two electrons is u.

Express your answer in terms of m, u, q, r_{min}, and k (where $k = \frac{1}{4\pi\epsilon_0}$).

Hint 1. Find the final kinetic energy

What is the final kinetic energy (both electrons)?

Express your answer in terms of u, and m.

ANSWER:

K_{final} =

Hint 2. Find the final potential energy

What is the final potential energy of this 2-electron system?

Express your answer in terms of k, q, and r_{min}.

ANSWER:

U_{final} =

ANSWER:

E_{final} =

Hint 4. Find the initial momentum

What is the total momentum $\vec{p}_{\text{initial}}$ of the two electrons when they are initially released?

Express your answer as a vector in terms of m, v, and $\hat{i}$.

ANSWER:

$\vec{p}_{\text{initial}}$ =

Hint 5. Find the final momentum

What is the total momentum $\vec{p}_{\text{final}}$ of the two electrons when they reach their minimum separation r_{min}? Assume that the (identical) velocity of the two electrons is $\vec{u}$.

Express your answer as a vector in terms of m and $\vec{u}$.

ANSWER:

$\vec{p}_{\text{final}}$ =

A $\rightarrow v$ $\quad$ $3v \leftarrow$ B

define positive direction $(\hat{x})$

Hint 6. Some math help

From the momentum equations, $|\vec{u}| = v$; that is, $u = v$. Substitute for u in the energy conservation equation to find r_{min}.

ANSWER:

$r_{\text{min}} = \dfrac{kq^2}{4mv^2}$

energy conservation $\quad$ i: infinity $\quad$ f: closest

$E_i = E_f$

$U_i + K_i = U_f + K_f$

same speed when closest

$0 + \frac{1}{2}mv^2 + \frac{1}{2}m(3v)^2 = k\frac{qq}{r_{min}} + \frac{1}{2}mv_f^2 + \frac{1}{2}mv_f^2$

$5mv^2 = k\frac{q^2}{r_{min}} + mv_f^2 \Rightarrow 4mv^2 = k\frac{q^2}{r_{min}}$

momentum conservation

$p_i = p_f$

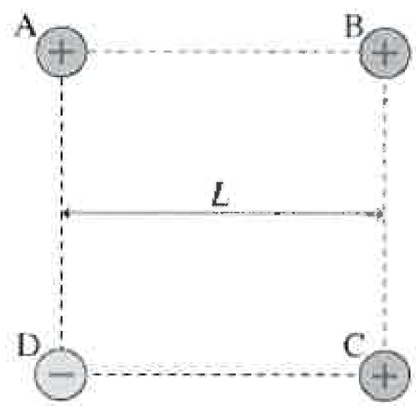
$mv - 3mv = 2mv_f \Rightarrow v_f = -v$

$r_{min} = \dfrac{kq^2}{4mv^2}$

Energy Stored in a Charge Configuration

Four point charges, A, B, C, and D, are placed at the corners of a square with side length L. Charges A, B, and C have charge $+q$, and D has charge $-q$.

Throughout this problem, use k in place of $\frac{1}{4\pi\epsilon_0}$.



$U_{total} = U_{AB} + U_{BC} + U_{CD} + U_{AC} + U_{BD} + U_{AD}$

$= k\frac{q^2}{L} + k\frac{q^2}{L} - k\frac{q^2}{L} + k\frac{q^2}{\sqrt{2}L} - k\frac{q^2}{\sqrt{2}L} - k\frac{q^2}{L}$

$= 0$

Part A

If you calculate W, the amount of work it took to assemble this charge configuration if the point charges were initially infinitely far apart, you will find that the contribution for each charge is proportional to $\frac{kq^2}{L}$. In the space provided, enter the numeric value that multiplies the above factor, in W.

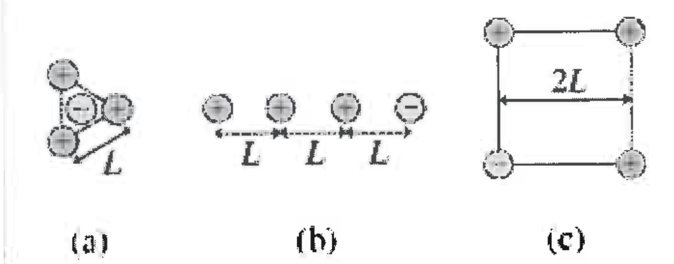
You did not open hints for this part.

ANSWER:

$W =$ 0 $\times \left(\frac{kq^2}{L}\right)$

Part B

Which of the following figures depicts a charge configuration that requires less work to assemble than the configuration in the problem introduction? Assume that all charges have the same magnitude q.



ANSWER:

- [x] figure a
- figure b
- figure c

Electric Force and Potential: Spherical Symmetry

Learning Goal:

To understand the electric potential and electric field of a point charge in three dimensions

Consider a positive point charge q, located at the origin of three-dimensional space. Throughout this problem, use k in place of $\frac{1}{4\pi\epsilon_0}$.

Part A

Due to symmetry, the electric field of a point charge at the origin must point _____ from the origin.

Answer in one word.

ANSWER:

radial

Part B

Find $E(r)$, the magnitude of the electric field at distance r from the point charge q.

Express your answer in terms of r, k, and q.

ANSWER:

$E(r)$ = $\frac{kq}{r^2}$

Part C

Find $V(r)$, the electric potential at distance r from the point charge q.

Express your answer in terms of r, k, and q.

ANSWER:

$V(r)$ = $\frac{kq}{r}$

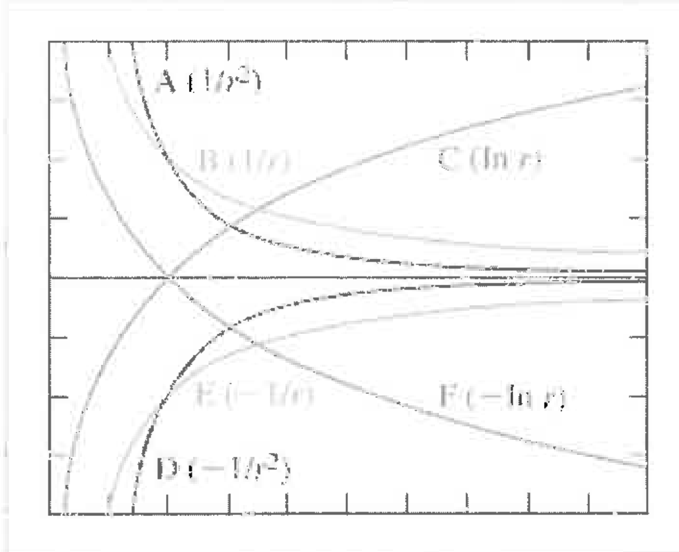
Part D

Which of the following is the correct relationship between the magnitude of a radial electric field $E(r)$ and its associated electric potential $V(r)$? More than one answer may be correct for the particular case of a point charge at the origin, but you should choose the correct *general* relationship.

ANSWER:

- ○ $E(r) = \frac{dV(r)}{dr}$
- ○ $E(r) = \frac{V(r)}{r}$
- ⊗ $E(r) = -\frac{dV(r)}{dr}$
- ○ $E(r) = -\frac{V(r)}{r}$

Now consider the figure, which shows several functions of the variable r.



Part E

Which curve could indicate the *magnitude of* the electric field due to a charge q located at the origin ($r = 0$)?

You did not open hints for this part.

ANSWER:

- A
- B
- C
- D
- E
- F

Part F

Which curve could indicate the electric potential due to a *positive* charge q located at the origin ($r = 0$)?

You did not open hints for this part.

ANSWER:

- ○ A
- ⊗ B
- ○ C
- ○ D
- ○ E
- ○ F

Part G

Which curve could indicate the electric potential due to a *negative* charge q located at the origin ($r = 0$)?

ANSWER:

- ○ A
- ○ B
- ○ C
- ○ D
- ⊗ E
- ○ F

Part H

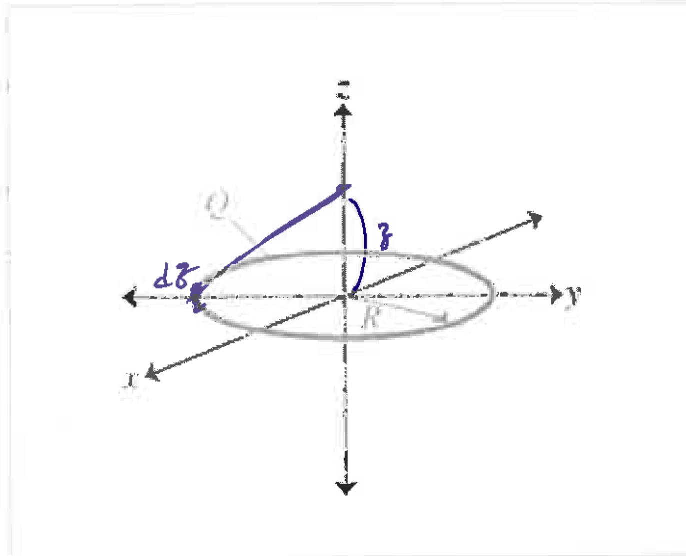
For either a positive or a negative charge, the electric field points from regions of ______ electric potential.

ANSWER:

- ⊗ higher to lower
- ○ lower to higher

Potential of a Charged Ring

A ring with radius R and a uniformly distributed total charge Q lies in the *xy* plane, centered at the origin.



Part A

What is the potential $V(z)$ due to the ring on the z axis as a function of z?

Express your answer in terms of Q, z, R, and ϵ_0 or $k = \frac{1}{4\pi\epsilon_0}$.

You did not open hints for this part.

ANSWER:

$$V(z) = \frac{kQ}{\sqrt{R^2+z^2}}$$

V is scalar

$$dV = k\frac{dz}{\sqrt{R^2+z^2}} = k\frac{\lambda \cdot dl}{\sqrt{R^2+z^2}}$$

$$V = \int dV = \frac{k\lambda}{\sqrt{R^2+z^2}}\int dl = \frac{k\lambda \cdot 2\pi R}{\sqrt{R^2+z^2}}$$

$$\boxed{\star\ \lambda = \frac{Q}{2\pi R}} \Rightarrow V = \frac{kQ}{\sqrt{R^2+z^2}}$$

Part B

What is the magnitude of the electric field $\vec{E}$ on the z axis as a function of z, for $z > 0$?

Express your answer in terms of some or all of the quantities Q, z, R, and ϵ_0 or $k = \frac{1}{4\pi\epsilon_0}$.

You did not open hints for this part.

ANSWER:

$$|\vec{E}(z)| = \frac{kQz}{(R^2+z^2)^{3/2}}$$

$\vec{E} = -\vec{\nabla} \cdot V$ ★ $V(z)$ is only a function of z

$$= -\left(\frac{\partial V}{\partial x}\hat{i} + \frac{\partial V}{\partial z}\hat{j} + \frac{\partial V}{\partial z}\hat{k}\right)$$

$$= -\left(0\,\hat{i} + 0\,\hat{j} - \frac{1}{2}\frac{kQ}{(R^2+z^2)^{-3/2}} \cdot 2z\,\hat{k}\right)$$

$$= \frac{kQz}{(R^2+z^2)^{3/2}}\,\hat{k}$$

± Equipotential Surfaces in a Capacitor

Part A

Is the electric potential energy of a particle with charge q the same at all points on an equipotential surface?

You did not open hints for this part.

ANSWER:

- [x] Yes
- [ ] No

Part B

What is the work required to move a charge around on an equipotential surface at potential V with constant speed?

You did not open hints for this part.

ANSWER:

Work = 0

Part C

What is the work done by the electric field on a charge as it moves along an equipotential surface at potential V?

You did not open hints for this part.

ANSWER:

Work done by the electric field = 0

Part D

The work W_E done by the uniform electric field $\vec{E}$ in displacing a particle with charge q along the path $\vec{d}$ is given by

$$W_E = q\vec{E}\cdot\vec{d} = q|\vec{E}||\vec{d}|\cos\theta,$$

where θ is the angle between $\vec{E}$ and $\vec{d}$. Since in general, $\vec{E}$ is not equal to zero, for points on an equipotential surface, what must θ be for W_E to equal 0?

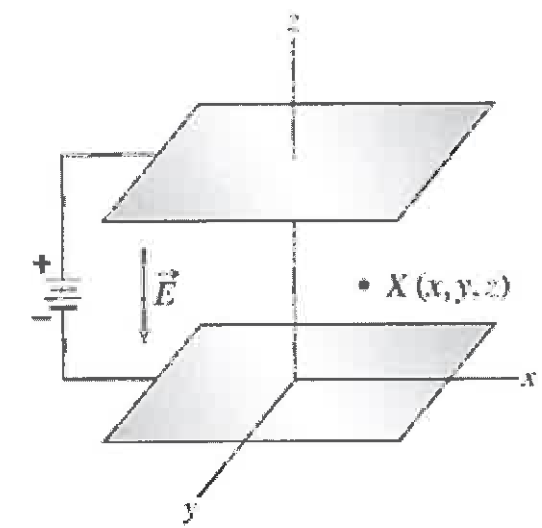
$\cos\frac{\pi}{2} = 0$

Express your answer in radians.

ANSWER:

θ = $\frac{\pi}{2}$ rad

Now assume that a parallel-plate capacitor is attached across the terminals of a battery as shown in the figure. The electric field $\vec{E}$ in the region between the plates points in the negative z direction, from higher to lower voltage.



Part E

Find the electric potential $V(x,y,z)$ at a point $\mathrm{X} = (x,y,z)$ inside the capacitor if the origin of the coordinate system $\mathrm{O} = (0,0,0)$ is at potential 0.

Express your answer in terms of some or all of the variables E, x, y, and z.

Hint 1. The relation between electric potential and the electric field

ield $\vec{E}$ in a region of space is

$$V = -\int \vec{E}\cdot d\vec{\ell},$$

where the line integral may be taken along any path ℓ.

Hint 2. Expressing an infinitesimal length element

In general, a small length vector along the path of choice can be written as

$$d\vec{\ell} = dx\,\hat{\imath} + dy\,\hat{\jmath} + dz\,\hat{k}.$$

Substitute this expression into the integral for V.

Hint 3. Analysis of the equation

Recall that if $\vec{E}$ and $\hat{w}$ are perpendicular (where $\hat{w}$ is one of the Cartesian coordinate axes), then $\vec{E}\cdot\hat{w} = 0$, since $\theta = \pi/2$. According to the setup of this part, only one of the directions ($\hat{\imath}$, $\hat{\jmath}$, $\hat{k}$) will not be perpendicular to the electric field as defined.

ANSWER:

$V(x,y,z)$ = $|\vec{E}|\cdot z$

$\Delta V = -\int_1^2 \vec{E}\cdot d\vec{\ell}$

$V_2 - V_1 = -\int_1^2 (-|\vec{E}|\cdot|d\vec{\ell}|)$

0

($\vec{E}$ and $d\vec{\ell}$ has $180°$)

$\vec{E}$ 2 x (x,y,z) 1 $V_1 = 0$

$\int_1^2 |\vec{E}|\cdot dz = |\vec{E}|(z_2 - z_1)$

$= |\vec{E}|\cdot z$

$|d\vec{\ell}| = dz$

$\Rightarrow V_2 = |\vec{E}|\cdot z$

Part F

What is the distance Δz between two surfaces separated by a potential difference ΔV?

Express your answer in terms of E and ΔV.

Hint 1. How to approach the problem

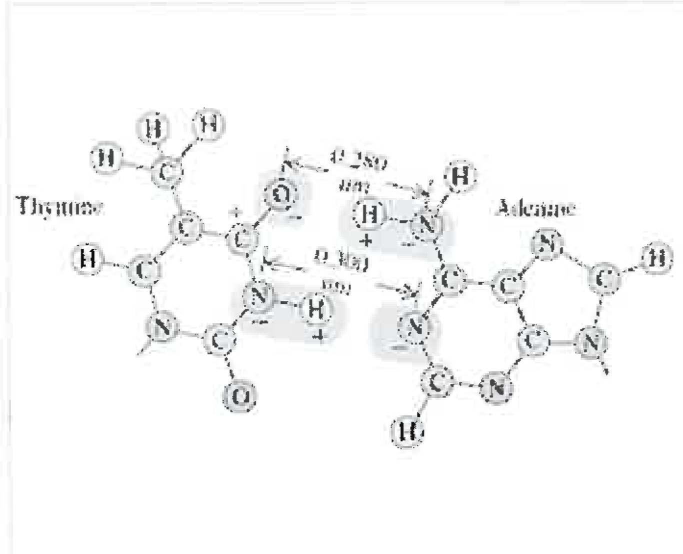
Use the equation you found in Part E to find equations that represent the potentials V_1 and V_2 of the planes located at z_1 and z_2. Use these expressions to find an equation for Δz.

ANSWER:

$\Delta z = \dfrac{\Delta V}{|\vec{E}|}$

Exercise 23.6

The two sides of the DNA double helix are connected by pairs of bases (adenine, thymine, cytosine, and guanine). Because of the geometric shape of these molecules, adenine bonds with thymine and cytosine bonds with guanine. The figure shows the thymine–adenine bond. Each charge shown is $\pm e$ and the $\mathrm{H-N}$ distance is 0.110 nm.



Part A

Calculate the electric potential energy of the adenine–thymine bond, using combinations of molecules $\mathrm{O-H-N}$ and $\mathrm{N-H-N}$.

Express your answer with the appropriate units.

ANSWER:

$U_{\text{adenine-thymine}} = -9.8 \times 10^{-19}$ J

$U_{total} = U_{OHN} + U_{NHN}$

$= U_{OH} + U_{ON} + U_{NH} + U_{NN}$

$= k\frac{-e^2}{0.17\times10^{-9}} + k\frac{e^2}{(0.28\times10^{-9})} + k\frac{-e^2}{0.19\times10^{-9}} + k\frac{e^2}{0.3\times10^{-9}}$

$= ke^2\left(-\frac{1}{0.17\times10^{-9}} + \frac{1}{0.28\times10^{-9}} - \frac{1}{0.19\times10^{-9}} + \frac{1}{0.3\times10^{-9}}\right)$

$= 9\times10^9 \times (1.6\times10^{-19})^2 (-4.2\times10^9)$

$= -98 \times 10^{-20}$

Part B

Compare this energy with the potential energy of the proton–electron pair in the hydrogen atom. The electron in the hydrogen atom is 0.0529 nm from the proton.

ANSWER:

$\frac{U_{\text{adenine-thymine}}}{U_{\text{H}}} = \frac{-9.8\times10^{-19}}{-4.36\times10^{-18}} = 2.2\times10^{-1}$

$U_H = k\frac{-e^2}{0.0529\times10^{-9}}$

$= 9\times10^9 \frac{-(1.6\times10^{-19})^2}{0.0529\times10^{-9}}$

$= -436 \times 10^{-20}$

$= -4.36\times10^{-18}$

Exercise 23.23

Part A

An electron is to be accelerated from a velocity of 1.00×10^6 m/s to a velocity of 9.00×10^6 m/s . Through what potential difference must the electron pass to accomplish this?

ANSWER:

$\Delta K = \Delta U = q \Delta V$

$V_1 - V_2 = -227.5 \text{ V}$

$\frac{1}{2} m v_f^2 - \frac{1}{2} m v_i^2 = q \Delta V$

$\frac{1}{2} \times 9.1 \times 10^{-31} \left((9\times10^6)^2 - (1\times10^6)^2 \right) = -1.6\times10^{-19} \Delta V$

V

$\Delta V = -227.5 \text{ V}$

Part B

Through what potential difference must the electron pass if it is to be slowed from $9.00\times10^6 \mathrm{m/s}$ to a halt?

ANSWER:

$\Delta K = \Delta U = q \Delta V$

$\frac{1}{2} m (v_f^2 - v_i^2) = -e \Delta V$

$\frac{1}{2} \times 9.1\times10^{-31} \left(0 - (9\times10^6)^2 \right) = -1.6\times10^{-19} \Delta V$

$V_1 - V_2 = 230 \text{V}$ V

$\Delta V = 230 \text{ V}$

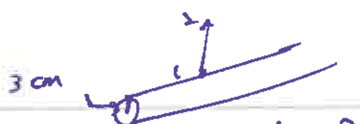
Exercise 23.32

A very long insulating cylinder of charge of radius 3.00cm carries a uniform linear density of 15.0$\mathrm{nC/m}$.

Part A

If you put one probe of a voltmeter at the surface, how far from the surface must the other probe be placed so that the voltmeter reads 200V ?

ANSWER:



$$\Delta V = -\int \vec{E} \cdot d\vec{l}$$
$$= -\int_1^2 |\vec{E}||d\vec{l}|$$
$$= -\int_1^2 |\vec{E}|\, dr$$
$$= -\int_1^2 \frac{\lambda}{2\pi\epsilon_0} \frac{1}{r}\, dr$$
$$= -\frac{\lambda}{2\pi\epsilon_0} \ln \frac{r_2}{r_1}$$

$|\vec{E}| = \frac{1}{2\pi\epsilon_0} \frac{\lambda}{r}$

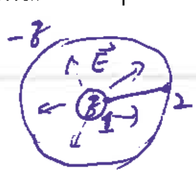
$d = 6.3 - 3 = 3.3 \text{ cm}$ cm

$$-200 = -\frac{15\times10^{-9}}{2\pi \cdot 8.85\times10^{-12}} \ln \frac{r_2}{3\times10^{-2}}$$

$\Rightarrow r_2 = 0.063 \text{ m} = 6.3 \text{ cm}$

Exercise 23.48

A metal sphere with radius r_a = 1.40cm is supported on an insulating stand at the center of a hollow, metal, spherical shell with radius r_b = 9.80cm . Charge $+q$ is put on the inner sphere and charge $-q$ on the outer spherical shell. The magnitude of q is chosen to make the potential difference between the spheres 470V , with the inner sphere at higher potential.



$\vec{E} = k \frac{q}{r^2} \hat{r}$

Part A

Calculate q.

$$\Delta V = -\int_1^2 \vec{E} \cdot d\vec{l} = -\int_1^2 \frac{kq}{r^2}\, dr = -kq\left(-\frac{1}{r}\right)\Big|_1^2$$
$$= kq\left(\frac{1}{r_2} - \frac{1}{r_1}\right)$$

ANSWER:

$$-470 = 9\times10^9 \cdot q \left(\frac{1}{9.8\times10^{-2}} - \frac{1}{1.4\times10^{-2}} \right) \Rightarrow q = 8.5\times10^{-10} \text{ C}$$

$q =$ 8.5×10^{-10} C

Part B

Are the electric field lines and equipotential surfaces mutually perpendicular?

ANSWER:

- [x] Yes
- [ ] No

Part C

Are the equipotential surfaces closer together when the magnitude of $\vec{E}$ is largest?

ANSWER:

- [x] Equipotential surfaces closer together when the magnitude of $\vec{E}$ is largest.
- [ ] Equipotential surfaces closer together when the magnitude of $\vec{E}$ is smallest.

Problem 23.89 (see attached sheets)

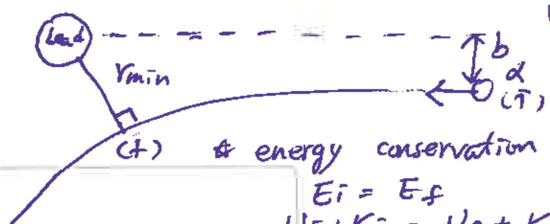
An alpha particle with kinetic energy 11.0MeV makes a collision with lead nucleus, but it is not "aimed" at the center of the lead nucleus, and has an initial nonzero angular momentum (with respect to the stationary lead nucleus) of magnitude $L = p_0 b$, where p_0 is the magnitude of the initial momentum of the alpha particle and b=1.50×10^{-12}m . (Assume that the lead nucleus remains stationary and that it may be treated as a point charge. The atomic number of lead is 82. The alpha particle is a helium nucleus, with atomic number 2.)

Part A

What is the distance of closest approach?

ANSWER:

$r =$ 1.51×10^{-12} m



$E_k = \frac{p^2}{2m}$

$E_{k_0} = \frac{P_0^2}{2m}$

$P_0 = \sqrt{2mE_k}$

$= \sqrt{2 \times 6.6 \times 10^{-27} \times 11 \times 10^6 \times 1.6 \times 10^{-19}}$

$= \sim$

energy conservation

$E_i = E_f$

$U_i + K_i = U_f + K_f$

$0 + \frac{P_0^2}{2m} = k\frac{q_1 q_2}{r_{min}} + \frac{p^2}{2m}$

angular momentum conservation

$\vec{L_i} = \vec{L_f}$

$P_0 b = \vec{r}_{min} \times \vec{P} = r_{min} \cdot P$

$P = \frac{P_0 b}{r_{min}}$

$\frac{P_0^2}{2m} = k\frac{q_1 q_2}{r_{min}} + \frac{P_0^2 b^2}{2m r_{min}^2}$

$P_0^2 r_{min}^2 - 2mkq_1q_2 r_{min} - P_0^2 b^2 = 0$

$r_{min} = \frac{2mkq_1q_2 \pm \sqrt{4m^2k^2q_1^2q_2^2 + 4P_0^4 b^2}}{2P_0^2}$

Part B

Repeat for b=1.30×10^{-13}m .

ANSWER: just plug in ~~m~~ number, you will get answer

r = 1.41×10^{-13} m m

Part C

Repeat for b=1.40×10^{-14}m .

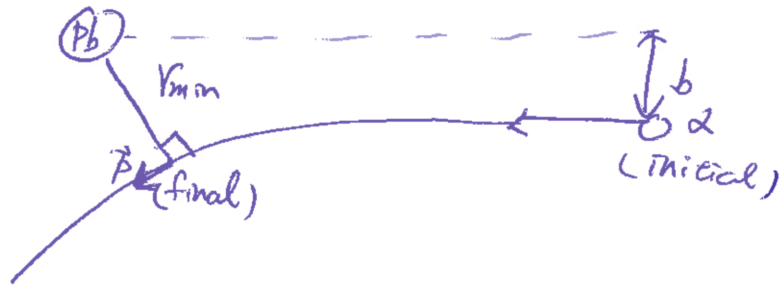
ANSWER:

r = 2.85×10^{-14} m

Score Summary:

Your score on this assignment is 0.0%.
You received 0 out of a possible total of 15 points.

re write P. 23.89



☆ $E_k = \frac{1}{2}mv^2 = \frac{p^2}{2m}$

☆ energy conservation

$E_i = E_f$

$U_i + K_i = U_f + K_f$

$$0 + \frac{P_0^2}{2m} = k\frac{q_1 q_2}{r_{min}} + \frac{P^2}{2m} \quad \text{①}$$

☆ angular momentum conservation

$\vec{L}_i = \vec{L}_f$

$$P_0 b = \vec{r}_{min} \times \vec{P} = r_{min} \cdot P \cdot \underbrace{\sin\left(\frac{\pi}{2}\right)}_{=1}$$

$$\Rightarrow P = \frac{P_0 b}{r_{min}}$$

plug-in to ①

$$\frac{P_0^2}{2m} = k\frac{q_1 q_2}{r_{min}} + \frac{P_0^2 b^2}{2m r_{min}^2}$$

$$\Rightarrow P_0^2 r_{min}^2 - 2mkq_1q_2 r_{min} - P_0^2 b^2 = 0$$

$$r_{min} = \frac{2mkq_1q_2 \pm \sqrt{4m^2k^2q_1^2q_2^2 + 4P_0^4 b^2}}{2P_0^2}$$

$$E_i = K_i = \frac{P_0^2}{2m} \Rightarrow P_0 = \sqrt{2mK_i}$$

$$m_\alpha = 6.6\times10^{-27}\,kg$$

$$K_i = 11\times10^{6}\,eV = 11\times10^{6}\times1.6\times10^{-19}\,J = 1.76\times10^{-12}\,J$$

$$P_0 = \sqrt{2\times6.6\times10^{-27}\times1.76\times10^{-12}} = 1.52\times10^{-19}\ kg\cdot m/s$$

$$q_1 = q_{Pb} = 82\times1.6\times10^{-19}\ C$$

$$q_2 = q_\alpha = 2\times1.6\times10^{-19}\ C$$

$$r_{min} = \frac{2\times6.6\times10^{-27}\times9\times10^{9}\times82\times1.6\times10^{-19}\times2\times1.6\times10^{-19} \pm \sqrt{A}}{2\times(1.52\times10^{-19})^2}$$

where

$$A = 4\times(6.6\times10^{-27})^2\cdot(9\times10^{9})^2\times(82\times1.6\times10^{-19})^2\times(2\times1.6\times10^{-19})^2 + 4\times(1.52\times10^{-19})^4\times\underbrace{(1.5\times10^{-12})^2}_{b}$$

change this gives answers for part B, C

$$= 4.8\times10^{-99}$$

$$r_{min} = \frac{4.99\times10^{-52} \pm \sqrt{4.8\times10^{-99}}}{4.6\times10^{-38}} = 1.08\times10^{-14} \pm 1.5\times10^{-12}$$

r_{min} should be >0 $\Rightarrow$ "+" $\Rightarrow$ $r_{min} = 1.51\times10^{-12}\ m$