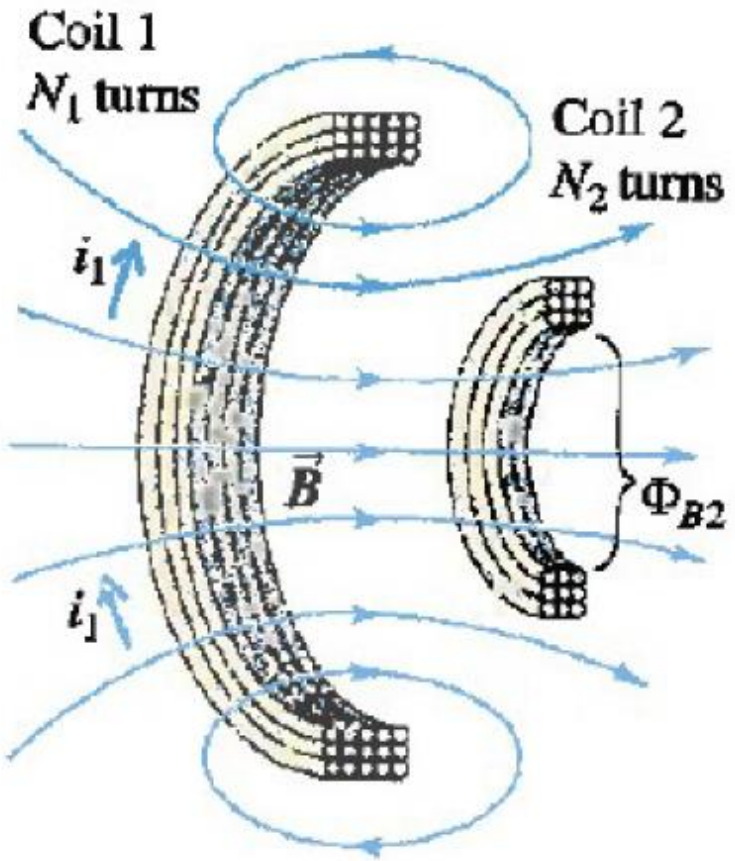
Chapter 30: Inductance

- Mutual Inductance
- Self Inductance
- R-L, L-C, R-L-C circuits

Mutual Inductance

$$\epsilon_1 = -N_1 \frac{d\Phi_{B1}}{dt} \qquad \epsilon_2 = -N_2 \frac{d\Phi_{B2}}{dt}$$



$$\Phi_{B2} = \oint \vec{B}_1 \cdot d\vec{A}$$

$$N_2 \Phi_{B2} = M_{21} I_1$$

$$\epsilon_1 = -M \frac{dI_2}{dt} \qquad \epsilon_2 = -M \frac{dI_1}{dt}$$

Unit for the mutual inductance: henry (H)

$$1\,H = 1\,Wb/A = 1\,\mathrm{V}\cdot s/A = 1\,\Omega\cdot s = 1\,J/A^2$$

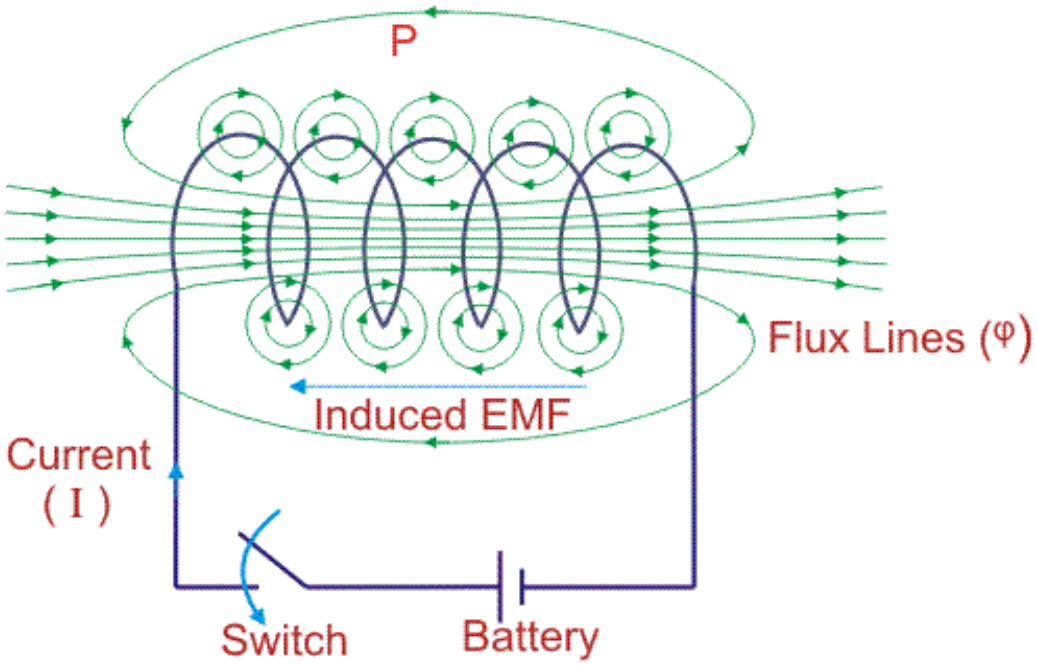
Example

In one form of Tesla coil (a high-voltage generator popular in science museums), a long solenoid with length *l* and cross-sectional area *A* is closely wound with N_1 turns of wire. A coil with N_2 turns surrounds it at its center. Find the mutual inductance M.

Self Inductance

$$L = \frac{N \Phi_B}{I}$$

$$\epsilon = -N \frac{d \Phi_B}{dt} = -L \frac{dI}{dt}$$

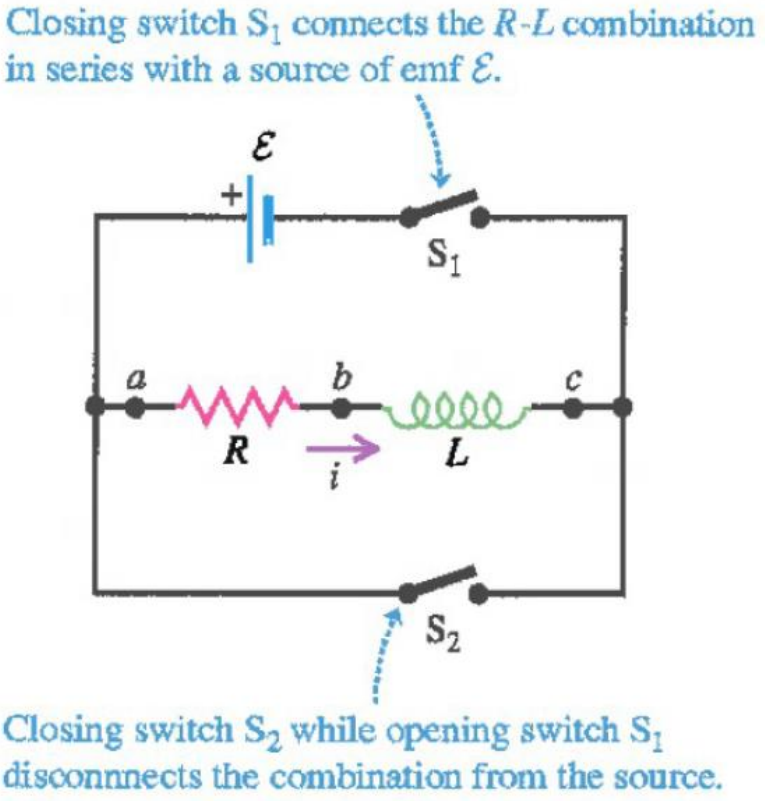


R-L circuit

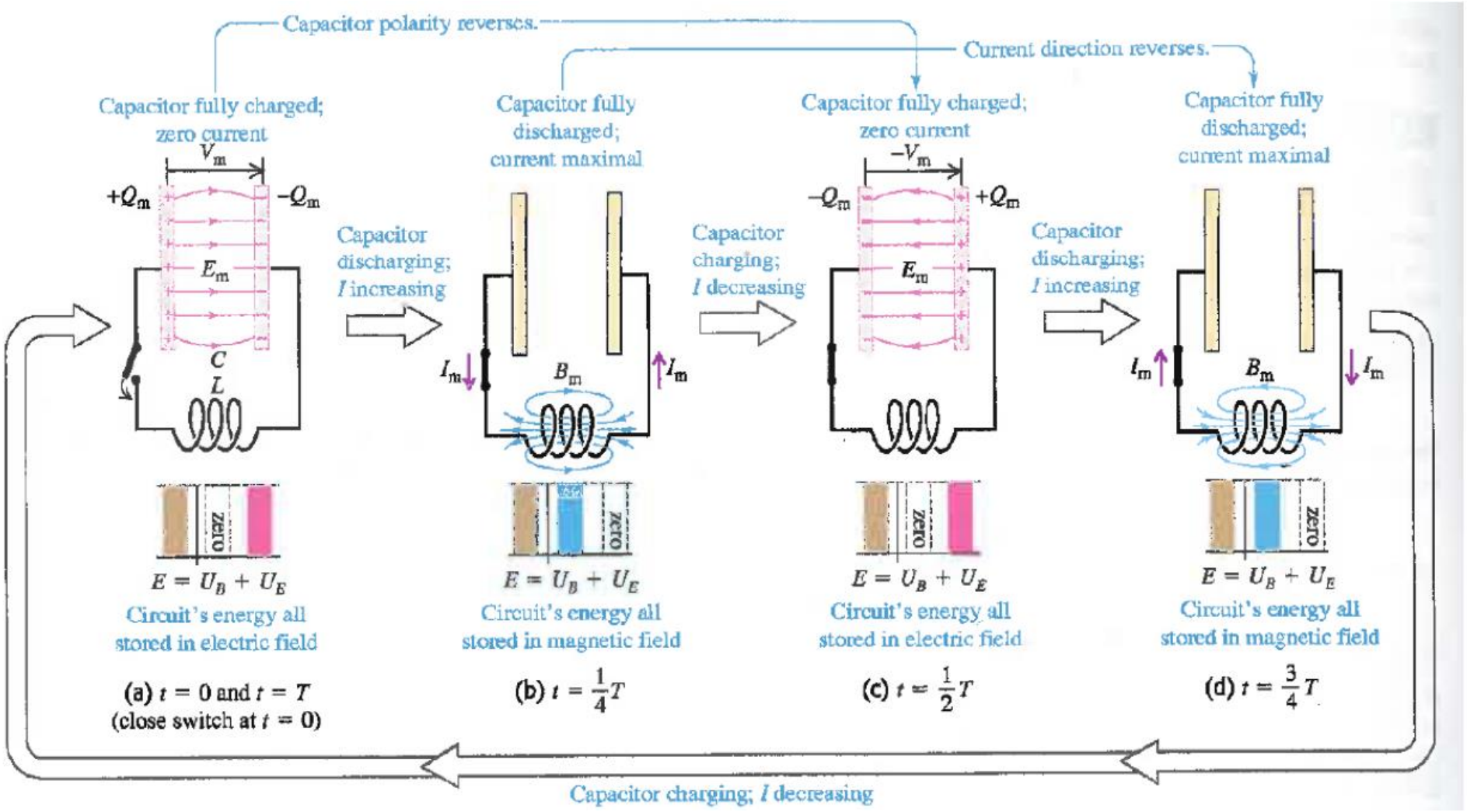
$$I(t)=\frac{\epsilon}{R}\left(1-e^{-(R/L)t}\right)$$

$$I(t)=I_0 e^{-(R/L)t}$$

Time constant: $\tau=\frac{L}{R}$



L-C circuit


Capacitor polarity reverses.
Current direction reverses.
Capacitor fully charged; zero current
Capacitor fully discharged; current maximal
Capacitor fully charged; zero current
Capacitor fully discharged; current maximal
Capacitor discharging; I increasing
Capacitor charging; I decreasing
Capacitor discharging; I increasing
Capacitor charging; I decreasing
zero
$E = U_B + U_E$
Circuit's energy all stored in electric field
Circuit's energy all stored in magnetic field
(a) $t = 0$ and $t = T$ (close switch at $t = 0$)
(b) $t = \frac{1}{4}T$
(c) $t = \frac{1}{2}T$
(d) $t = \frac{3}{4}T$

L-C circuit

$$q(t) = Q\cos(\omega t + \phi)$$
$$I(t) = -\omega Q\sin(\omega t + \phi)$$

$$\omega = 2\pi f = \sqrt{\frac{1}{LC}}$$

R-L-C circuit

$$q(t) = A e^{-(R/2L)t} \cos\left(\sqrt{\frac{1}{LC} - \frac{R^2}{4L^2}}\, t + \phi\right)$$

$$\omega' = \sqrt{\frac{1}{LC} - \frac{R^2}{4L^2}}$$