Chapter 5
The Wavelike Properties of Particles
The de Broglie Hypothesis (1925)

Electrons could be considered as wave and the frequency and the wavelength are related to energy and momentum as:

\[ f = \frac{E}{h} \]
\[ \lambda = \frac{h}{p} \]

This gives physical interpretation of Bohr’s quantization of the angular momentum of the electron in hydrogenlike atoms.
Review of $E$ and $P$ vs $f$ and $\lambda$

Massive particle

\[ E = \gamma mc^2 = \sqrt{(mc^2)^2 + (pc)^2} \]

Massless particle (E&M wave)

\[ E = pc \]

Non-relativistic massive particle

\[ \lambda = \frac{h}{p} \]
\[ E \approx E_k = \frac{p^2}{2m} \]
\[ p = \sqrt{2mE_k} \]

Energy and momentum for massless particle

\[ E = hf \]
\[ p = \frac{E}{c} = \frac{hf}{c} = \frac{h}{\lambda} \]
\[ f = \frac{E}{h} \]
\[ \lambda = \frac{h}{p} \]
Example

• Compute the de Broglie wavelength of an electron whose kinetic energy is 10 eV.
The Davisson-Germer Experiment (1927)
Interference/Diffraction with wave

\[ n\lambda = 2d\sin\theta \]
Crystal lattice of atoms

\[ n\lambda = 2d \sin \theta \]

D is a intrinsic property of a particular crystal, a single value

d varies depend on the plane you are considering
Electron Diffraction

\[ n\lambda = 2d \sin \theta \]

*d*: spacing between certain set of planes

\[ n\lambda = D \sin 2\alpha \]

*D*: spacing between lattice points
Example

• Calculate the constructive diffraction angle, $\varphi$, of Ni (D = 0.215 nm) for this particular crystal planes (indicated by red line), when using electrons with 54 eV energy. Assuming the crystal is square/cubic.
Electron Diffraction of Al Foil (1927)

State-of-the-art electron diffraction instrument: Low Energy Electron Diffraction optics

Cu(001) crystal with 134 eV electron
Diffraction with Other Particles

Cu with neutron

NaCl with neutron

Oxygen nuclei with proton

LiF with He

All matter has wavelike as well as particlelike properties.
Example

- Compute the wavelength of a neutron with kinetic energy of 0.0568 eV. Cu has lattice constant (atom-atom distance) of 0.36 nm. If you use this neutron beam to do the diffraction experiment, what would be the first angle you could see the diffraction?
Wave Packets

• The matter wave describe the probability of finding the particle

Wave Equation

\[
\frac{\partial^2 y}{\partial x^2} = \frac{1}{v^2} \frac{\partial^2 y}{\partial t^2}
\]

\[
y(x, t) = y_0 \cos(kx - \omega t)
\]

Wave number \( k = \frac{2\pi}{\lambda} \)

Angular (phase) frequency \( \omega = \frac{2\pi}{T} \)

Phase velocity \( v_p = f\lambda = \frac{\lambda}{T} = \frac{\omega}{k} \)
Wave Packet

\[ y(x, t) = y_0 \cos(k_1 x - \omega_1 t) + y_0 \cos(k_2 x - \omega_2 t) \]

\[ y(x, t) = 2y_0 \cos\left(\frac{1}{2} \Delta k x - \frac{1}{2} \Delta \omega t\right) \sin(k x - \bar{\omega} t) \]

http://www.acs.psu.edu/drussell/Demos/superposition/beats.gif
https://upload.wikimedia.org/wikipedia/commons/thumb/b/bd/Wave_group.gif/440px-Wave_group.gif
https://upload.wikimedia.org/wikipedia/commons/c/c7/Wave_opposite-group-phase-velocity.gif
Wave Packet (adding more than two waves)

\[ v_g = \frac{d\omega}{dk} \]
Group velocity and phase velocity

Phase velocity \( v_p = \frac{\omega}{k} \)

Group velocity \( v_g = \frac{d\omega}{dk} \)

Directly related to particle velocity

- Non-dispersive medium: phase velocity is independent of \( k \) (sound waves in air, E&M wave in vacuum)
- Dispersive medium: phase velocity varies with \( k \) (E&M wave in medium, water wave, electron wave)

https://upload.wikimedia.org/wikipedia/commons/thumb/b/bd/Wave_group.gif/440px-Wave_group.gif
https://upload.wikimedia.org/wikipedia/commons/c/c7/Wave_opposite-group-phase-velocity.gif
Particle Wave Packets

\[ E = hf = \hbar \omega \]
\[ p = \frac{h}{\lambda} = \hbar k \]

\[ E = \hbar \omega = \frac{p^2}{2m} = \frac{\hbar^2 k^2}{2m} \]
\[ \omega = \frac{\hbar k^2}{2m} \]

\[ v_p = \frac{\omega}{k} = f \lambda = \frac{E}{p} = \frac{p}{2m} = \frac{mv}{2m} = \frac{v}{2} \]

\[ v_g = \frac{d\omega}{dk} = \frac{dE}{dp} = \frac{p}{m} = \frac{mv}{m} = v \]
Probabilistic Interpretation of the Wave Function

https://www.youtube.com/watch?v=7u_UQG1La1o
Uncertainty Relation (Classical)

\[ \Delta k \Delta x \sim 1 \]
\[ \Delta \omega \Delta t \sim 1 \]
Example

• The frequency of the alternating voltage produced at electric generating stations is carefully maintained at 60.00 Hz (in North America). The frequency is monitored on a digital frequency meter in the control room. For how long must the frequency be measured and how often can the display be updated if the reading is to be accurate to within 0.01 Hz?
Uncertainty Relation (Quantum)

\[ \Delta p \Delta x \sim \hbar \]
\[ \Delta E \Delta t \sim \hbar \]
\[ \Delta p \Delta x \geq \frac{\hbar}{2} \]
\[ \Delta E \Delta t \geq \frac{\hbar}{2} \]

Particle confined in a 1D box with length of L

\[ \Delta p \Delta x \geq \hbar \]
\[ \Delta p \geq \frac{\hbar}{L} \]

\[ (\Delta p)^2 = (p - \bar{p})^2_{ave} = (p^2 - 2p\bar{p} + \bar{p}^2)_{ave} = \bar{p}^2 - \bar{p}^2 \]

\[ \bar{E} = \frac{\bar{p}^2}{2m} = \frac{(\Delta p)^2}{2m} \geq \frac{\hbar^2}{2mL^2} \]
Example

• If the particle in a one-dimensional box of length $L = 0.1$ nm (about the diameter of an atom) is an electron, what will be its zero-point energy?
Lifetime of an energy state

Uncertainty Principle for energy and time  \[ \Delta E \Delta t \sim \hbar \]

If for a particular atomic spectrum transition, the lifetime, \( \tau \), is in the order of 10 ns (\( 10^{-8} \text{s} \)), there is an energy uncertainty as:

\[ \Delta E \sim \frac{\hbar}{\tau} \approx 10^{-7} \text{eV} \]

Thus, the uncertainty in frequency of the emitted light is:

\[ \Delta f = \frac{\Delta E}{\hbar} \approx \frac{1}{\tau \hbar} \approx \frac{1}{2\pi \tau} \approx 10^7 \text{Hz} \]

Then, the uncertainty in wavelength is:

\[ \frac{\Delta \lambda}{\lambda} \approx \frac{\Delta E}{E - E_0} \]
Example

- When you measure a spectrum emitted by He atom, you found out there is a peak width in intensity vs energy spectrum is around 0.1 meV. Assuming this peak width originates from the energy uncertainty of the excited state of He atom, find out the lifetime of the excited state of He atom.