Do the following ten problems and be prepared to discuss them in class.

1. Suppose that the interstellar medium contains dust grains with uniform number density $10^{-6}$ m$^{-3}$, all with the same radius, $10^{-7}$ m, and extinction efficiency $Q_{\text{ext}} = 0.5$ at wavelength $\lambda_0$. Show that the extinction in magnitudes at wavelength $\lambda_0$ for a 1 kpc distant star is $\sim 0.5$ mag.

2. How long does it take grains to grow? Suppose at time $t = 0$ the grain radius is $r(0)$, and that the grain grows by the addition of species $i$ (an atom or molecule) which has mass $m_i$, and mean thermal velocity $\bar{v}_i$. Let $s$ be the bulk density of the material and $\epsilon$ the sticking coefficient.
   a) Show that the grain radius as function of time is $r(t) = r(0) + \frac{\epsilon n_i m_i \bar{v}_i}{4s} t$.
   b) What assumption is made concerning $n_i$?
   c) How long would it take for a grain of radius $10^{-7}$ m to grow to a radius of $2 \times 10^{-7}$ m by adding a mantle of CO? Assume $n_{\text{CO}} = 10^6$ m$^{-3}$, $T = 10$ K, $\epsilon = 1$, and $s_{\text{CO}} = 10^3$ kg m$^{-3}$.

3. Show that the lifetime for interstellar grains by sputtering in supernova blast waves is of order $10^9$ yr. Assume a uniform interstellar gas, H-atom density $10^6$ m$^{-3}$, a Galactic supernova rate of one every 30 yr, and that a supernova of energy $5 \times 10^{43}$ J may set up shock waves that destroy all grains in a mass of gas $\approx 300 \, M_\odot$.

4. Suppose that the heat input into an interstellar grain is entirely by UV absorption. Assume that the UV flux is $10^{10}$ photons m$^{-2}$ s$^{-1}$ nm$^{-1}$, that the bandwidth is 100 nm, and that the mean photon energy is 9 eV.
   a) If the grain cools by radiating like a blackbody with an efficiency of 0.1%, show that the grain temperature is $\sim 12.6$ K.
   b) For standard-sized grains of this temperature, estimate the flux of the re-radiated infrared radiation at 60 $\mu$m in a 20' beam looking toward the Galactic pole. Take the path length to be 100 pc and ISM density to be the standard $10^6$ H atom m$^{-3}$. Express your result in Jy. Assume (but check) that the optical depth is $\ll 1$. 
5. A large velocity may be obtained by grains if they are blown by radiation pressure through an envelope of cool gas surrounding a star. The radiation acceleration is usually much greater than gravity, and acceleration outwards is impeded only by collisions with the gas. Balancing these two forces for grains of radius $a$ at a distance $R$ from a star of radius $R_*$ gives for the velocity $v_g$ of the grains, of number density $n(R)$,

$$E \left( \frac{R_*}{R} \right)^2 Q_{pr} \frac{\pi a^2}{c} \approx n(R) \pi a^2 m_H v_g^2$$

where $E$ is the photon power per unit area emitted by the star and $Q_{pr} \approx 1$ is the radiation pressure efficiency of the grain. Show that the grain velocity at the radius of the star is $\sim 3$ km s$^{-1}$ if $T_* = 10^{3.5}$ K and $n(R_*) = 10^{18}$ m$^{-3}$.

6. Suppose two 05 stars are located near the origin of a spherical shell of dust. The inner radius of the shell is approximately 1 A.U. and the outer radius is 5 pc.
   a) Plot the dust temperature as a function of distance from the stars. Assume that the dust cloud is optically thin and that the ratio of the optical absorbing coefficient to the IR emissivity coefficient is $10^3$.
   b) How would your plot quantitatively change if the central heating source were a single O5 star?
   c) How would your plot qualitatively change if the dust cloud were optically thicker (but the same size)? i.e., say $\tau \sim 5$ so that the effects of absorption and scattering are enhanced, and thus fewer and fewer photons reach the outskirts as compared to the optically thin case.
   d) What does it mean for the dust cloud to be optically thicker? In other words, what would have to be the physical difference in the clouds?
   e) We assume the dust particles are heated only by the energy directly coming from the central stars. Why is it ok to neglect the effects of the thermally re-radiated energy by neighboring dust particles?

7.) Dark globules, with a typical radius of 0.05 pc, contain enough dust to reduce background starlight at optical wavelengths by a factor of 50 or more.
   a) Calculate the number density of grains in a globule if the radius of the dust particles is 100 nm and the extinction efficiency is $Q = 0.5$.
   b) Calculate the total mass (in $M_\odot$) of the globule assuming a typical dust-to-gas mass ratio of 1:100.
8. Here we will study the way in which way grains align, perpendicular or parallel to the magnetic field.

a) Suppose grains are prolate spheroids with moments of inertia $I_\parallel$ and $I_\perp$ ($I_\parallel < I_\perp$). In thermodynamic equilibrium, we expect the rotational energy to be the same on both axes. Show that this means that the angular momenta $J_\parallel$ and $J_\perp$ are related by the equation

$$J_\parallel = J_\perp \left(\frac{I_\parallel}{I_\perp}\right)^{0.5}$$

so that

$$J_\parallel < J_\perp$$

and the particle therefore tends to rotate on the short axis. If there is a magnetic field present, particles with vector $\mathbf{J}$ not parallel to the field $\mathbf{B}$ will have a magnetic moment $\mathbf{M}$ induced, which will dissipate energy as the particle rotates. Eventually a minimum of energy will be reached where $\mathbf{J} \parallel \mathbf{B}$. Thus the grains end up perpendicular to the field lines.

b) How fast do grains rotate? Use the equipartition theory to estimate the angular speed $\omega$ of a grain (assume spherical symmetry) in a gas of temperature $T$ (approximately 100 K in an H I cloud). Why should it be the gas temperature, rather than the grain temperature, which is used in this calculation?

9. Assuming that dust grains in the planetary nebula NGC 7027 emit as blackbodies, the 9.2$\mu$m-to-18.0$\mu$m ratio can be used to calculate the dust temperature across the nebula.\(^1\)

a) Plot $I_\nu(9.2\mu m)/I_\nu(18.0\mu m)$ as a function of dust temperature.\(^2\) What is the grain temperature in the hottest part of the nebula?

b) Theoretically calculate the expected temperature of a dust grain that lies in the hottest part of the nebula. Assume that the only source of heating is the hot central star (whose position is marked with a cross) and that the grains are in thermal equilibrium with the environment. Assume:

Distance to NGC 7027 = 880 pc
Luminosity of central star = 7700$L_\odot$
grain radius = 0.1$\mu$m
grain albedo (fraction of incident energy that is reflected) $A = 0.1$
grain emissivity $\epsilon = 0.1$

c) Compare and comment upon the temperatures calculated in parts a) and b).

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\(^1\)The infrared flux density ratio map is provided at physics.uwyo.edu/~ddale/teach/NGC7027.jpg.

\(^2\)The y-axis scaling for the figure quantifies this ratio, where the intensities are in units of Jansky per steradian.
10. Suppose that dust produced extinction $A(\lambda)$ proportional to the frequency of light. What would be the value of $R_V$?