2/17/2014

HW 4

W

HW4

Due: 11:00pm on Sunday, February 16, 2014

You will receive no credit for items you complete after the assignment is due. Grading Policy

Introduction to Capacitance

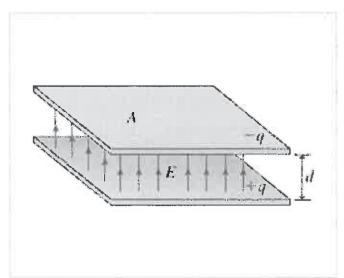
Learning Goal:

To understand the meaning of capacitance and ways of calculating capacitance

When a positive charge q is placed on a conductor that is insulated from ground, an electric field emanates from the conductor to ground, and the conductor will have a nonzero potential V relative to ground. If more charge is placed on the conductor, this voltage will increase proportionately. The ratio of charge to voltage is called the *capacitance* C of this conductor: C = q/V.

Capacitance is one of the central concepts in electrostatics, and specially constructed devices called *capacitors* are essential elements of electronic circuits. In a capacitor, a second conducting surface is placed near the first (they are often called *electrodes*), and the relevant voltage is the voltage between these two electrodes.

This tutorial is designed to help you understand capacitance by assisting you in calculating the capacitance of a parallel-plate capacitor, which consists of two plates each of area A separated by a small distance d with air or vacuum in between. In figuring out the capacitance of this configuration of conductors, it is important to keep in mind that the voltage difference is the line integral of the electric field between the plates.



Part A

What property of objects is best measured by their capacitance?

ANSWER:

- O ability to conduct electric current
- O ability to distort an external electrostatic field
- X ability to store charge

Part B

Assume that charge -q is placed on the top plate, and +q is placed on the bottom plate. What is the magnitude of the electric field E between the plates?

Express E in terms of q and other quantities given in the introduction, in addition to ϵ_0 and any other constants needed.

You did not open hints for this part.

ANSWER:

$$E = \frac{8}{AE_0}$$

Part C

What is the voltage V between the plates of the capacitor?

Express V in terms of the quantities given in the introduction and any required physical constants.

You did not open hints for this part.

ANSWER:

$$V = \frac{8d}{A\epsilon_0}$$

Part D

Now find the capacitance C of the parallel-plate capacitor.

Express C in terms of quantities given in the introduction and constants like ϵ_0 .

ANSWER:

$$C = \frac{\epsilon_0 A}{d}$$

Part E

Consider an air-filled charged capacitor. How can its capacitance be increased?

You did not open hints for this part.

ANSWER:

- Increase the charge on the capacitor.
- O Decrease the charge on the capacitor.
- O Increase the spacing between the plates of the capacitor.
- Decrease the spacing between the plates of the capacitor.
- O Increase the length of the wires leading to the capacitor plates.

Part F

Consider a charged parallel-plate capacitor. Which combination of changes would quadruple its capacitance?

ANSWER:

- O Double the charge and double the plate area.
- O Double the charge and double the plate separation.
- O Halve the charge and double the plate separation.
- O Halve the charge and double the plate area.
- Alve the plate separation; double the plate area.
- O Double the plate separation; halve the plate area.

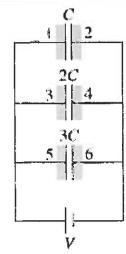
Capacitors in Parallel

Learning Goal:

To understand how to calculate capacitance, voltage, and charge for a parallel combination of capacitors.

Frequently, several capacitors are connected together to form a collection of capacitors. We may be interested in determining the overall capacitance of such a collection. The simplest configuration to analyze involves capacitors connected in series or in parallel. More complicated setups can often (though not always!) be treated by combining the rules for these two cases. Consider the example of a parallel combination of capacitors: Three capacitors are connected to each other and to a battery

as shown in the figure. The individual capacitances are $C,\,2C,\,$ and $3C,\,$ and the battery's voltage is $V.\,$



Part A

If the potential of plate 1 is V, then, in equilibrium, what are the potentials of plates 3 and 6? Assume that the negative terminal of the battery is at zero potential.

You did not open hints for this part.

ANSWER:

- O V and V
- O 2V and 3V
- \bigwedge V and 0
- $\bigcirc \ rac{V}{2} \ {
 m and} \ rac{V}{3}$

Part B

If the charge of the first capacitor (the one with capacitance C) is Q, then what are the charges of the second and third capacitors?

You did not open hints for this part.

ANSWER:

- $\nearrow \hspace{-0.5cm} \bigcirc \hspace{$
- \bigcirc $\frac{Q}{2}$ and $\frac{Q}{3}$
- $\bigcirc \ Q$ and Q
- O 0 and 0

Part C

Suppose we consider the system of the three capacitors as a single "equivalent" capacitor. Given the charges of the three individual capacitors calculated in the previous part, find the total charge Q_{tot} for this equivalent capacitor.

Express your answer in terms of ${\cal V}$ and ${\cal C}.$

$$C = \frac{Q}{V}$$

ANSWER:

$$Q_{\text{tot}} = 6Q = 6CV$$

Part D

2/17/2014 H

Using the value of $Q_{
m tot}$, find the equivalent capacitance $C_{
m eq}$ for this combination of capacitors.

Express your answer in terms of C.

You did not open hints for this part.

ANSWER:

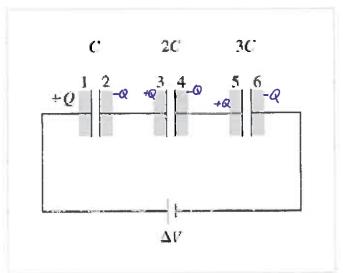
$$C_{\text{eq}} = \frac{Q_{\text{tore}}}{V} = \frac{6Q}{V} = 6C$$

Capacitors in Series

Learning Goal:

To understand how to calculate capacitance, voltage, and charge for a combination of capacitors connected in series.

Consider the combination of capacitors shown in the figure. Three capacitors are connected to each other in series, and then to the battery. The values of the capacitances are C, 2C, and 3C, and the applied voltage is ΔV . Initially, all of the capacitors are completely discharged; after the battery is connected, the charge on plate 1 is Q.



Part A

What are the charges on plates 3 and 6?

You did not open hints for this part.

ANSWER:

$$\bigcirc +Q$$
 and $+Q$

$$\bigcirc \ -Q$$
 and $-Q$

$$oxtimeq +Q$$
 and $-Q$

$$\bigcirc \ -Q \ {
m and} \ +Q$$

$$\bigcirc$$
 0 and $+Q$

$$\bigcirc$$
 0 and $-Q$

Part B

If the voltage across the first capacitor (the one with capacitance C) is $\Delta V_{\rm I}$, then what are the voltages across the second and third capacitors?

You did not open hints for this part.

$$C = \frac{Q}{V} \Rightarrow V = \frac{Q}{C}$$

ANSWER:

O
$$2\Delta V_1$$
 and $3\Delta V_1$

$$igotimes rac{1}{2} \Delta V_1$$
 and $rac{1}{3} \Delta V_1$

O
$$\Delta V_1$$
 and ΔV_1

$$oldsymbol{\mathsf{O}}$$
 0 and ΔV_1

$$\Delta V_1 = \frac{Q}{C}$$

$$4V_2 = \frac{Q_2}{C_2} = \frac{Q}{2C} = \frac{4V_1}{2}$$

$$\Delta V_3 = \frac{Q_3}{C_3} = \frac{Q}{3C} = \frac{\Delta V_1}{3}$$

Part C

Find the voltage ΔV_1 across the first capacitor.

Express your answer in terms of ΔV .

$$aV = aV_1 + 6V_2 + 6V_3$$

$$= 6V_1 + \frac{aV_1}{2} + \frac{aV_1}{3}$$

$$= \frac{6+3+2}{4} \quad aV_1 = \frac{11}{6} aV_1$$

You did not open hints for this part.

ANSWER:

$$\Delta V_1 = \frac{6}{11}$$

Find the charge ${\it Q}$ on the first capacitor.

Express your answer in terms of C and ΔV_1 :

ANSWER:

$$Q = C \otimes V_{i}$$

Part E

Using the value of Q just calculated, find the equivalent capacitance C_{eq} for this combination of capacitors in series.

Express your answer in terms of C.

You did not open hints for this part.

ANSWER:

$$C_{\text{eq}} = \frac{Q}{4V} = \frac{Q}{\frac{11}{6}\Delta V_1} = \frac{C}{\frac{17}{6}} = \frac{6C}{11}$$

Finding the Capacitance

A parallel-plate capacitor is filled with a dielectric whose dielectric constant is K, increasing its capacitance from C_1 to KC_1 . A second capacitor with capacitance C_2 is then connected in series with the first, reducing the net capacitance back to C_1 .

Part A

What is the capacitance C_2 of the second capacitor?

Express your answer in terms of K, C_1 , and constants.

You did not open hints for this part.

P

$$C_1 = C_{0g}$$

$$\frac{1}{C_1} = \frac{1}{C_2} = \frac{1}{kC_1} + \frac{1}{C_2}$$

ANSWER:

$$C_2 = \frac{|C_2|}{|C_1|} C_1$$

$$\frac{|\mathsf{K}^{\mathsf{C}}|}{|\mathsf{K}^{\mathsf{C}}|} = \frac{1}{\mathsf{C}_2}$$

Force between Capacitor Plates

Consider a parallel-plate capacitor with plates of area A and with separation d.

Part A

Find F(V), the magnitude of the force each plate experiences due to the other plate as a function of V, the potential drop across the capacitor.

Express your answer in terms of given quantities and ϵ_0 .

 $\vec{E}_0 = \frac{\vec{b}}{260} = \frac{9^0A}{260}$

You did not open hints for this part.

ANSWER:

$$F(V) = \frac{60 \text{AV}^2}{2 \text{AV}^2}$$

$$\vec{F} = \mathbf{Q} \vec{E}_0 = 2 \cdot \mathbf{E} \cdot \mathbf{A} \cdot \mathbf{E} \cdot \mathbf{I}^2$$

$$= 2 \cdot \mathbf{E} \cdot \mathbf{A} \cdot \mathbf{E} \cdot \mathbf{I}^2$$

$$= \frac{6 \cdot \mathbf{A} \cdot \mathbf{V}^2}{2 \cdot \mathbf{I}^2}$$

The Capacitor as an Energy-Storing Device

Learning Goal:

To understand that the charge stored by capacitors represents energy; to be able to calculate the stored energy and its changes under different circumstances.

An air-filled parallel-plate capacitor has plate area A and plate separation d. The capacitor is connected to a battery that creates a constant voltage V.

Part A

Find the energy U_0 stored in the capacitor.

Express your answer in terms of A, d, V, and ϵ_0 . Remember to enter ϵ_0 as epsilon_0.

You did not open hints for this part.

$$C = \epsilon_0 \frac{A}{d}$$

ANSWER:

$$U_0 = \frac{1}{2} C V^2 = \frac{\epsilon_0 A V^2}{2d}$$

Part B

The capacitor is now disconnected from the battery, and the plates of the capacitor are then slowly pulled apart until the separation reaches 3d. Find the new energy U_1 of the capacitor after this process.

Express your answer in terms of A, d, V, and ϵ_0 .

You did not open hints for this part.

 $C' = \epsilon_0 \frac{A}{3d}$

ANSWER:

http://session.mastering.physics.com/myct/assig.nmentPrintView?displayMode=studentView&assig.nmentID=2627432

$$U_1 = \frac{3}{2} \frac{\epsilon_0 A V^2}{d}$$

Part C

Part A

The capacitor is now reconnected to the battery, and the plate separation is restored to d. A dielectric plate is slowly moved into the capacitor until the entire space between the plates is filled. Find the energy U_2 of the dielectric-filled capacitor. The capacitor remains connected to the battery. The dielectric constant is K.

Express your answer in terms of A, d, V, K, and ϵ_0 .

C"= KG A

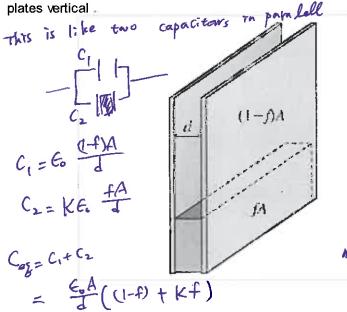
ANSWER:

$$U_2 = \frac{\text{k6 AV}^2}{\text{2d}}$$

$$U_2 = \frac{1}{2}CV^2 = \frac{K6AV^2}{2d}$$

Capacitors with Partial Dielectrics

Consider two parallel-plate capacitors identical in shape, one aligned so that the plates are horizontal, and the other with the



use Gauss ($\stackrel{\text{diff}}{=}$ { $\stackrel{\text{Evin}}{=}$ vacuum (upper haft) = $\frac{\sigma}{\epsilon_0}$ modified $\stackrel{\text{Ed in}}{=}$ dielectric (lower hulf) = $\frac{\sigma}{\kappa \epsilon_0}$ $\Delta V = -\int_{1}^{\infty} \stackrel{\text{Ed}}{=} d\vec{k} = -(\int_{1}^{\infty} \stackrel{\text{Ed}}{=} d\vec{k})$

A

=-()[三]·青+(三]·青)

 $=-\frac{d}{2}\frac{T}{E_0}\left(1+\frac{1}{K}\right)=-\frac{d}{2E_0}\left(1+\frac{1}{K}\right)\frac{Q}{A}$

The horizontal capacitor is filled halfway with a material that has dielectric constant K. What fraction f of the area of the vertical capacitor should be filled (as shown in the figure) with the same dielectric so that the two capacitors have equal $C = \frac{Q}{V} = \frac{2\epsilon_0 A}{J(1+\frac{1}{2})}$ capacitance?

Express your answer in terms of K.

 $\Rightarrow \frac{d}{d}((1-t)+kt) = \frac{d(1+t)}{260A} \Rightarrow (1-t+kt)(1+t)=1$ 1+六十千(K-1)(1+元)=2

You did not open hints for this part.

Video Tutor: Discharge Speed for Series and Parallel Capacitors

First, <u>launch the video</u> below. You will be asked to use your knowledge of physics to predict the outcome of an experiment. Then, close the video window and answer the question at right. You can watch the video again at any point.



Part A

The capacitors in each circuit are fully charged before the switch is closed. Rank, from longest to shortest, the length of time the bulbs (resistors) stay lit in each circuit.

You did not open hints for this part.

ANSWER:

10/17

Energy of a Capacitor in the Presence of a Dielectric

An dielectric-filled parallel-plate capacitor has plate area A and plate separation d. The capacitor is connected to a battery that creates a constant voltage V. The dielectric constant is K:

Part A

Find the energy U_1 of the dielectric-filled capacitor. The capacitor remains connected to the battery.

Express your answer in terms of A, d, V, K, and ϵ_0 .

 $C = k\epsilon_0 \frac{A}{d}$ $U_1 = \frac{1}{2} C V^2 = \frac{k\epsilon_0 A V^2}{2 d}$ You did not open hints for this part.

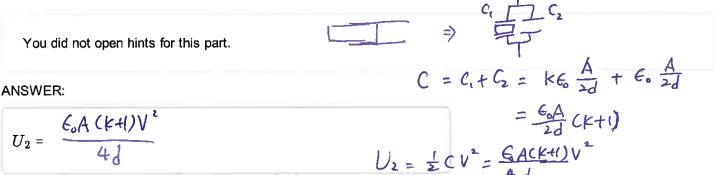
ANSWER:

$$U_1 = \frac{\text{KE. AV}^2}{2d}$$

Part B

The dielectric plate is now slowly pulled out of the capacitor, which remains connected to the battery. Find the energy U_2 of the capacitor at the moment when the capacitor is half filled with the dielectric.

Express your answer in terms of A, d, V, K, and ϵ_0 .



Part C

The capactor is now disconnected from the battery, and the dielectric plate is then slowly removed the rest of the way out of the capacitor. Find the new energy of the capacitor, U_3 .

Express your answer in terms of A, d, V, K, and ϵ_0 .

You did not open hints for this part.

after disconnection. $Q = CV = \frac{G_0A}{2d}(KH) \cdot V$ Q does'nt change after the dielectric is removed new capacitance after the dielectric is removed $C' = G_0A$

ANSWER:

$$U_3 = \frac{\epsilon_0 A(k+1)^2 V^2}{8 d}$$

$$U_3 = \frac{1}{2} \frac{Q^2}{C'} = \frac{1}{2} \cdot \frac{\epsilon_0^2 A^2(k+1)^2 V^2}{4 d^2} \cdot \frac{1}{\epsilon_0 A}$$

$$= \frac{\epsilon_0 A(k+1)^2 V^2}{8 d}$$

Part D

In the process of removing the remaining portion of the dielectric from the disconnected capacitor, how much work W is done by the external agent acting on the dielectric?

Express your answer in terms of A, d, V, K, and ϵ_0 .

You did not open hints for this part.

ANSWER:

$$W = U_3 - U_2 = \frac{\epsilon_0 A(K+1)V^2}{4d} (\frac{K+1}{2} - 1)$$

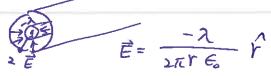
Exercise 24.11

A capacitor is made from two hollow, coaxial, iron cylinders, one inside the other. The inner cylinder is negatively charged and the outer is positively charged; the magnitude of the charge on each is 16.0 pC. The inner cylinder has a radius of 0.450 mm, the outer one has a radius of 7.40 mm, and the length of each cylinder is 20.0 cm.

Part A

What is the capacitance?

Use $8.854 \times 10^{-12} \mathrm{F/m}$ for the permittivity of free space.



ANSWER:

$$C = \frac{Q}{V} = \frac{16 \times 10^{-12}}{4.026} = 3.97 \times 10^{-12}$$
 F

$$= -\int_{1}^{2} |\vec{E}| |\vec{d}|$$

$$= \int_{1}^{2} \frac{\lambda}{2\pi r E_{0}} dr$$

Part B

$$= \frac{2}{2\pi\epsilon} \ln \frac{V_2}{V_1} = \frac{Q_1}{2\pi\epsilon} \ln \frac{V_2}{V_1}$$

What applied potential difference is necessary to produce these charges on the cylinders?

ANSWER:

V

Exercise 24.18

Two capacitors are connected in series. Let $3.00 \mu F$ be the capacitance of first capacitor, $5.40 \mu F$ the capacitance of the second capacitor, and V_{ab} = 54.0V the potential difference across the system.

Part A

Calculate the charge on each capacitor.

ANSWER:

$$Q_1 = \{0\}$$

$$Q_1 = Q_2$$

$$V_1+V_2=V=\frac{Q}{C_1}+\frac{Q}{C_2}$$

$$Q = \frac{V}{\frac{1}{C_1} + \frac{1}{C_2}}$$

Part B

ANSWER:

 μC

 μC

Part C

Calculate the potential difference across each capacitor.

ANSWER:

ANSWER:
$$V_1 = \frac{Q}{C_1} = \frac{104 \times 10^{-6}}{3.0 \times 10^{-6}} = \frac{V_1}{3.0 \times 10^{-6}} = \frac{104 \times 10^{-6}}{3.0 \times$$

Part D

ANSWER:

$$V_2 = \begin{array}{c} 1 \\ 19 \\ 3 \end{array}$$

Exercise 24.28

A parallel-plate vacuum capacitor has 7.06J of energy stored in it. The separation between the plates is 3.20mm all the separation is decreased to 1.75mm,

Part A

what is the energy now stored if the capacitor was disconnected from the potential source before the separation of the plates was changed? Q = constant

ANSWER:

$$U = 0.547 \times U_0 = 3.86 \quad (J)$$

$$C_0 = \epsilon_0 \frac{A}{d_0}$$

$$C' = \epsilon_0 A$$

$$J \qquad U = \frac{1}{2} \frac{Q^2}{C'}$$

$$C = \epsilon_0 \frac{A}{d_0} \qquad \frac{U}{U_0} = \frac{C_0}{C'} = \frac{d}{d_0} = \frac{1.75 \times 10^3}{3.2 \times 10^3}$$

U. = 1 Q

Part B

What is the energy now stored if the capacitor remained connected to the potential source while the separation of the plates was changed? V is constant

ANSWER:

ANSWER:

$$U = 1.83 \ U_0 = 12.9 \ (J)$$

$$U = \frac{1}{2} \ C' V^2$$

$$U_0 = \frac{1}{2} \ C' V^2$$

$$U_0 = \frac{1}{2} \ C' V^2$$

$$U_0 = \frac{1}{2} \ C' V^2$$

Exercise 24.36

A parallel-plate capacitor has a capacitance of $C_0 = 3.80 \mathrm{pF}$ when there is air between the plates. The separation between the plates is 2.60mm.

Part A

What is the maximum magnitude of charge that can be placed on each plate if the electric field in the region between the plates is not to exceed $3.00 \times 10^4 \text{V/m}$? Emax 0 = Vmax = Qmax

ANSWER:

$$Q = 2.96 \times 10^{-10}$$

Part B

A dielectric with a dielectric constant of 3.10 is inserted between the plates of the capacitor, completely filling the volume between the plates. Now what is the maximum magnitude of charge on each plate if the electric field between the plates is not to exceed $3.00 \times 10^4 V/m$? Qmax = | Emax | · d · C' ; C' = K Co

ANSWER:

$$Q = 9.2 \times 10^{-10}$$

$$Q = 9.2 \times 10^{-10}$$

$$Q = 9.2 \times 10^{-10}$$

Exercise 24.45

A parallel-plate capacitor has the volume between its plates filled with plastic with dielectric constant K. The magnitude of the charge on each plate is Q. Each plate has area A, and the distance between the plates is d.

Part A

Use Gauss's law to calculate the magnitude of the electric field in the dielectric.

Express your answer in terms of the given quantities and appropriate constants.

ANSWER:

$$E = \frac{Q}{K \epsilon_0 A}$$

se constants.

Remodified Gavss's law

$$\oint K \vec{E} \cdot d\vec{A} = \frac{Q \text{ free-encl}}{E}$$

$$|\vec{E}| = \frac{Q}{KE} \cdot \vec{A}$$

Part B

Use the electric field determined in part A to calculate the potential difference between the two plates.

Express your answer in terms of the given quantities and appropriate constants.

ANSWER:

HW4
$$\overrightarrow{E}$$

$$= -\int_{1}^{2} \overrightarrow{E} \cdot d\overrightarrow{R}$$

$$= -\int_{1}^{2} \overrightarrow{E} \cdot d\overrightarrow{R}$$

$$\Delta V = -\int_{1}^{2} \vec{E} \cdot d\vec{k}$$

$$= -\int_{1}^{2} \frac{\vec{Q}}{|\vec{Q}|} d\vec{k}$$

$$= -\frac{\vec{Q}}{|\vec{Q}|} d\vec{k}$$

$$= -\frac{\vec{Q}}{|\vec{Q}|} d\vec{k}$$

Part C

Use the result of part B to determine the capacitance of the capacitor.

Express your answer in terms of the given quantities and appropriate constants.

ANSWER:

$$C = \frac{Q}{V} = \frac{\langle \epsilon, A \rangle}{d}$$

Problem 24.53

A $20.0\mu\mathrm{F}$ capacitor is charged to a potential difference of $800\mathrm{V}$. The terminals of the charged capacitor are then connected to those of an uncharged $9.00 \mu F$ capacitor.

Part A

⇒ Q = CV = 20×10⁶×800

= 1.6x10=

ANSWER:

Q= 1.6 x 102

Part B

Part C

Compute the final potential difference across capacitor.

Compute the original charge of the system.

 \mathbf{C}

ANSWER:

 $\Delta V = 550 \text{ V}$

 $\frac{Q_1}{C_1} = \frac{Q_2}{C_2}$

 $V = Q_1 + Q_2 = Q \leftarrow charges are shared$

 $Q_1 + \frac{C_2}{G_1}Q_1 = Q$ $Q_1 = Q \times \frac{c_1}{c_1 + c_2} = 1.6 \times 10^{-2} \times \frac{20 \times 10^{-6}}{60 + 9.0 \times 10^{-6}}$

Compute the final energy of the system.

ANSWER:

= 1.1x10 C

 $V_i = \frac{Q_i}{C_i} = \frac{1.1 \times 10^{-2}}{1.0 \times 10^{-6}} = 550 \text{ V}$

$$U = 4,39 (J)$$

$$= \frac{1}{2} C_1 V_1^2 + \frac{1}{2} C_2 V_2^2$$

$$= \frac{1}{2} V^2 (C_1 + C_2) = \frac{1}{2} \times 550^2 (20 + 9) \times 10^6$$

Part D

Compute the decrease in energy when the capacitors are connected.

$$\Delta U = \left| V_{f} - U_{7} \right| = \left| 4.39 - 6.4 \right| = \sim 2$$
 J

$$D_{i} = \frac{1}{2} CV^{2}$$

$$= \frac{1}{2} \times 20 \times 10^{6} \times 860^{2}$$

$$= 6.4$$

Score Summary:

Your score on this assignment is 0.0%.

You received 0 out of a possible total of 15 points.