The Electric Field inside a Conductor

Learning Goal:

To understand how the charges within a conductor respond to an externally applied electric field.

To illustrate the behavior of charge inside conductors, consider a long conducting rod that is suspended by insulating strings (see the figure). Assume that the rod is initially electrically neutral, and that it remains so for this discussion. The rod is positioned along the x axis, and an external electric field that points in the positive x direction (to the right) can be applied to the rod and the surrounding region. The atoms in the rod are composed of positive nuclei (indicated by plus signs) and negative electrons (indicated by minus signs). Before application of the electric field, these atoms were distributed evenly throughout the rod.

Part A

What is the force felt by the electrons and the nuclei in the rod when the external field described in the problem introduction is applied? (Ignore internal fields in the rod for the moment.)

You did not open hints for this part.

ANSWER:

- Both electrons and nuclei experience a force to the right.  
- The nuclei experience a force to the right and the electrons experience a force to the left.  
- The electrons experience a force to the left but the nuclei experience no force.  
- The electrons experience no force but the nuclei experience a force to the right.
What is the motion of the negative electrons and positive atomic nuclei caused by the external field?

You did not open hints for this part.

ANSWER:

- Both electrons and nuclei move to the right.
- The nuclei move to the right and the electrons move to the left through equal distances.
- The electrons move to the left and the nuclei are almost stationary.
- The electrons are almost stationary and the nuclei move to the right.

Part C

Imagine that the rightward current flows in the rod for a short time. As a result, what will the net charge on the right and left ends of the rod become?

**Hint 1.** How to approach this part

Remember that the rod as a whole must remain electrically neutral even if the charges are redistributed. This is because applying an electric field does not change the charge on the rod, only redistributes it.

ANSWER:

- left end negative and right end positive
- left end negative and right end negative
- left end negative and right end nearly neutral
- left end nearly neutral and right end positive
- both ends nearly neutral

Part D

The charge imbalance that results from this movement of charge will generate an additional electric field in the region within the rod. In what direction will this field point?

**Hint 1.** Direction of the electric field

The electric field point away from positive charges and towards negative ones.

ANSWER:
PhET Tutorial: Charges and Electric Fields

Learning Goal:
To understand the spatial distribution of the electric field for a variety of simple charge configurations.

For this problem, use the PhET simulation Charges and Fields. This simulation allows you to place multiple positive and negative point-charges in any configuration and look at the resulting electric field.

Start the simulation. You can click and drag positive charges (red) or negative charges (blue) into the main screen. If you select Show E-field in the green menu, red arrows will appear, showing the direction of the electric field. Faint red arrows indicate that the electric field is weaker than at locations where the arrows are brighter (this simulation does not use arrow length as a measure of field magnitude).
Feel free to play around with the simulation. When you are done, click Clear All before beginning Part A.

Part A
Select Show E-field and grid in the green menu. Drag one positive charge and place it near the middle of the screen, right on top of two intersecting bold grid lines. You should see something similar to the figure below.
ANSWER:

The electric field produced by the positive charge

☐ is directed radially away from the charge at all locations near the charge.
☐ wraps circularly around the positive charge.
☐ is directed radially toward the charge at all locations near the charge.

Part B

Now, let's look at how the distance from the charge affects the magnitude of the electric field. Select **Show numbers** on the green menu, and then click and drag one of the orange E-Field Sensors. You will see the magnitude of the electric field given in units of V/m (volts per meter, which is the same as newtons per coulomb).

Place the E-Field Sensor 1 m away from the positive charge (1 m is two bold grid lines away if going in a horizontal or vertical direction), and look at the resulting field strength.

Consider the locations to the right, left, above, and below the positive charge, all 1 m away.

ANSWER:

\[ E = \frac{1}{4\pi\varepsilon_0} \frac{q}{r^2} \hat{r}, \text{ same } |\hat{r}|, \text{ same magnitude} \]

For these four locations, the magnitude of the electric field is

☐ greatest to the left of the charge.
☐ greatest below the charge.
☐ the same.
☐ greatest to the right of the charge.
☐ greatest above the charge.
Part C

What is the magnitude of the electric field 1 m away from the positive charge compared to the magnitude of the electric field 2 m away?

**Hint 1. How to approach the problem**

Use an E-Field Sensor to determine the field strength both at 1 m away and at 2 m away from the charge. Then, take the ratio of the two field strengths.

**ANSWER:**

\[ |E_1| = \frac{1}{4\pi \varepsilon_0} \frac{q}{r^2} \quad |E_2| = \frac{1}{4\pi \varepsilon_0} \frac{q}{(2r)^2} = 4 |E_2| \]

The magnitude of the electric field 1 m away from the positive charge is four times the magnitude of the electric field 2 m away.

Part D

If the field strength is \( E = 9 \text{ V/m} \) a distance of 1 m from the charge, what is the field strength \( E \) a distance of 3 m from the charge?

**Hint 1. How to approach the problem**

The magnitude of the electric field is inversely proportional to distance squared \( (E \propto 1/r^2) \). So if the distance is increased by a factor of three, the field strength must decrease by a factor of three squared. You could use the simulation to make a measurement (you might have to drag the charge away from the center so you have enough room to get 3 m away).

**ANSWER:**

\[ |E| = \frac{1}{4\pi \varepsilon_0} \frac{q}{r^2} \quad 9 = \frac{1}{4\pi \varepsilon_0} \frac{q}{1} \quad \Rightarrow \frac{q}{4\pi \varepsilon_0} = 9 \quad |E'| = \frac{q}{4\pi \varepsilon_0} \frac{1}{3^2} \]

\[ E = \frac{9}{9} = 1 \text{ V/m} \]

Part E

Remove the positive charge by dragging it back to the basket, and drag a negative charge (blue) toward the middle of the screen. Determine how the electric field is different from that of the positive charge.

Which statement best describes the differences in the electric field due to a negative charge as compared to a positive charge?
Part F

Now, remove the negative charge, and drag two positive charges, placing them 1 m apart, as shown below.

Let's look at the resulting electric field due to both charges. Recall that the electric field is a vector, so the net electric field is the vector sum of the electric fields due to each of the two charges.

Where is the magnitude of the electric field roughly equal to zero (other than very far away from the charges)?

**ANSWER:**

- The electric field is nonzero everywhere on the screen.
- The electric field is roughly zero near the midpoint of the two charges.
- The electric field is zero at any location along a vertical line going through the point directly between the two charges.
Consider a point 0.5 m above the midpoint of the two charges. As you can verify by removing one of the positive charges, the electric field due to only one of the positive charges is about 18 V/m. What is the magnitude of the total electric field due to both charges at this location?

**ANSWER:**

- zero
- 36 V/m
- 25 V/m

\[ |E|_{\text{total}} = 18 \cos \theta + 18 \cos \theta = 36 \cos \theta \]

\[ = 36 \times \frac{0.5}{\sqrt{(0.5)^2 + 0.5^2}} \]

\[ = 25.46 \text{ V/m} \]

**Part H**

Make an electric dipole by replacing one of the positive charges with a negative charge, so the final configuration looks like the figure shown below.

**ANSWER:**

- The electric field at the midpoint is directed to the left.
- The electric field starts at "+" ends at "-".

**Part I**

Make a small dipole by bringing the two charges very close to each other, where they are barely touching. The midpoint of the two charges should still be on one of the grid point intersections (see figure below).
Measure the strength of the electric field 0.5 m directly above the midpoint as well as 1 m directly above. Does the strength of the electric field decrease as 1 over distance squared \((1/r^2)\)?

**Hint 1.** How to approach the problem

If the strength of the field is decreasing as \(1/r^2\), then the ratio of the magnitudes of the electric field measured at two distances, say 0.5 m away and 1 m away, would be

\[ E_{r=0.5}/E_{r=1} = (1/0.5)^2/(1/1)^2 = 4. \]

Compare this value to the value you measure with an E-Field Sensor.

**ANSWER:**

- Yes, it does.
- No, it decreases more quickly with distance.
- No, it decreases less quickly with distance.

**Part J**

Make a long line of positive charges, similar to that shown in the figure below. Try to place all of the charges centered along a horizontal grid line. Feel free to look at the electric field, as it is interesting.
Measure the strength of the electric field 1 m directly above the middle as well as 2 m directly above. Does the strength of the electric field decrease as 1 over distance squared (1/r²)?

**ANSWER:**

- ☐ No, it decreases more quickly with distance.
- ☑ Yes, it does.
- ☐ No, it decreases less quickly with distance.

**PhET Interactive Simulations**  
**University of Colorado**  
**http://phet.colorado.edu**

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**Video Tutor: Charged Rod and Aluminum Can**

First, launch the video below. You will be asked to use your knowledge of physics to predict the outcome of an experiment. Then, close the video window and answer the questions at right. You can watch the video again at any point.
Part A

Consider the situation in the figure below, where two charged rods are placed a distance \( d \) on either side of an aluminum can. What does the can do?

![Diagram of a charged can with two rods on either side](image)

You did not open hints for this part.

**ANSWER:**

- Rolls to the right
- Rolls to the left
- Stays still

Part B

Now, consider the situation shown in the figure below. What does the can do?
ANSWER:

- Rolls to the right
- Stays still
- Rolls to the left

Part C

Using the setup from the first question, imagine that you briefly touch the negatively charged rod to the can. You then hold the two rods at equal distances on either side of the can. What does the can do?

You did not open hints for this part.

ANSWER:

- Rolls toward the positively charged rod
- Does not move
- Rolls away from the positively charged rod

A Test Charge Determines Charge on Insulating and Conducting Balls

Learning Goal:

To understand the electric force between charged and uncharged conductors and insulators.

When a test charge is brought near a charged object, we know from Coulomb's law that it will experience a net force (either attractive or repulsive, depending on the nature of the object's charge). A test charge may also experience an electric force when brought near a neutral object. Any attraction of a neutral insulator or neutral conductor to a test charge must occur through induced polarization. In an insulator, the electrons are bound to their molecules. Though they cannot move freely throughout the insulator, they can shift slightly, creating a rather weak net attraction to a test charge that is brought close to the insulator's surface. In a conductor, free electrons will accumulate on the surface of the
conductor nearest the positive test charge. This will create a strong attractive force if the test charge is placed very close to the conductor’s surface.

Consider three plastic balls (A, B, and C), each carrying a uniformly distributed charge equal to either +Q, -Q or zero, and an uncharged copper ball (D). A positive test charge (T) experiences the forces shown in the figure when brought very near to the individual balls. The test charge T is strongly attracted to A, strongly repelled from B, weakly attracted to C, and strongly attracted to D.

Assume throughout this problem that the balls are brought very close together.

Part A

What is the nature of the force between balls A and B?

You did not open hints for this part.

**ANSWER:**

- strongly attractive
- strongly repulsive
- weakly attractive
- neither attractive nor repulsive

Part B

What is the nature of the force between balls A and C?

**Hint 1.** What is the charge on ball C?

Recall that ball C is composed of insulating material, which means that it can be polarized, but the charges inside are otherwise not free to move around inside the ball. Since the test charge experiences only a weak force due to ball C, if we compare to ball A we conclude that the charge on ball C must be

**ANSWER:**
Part C

What is the nature of the force between balls A and D?

Hint 1. What are the surface charges on ball D?

Recall that copper is a conductor, in which charges can freely flow. When ball D is brought close to ball A, what will be the nature of the surface charge density on the side of ball D that is closest to ball A?

ANSWER:

○ positive
○ negative
○ zero

Part D

What is the nature of the force between balls D and C?

ANSWER:
Coulomb's Law Tutorial

Learning Goal:
To understand how to calculate forces between charged particles, particularly the dependence on the sign of the charges and the distance between them.

Coulomb's law describes the force that two charged particles exert on each other (by Newton's third law, those two forces must be equal and opposite). The force \( \mathbf{F}_{21} \) exerted by particle 2 (with charge \( q_2 \)) on particle 1 (with charge \( q_1 \)) is proportional to the charge of each particle and inversely proportional to the square of the distance \( r \) between them:

\[
\mathbf{F}_{21} = \frac{k q_2 q_1}{r^2} \mathbf{\hat{r}}_{21},
\]

where \( k = \frac{1}{4\pi\varepsilon_0} \) and \( \mathbf{\hat{r}}_{21} \) is the unit vector pointing from particle 2 to particle 1. The force vector will be parallel or antiparallel to the direction of \( \mathbf{\hat{r}}_{21} \), parallel if the product \( q_1 q_2 > 0 \) and antiparallel if \( q_1 q_2 < 0 \); the force is attractive if the charges are of opposite sign and repulsive if the charges are of the same sign.

Part A

Consider two positively charged particles, one of charge \( q_0 \) (particle 0) fixed at the origin, and another of charge \( q_1 \) (particle 1) fixed on the \( y \)-axis at \((0, d_1, 0)\). What is the net force \( \mathbf{F} \) on particle 0 due to particle 1?

Express your answer (a vector) using any or all of \( k, q_0, q_1, d_1, \mathbf{i}, \mathbf{j}, \) and \( \mathbf{k} \).

**ANSWER:** The negative sign is because you know \( q_1 \) will repel \( q_0 \) to the left, which is \( -\mathbf{j} \) direction.

\[
\mathbf{F} = -k \frac{q_0 q_1}{d_1^2} \mathbf{j}
\]

Part B
Now add a third, negatively charged, particle, whose charge is \(-q_2\) (particle 2). Particle 2 fixed on the y-axis at position \((0, d_2, 0)\). What is the new net force on particle 0, from particle 1 and particle 2?

Express your answer (a vector) using any or all of \(k, q_0, q_1, q_2, d_1, d_2, \hat{i}, \hat{j}\), and \(\hat{k}\).

\[
\vec{F} = \left( -k \frac{q_0 q_1}{d_1^2} + k \frac{q_0 q_2}{d_2^2} \right) \hat{j}
\]

**Part C**

Particle 0 experiences a repulsion from particle 1 and an attraction toward particle 2. For certain values of \(d_1\) and \(d_2\), the repulsion and attraction should balance each other, resulting in no net force. For what ratio \(d_1/d_2\) is there no net force on particle 0?

Express your answer in terms of any or all of the following variables: \(k, q_0, q_1, q_2\).

**ANSWER:**

\[
d_1/d_2 = \frac{\sqrt{\frac{q_1}{q_2}}}{\sqrt{\frac{q_2}{q_1}}}
\]

**Part D**

Now add a fourth charged particle, particle 3, with positive charge \(q_3\), fixed in the yz-plane at \((0, d_2, d_2)\). What is the net force \(\vec{F}\) on particle 0 due solely to this charge?

Express your answer (a vector) using \(k, q_0, q_3, d_2, \hat{i}, \hat{j}\), and \(\hat{k}\). Include only the force caused by particle 3.
\[ |\vec{F}| = k \frac{\varrho_0 \varrho_3}{r^2} = k \frac{\varrho_0 \varrho_3}{2d_z^2} \]

\[ \vec{F} = -k \frac{\varrho_0 \varrho_3}{2d_z^2} \hat{j} - k \frac{\varrho_0 \varrho_3}{2d_z^2} \hat{k} \]

You did not open hints for this part.

**ANSWER:**

\[ \vec{F} = -k \frac{\varrho_0 \varrho_3}{2d_z^2} \hat{j} - k \frac{\varrho_0 \varrho_3}{2d_z^2} \hat{k} \]

---

**The Trajectory of a Charge in an Electric Field**

An charge with mass \( m \) and charge \( q \) is emitted from the origin, \((x, y) = (0, 0)\). A large, flat screen is located at \( x = L \). There is a target on the screen at y position \( y_h \), where \( y_h > 0 \). In this problem, you will examine two different ways that the charge might hit the target. Ignore gravity in this problem.

**Part A**

Assume that the charge is emitted with velocity \( v_0 \) in the positive x direction. Between the origin and the screen, the charge travels through a constant electric field pointing in the positive y direction. What should the magnitude \( E \) of the electric field be if the charge is to hit the target on the screen?

**Express your answer in terms of \( m \), \( q \), \( y_h \), \( v_0 \), and \( L \).**

You did not open hints for this part.

**ANSWER:**

\[ E = \frac{2m v_0 y_h}{8 L^2} \]
Part B

Now assume that the charge is emitted with velocity $v_0$ in the positive $y$ direction. Between the origin and the screen, the charge travels through a constant electric field pointing in the positive $x$ direction. What should the magnitude $E'$ of the electric field be if the charge is to hit the target on the screen?

Express your answer in terms of $m$, $q$, $y_h$, $v_0$, and $L$.

You did not open hints for this part.

ANSWER:

\[
E' = \frac{2mL v_0^2}{q y_h^2}
\]

\[
y_h = v_0 t = v_0 \sqrt{\frac{2mL}{q E'}} \Rightarrow |E'| = \frac{2mL v_0^2}{q y_h^2}
\]

Charged Ring

Consider a uniformly charged ring in the $xy$ plane, centered at the origin. The ring has radius $a$ and positive charge $q$ distributed evenly along its circumference.

Part A

What is the direction of the electric field at any point on the $z$ axis?

You did not open hints for this part.

ANSWER:
Part B

What is the magnitude of the electric field along the positive z axis?

Use $k$ in your answer, where $k = \frac{1}{4\pi \varepsilon_0}$.

\[
E_3 = k \frac{d\bar{E} \cdot \cos \theta}{a^2 + \bar{z}^2} = k \frac{d\bar{E}}{a^2 + \bar{z}^2} \cdot \frac{3}{\sqrt{a^2 + \bar{z}^2}} = k \frac{d\bar{E} \cdot \frac{3}{\sqrt{a^2 + \bar{z}^2}}}{(a^2 + \bar{z}^2)^{3/2}}
\]

You did not open hints for this part.

ANSWER:

\[
E(z) = k \frac{\bar{z} \cdot \frac{3}{\sqrt{a^2 + \bar{z}^2}}}{(a^2 + \bar{z}^2)^{3/2}}
\]

Part C

Imagine a small metal ball of mass $m$ and negative charge $-q_0$. The ball is released from rest at the point $(0, 0, d)$ and constrained to move along the z axis, with no damping. If $0 < d \ll a$, what will be the ball's subsequent trajectory?

ANSWER:

○ repelled from the origin
○ attracted toward the origin and coming to rest
○ oscillating along the z axis between $z = d$ and $z = -d$
○ circling around the z axis at $z = d$

Part D

The ball will oscillate along the z axis between $z = d$ and $z = -d$ in simple harmonic motion. What will be the angular frequency $\omega$ of these oscillations? Use the approximation $d \ll a$ to simplify your calculation; that is, assume that $d^2 + a^2 \approx a^2$.

Express your answer in terms of given charges, dimensions, and constants.

Hint 1. Simple harmonic motion
Recall the nature of simple harmonic motion of an object attached to a spring. Newton's second law for the system states that

$$F_x = m \frac{d^2 x}{dt^2} = -k' x,$$

leading to oscillation at a frequency of

$$\omega = \sqrt{\frac{k'}{m}}.$$

(here, the prime on the symbol representing the spring constant is to distinguish it from $k = \frac{1}{4\pi\epsilon_0}$). The solution to this differential equation is a sinusoidal function of time with angular frequency $\omega$. Write an analogous equation for the ball near the charged ring in order to find the $\omega$ term.

**Hint 2.** Find the force on the charge

What is $F_z$, the $z$ component of the force on the ball on the ball at the point $(0, 0, d)$? Use the approximation $d^2 + a^2 \approx a^2$.

**Express your answer in terms of $q_0$, $k$, $q$, $d$, and $a$.**

**Hint 1.** A formula for the force on a charge in an electric field

The formula for the force $\vec{F}$ on a charge $q$ in an electric field $\vec{E}$ is

$$\vec{F} = q\vec{E}.$$

Therefore, in particular,

$$F_z = qE_z.$$

You have already found $E_z(z)$ in Part B. Use that expression in the equation above to find an expression for the $z$ component of the force $F_z$ on the ball at the point $(0, 0, d)$. Don’t forget to use the approximation given.

**ANSWER:**

\[
\vec{F} = q\vec{E} = -q_0 \cdot k \cdot \frac{q_0}{(a^2 + d^2)^{3/2}} \Rightarrow \ z = d
\]

\[
F_z = -k \cdot \frac{q_0 \cdot d}{(a^2 + d^2)^{3/2}} = -k \cdot \frac{q_0 \cdot d}{(a^2)^{3/2}} \quad (a^2 + d^2 \approx a^2)
\]

**ANSWER:**

\[
\omega = \sqrt{\frac{kq_0}{ma^2}}
\]

The Electric Field Produced by a Finite Charged Wire

A charged wire of negligible thickness has length $2L$ units and has a linear charge density $\lambda$. Consider the electric field $\vec{E}$ at the point $P$, a distance $d$ above the midpoint of the wire.
Part A

The field \( \vec{E} \) points along one of the primary axes. Which one?

You did not open hints for this part.

**ANSWER:**

- \( \hat{i} \)
- \( \hat{j} \) (correct)
- \( \hat{k} \)

Part B

What is the magnitude \( E \) of the electric field at point \( P \)? Throughout this part, express your answers in terms of the constant \( k \), defined by \( k = \frac{1}{4\pi\epsilon_0} \).

Express your answer in terms of \( L, \lambda, d, \) and \( k \).

**Hint 1. How to approach the problem**

To compute the field at point \( P \), divide the rod into infinitesimal segments and find the electric field produced by each. Then integrate to find the total electric field. The electric field due to an arbitrary infinitesimal segment of wire has both \( x \) and \( y \) components. You could integrate those two components separately, but because of symmetry, you already know that the total electric field points only in the \( y \) direction. So you need to work only with the \( y \) component of the field.

**Hint 2. Find the field due to an infinitesimal segment**
Find $dE_y(x)$, the $y$ component of the electric field produced at point $P$ by the infinitesimal segment of the wire between $x$ and $x + dx$.

**Express your answer in terms of $x$, $d$, $dx$, $\lambda$, and $k$.**

**Hint 1.** Find the total electric field

Find the magnitude $dE$ of the electric field at point $P$ produced by the infinitesimal wire segment between $x$ and $x + dx$.

**Express your answer in terms of $\lambda$, $d$, $dx$, $x$, and the constant $k$.**

**Hint 1. Formula for electric field due to a point charge**

The electric field in the direction $\vec{r}$ at position $\vec{r}$ away from an infinitesimal charge element $dq$ is given by $d\vec{E} = k \frac{dq}{r^2} \hat{r}$.

**Hint 2.** Find $|\vec{r}|$

What is the distance from an infinitesimal segment of wire at position $x$ to the point $P$?

**Express your answer in terms of $x$ and $d$.**

**Answer:**

$|\vec{r}| = \ldots$

**Hint 3.** Find an expression for $dq$

The wire has charge per unit length $\lambda$, and the length of an infinitesimal charge element is $dx$. Write an expression for $dq$, the total charge on the infinitesimal element $dx$.

**Express your answer in terms of $\lambda$ and $dx$.**

**Answer:**

$dq = \ldots$
\[ dE = \]

**Hint 2.** Find the \( y \) component of \( dE \)

What is \( dE_y \), the \( y \) component of the electric field at point \( P \) due to an infinitesimal segment of wire located at position \( x \)? Assume that the magnitude of the electric field at point \( P \) due to the infinitesimal wire segment is \( dE \).

**Express your answer in terms of \( dE \), \( x \), and \( d \).**

**ANSWER:**

\[ dE_y = \]

**ANSWER:**

\[ dE_y(x) = \]

**Hint 3.** A necessary integral

\[ \int \frac{1}{\sqrt{(a^2+x^2)^3}} \, dx = \frac{x}{a^2\sqrt{a^2+x^2}} \]

**ANSWER:**

\[ E = \frac{2k\lambda}{d\sqrt{d^2 + L^2}} \]

(see calculations two page above)
Dipole Motion in a Uniform Field

Consider an electric dipole located in a region with an electric field of magnitude $E$ pointing in the positive $y$ direction. The positive and negative ends of the dipole have charges $+q$ and $-q$, respectively, and the two charges are a distance $D$ apart. The dipole has moment of inertia $I$ about its center of mass. The dipole is released from angle $\theta = \theta_0$, and it is allowed to rotate freely.

---

**Part A**

What is $\omega_{\text{max}}$, the magnitude of the dipole's angular velocity when it is pointing along the $y$ axis?

**Express your answer in terms of quantities given in the problem introduction.**

You did not open hints for this part.

**ANSWER:**

\[
\omega_{\text{max}} = \sqrt{\frac{2 \beta d |E| (1 - \cos \theta_0)}{I}}
\]

\[
\Rightarrow \omega_{\text{max}} = \frac{2 \beta d |E| (1 - \cos \theta_0)}{I}
\]

---

**Part B**

If $\theta_0$ is small, the dipole will exhibit simple harmonic motion after it is released. What is the period $T$ of the dipole's oscillations in this case?

**Express your answer in terms of $\pi$ and quantities given in the problem introduction.**

You did not open hints for this part.

**ANSWER:**

When $\theta_0$ is small \( (\sin \theta_0 \approx \theta_0) \),

\[
\ddot{\theta} = -\frac{|\dot{P}| |E| \cdot \dot{\theta}}{I} \cdot \sin \theta = I \cdot \frac{d^2 \theta}{dt^2} = -\frac{\theta}{I} \frac{d^2 |E|}{dt^2}
\]

& SHM: \( \theta(t) = A \sin(\omega t) \)

\( \ddot{\theta} = -A \omega^2 \sin(\omega t) = -\omega^2 \theta \)

\( \Rightarrow W = \frac{\dot{d} |E|}{I} \)

\( W = \sqrt{\frac{\dot{d} |E|}{I}} = \frac{d |E|}{I} \Rightarrow \text{next page} \)
Charging an Insulator

This problem explores the behavior of charge on realistic (i.e. non-ideal) insulators. We take as an example a long insulating rod suspended by insulating wires. Assume that the rod is initially electrically neutral. For convenience, we will refer to the left end of the rod as end A, and the right end of the rod as end B. In the answer options for this problem, "weakly attracted/repelled" means "attracted/repelled with a force of magnitude similar to that which would exist between two balls, one of which is charged, and the other acquires a small induced charge". An attractive/repulsive force greater than this should be classified as "strongly attracted/repelled".

Part A

A small metal ball is given a negative charge, then brought near (i.e., within a few millimeters) to end A of the rod. What happens to end A of the rod when the ball approaches it closely this first time?

Select the expected behavior.

You did not open hints for this part.

ANSWER:

- strongly repelled
- strongly attracted
- weakly attracted
- weakly repelled
- neither attracted nor repelled

Now consider what happens when the small metal ball is repeatedly given a negative charge and then brought into contact with end A of the rod.

http://session.masteringphysics.com/myct/assignmentPrintView?displayMode=studentView&assignmentID=2627429
Part B

After several contacts with the charged ball, how is the charge on the rod arranged?

Select the best description.

You did not open hints for this part.

ANSWER:

○ positive charge on end B and negative charge on end A
○ negative charge spread evenly on both ends
☑ negative charge on end A with end B remaining almost neutral
○ positive charge on end A with end B remaining almost neutral
○ none of the above

Part C

How does end A of the rod react when the ball approaches it after it has already made several contacts with the rod, such that a fairly large charge has been deposited at end A?

Select the expected behavior.

ANSWER:

☑ strongly repelled
○ strongly attracted
○ weakly attracted
○ weakly repelled
○ neither attracted nor repelled

Exercise 21.19

Three point charges are arranged along the x-axis. Charge \( q_1 = +3.00 \, \mu C \) is at the origin, and charge \( q_2 = -5.00 \, \mu C \) is at \( x = 0.200 \, m \). Charge \( q_3 = -8.00 \, \mu C \).

Part A

Where is \( q_3 \) located if the net force on \( q_1 \) is 7.00 \( N \) in the \(-x\) direction?

ANSWER:

\[
\vec{F} = k \frac{q_1 q_2}{(a_2)^2} - k \frac{q_1 q_3}{x^2}
\]
\[ k \approx 9 \times 10^9 \ \text{Nm}^{-2}\text{C}^{-2} \]

\[ F = \frac{k(3\mu)(5\mu)}{(0.2)^2} - \frac{k(3\mu)(8\mu)}{x^2} \]

\[ m = -7 \ \text{(N)} \]

\[ x = 0.144 \ \text{m} \]

**Exercise 21.38**

A uniform electric field exists in the region between two oppositely charged plane parallel plates. A proton is released from rest at the surface of the positively charged plate and strikes the surface of the opposite plate, 1.52 cm distant from the first, in a time interval of \( 1.57 \times 10^{-8} \text{s} \).

**Part A**

Find the magnitude of the electric field.

**ANSWER:**

\[ E = 1.31 \times 10^2 \ \text{N/C} \]

\[ 1.52 \times 10^{-2} = 1.18 \times 10^{14} \ |E| \]

**Part B**

Find the speed of the proton when it strikes the negatively charged plate.

**ANSWER:**

\[ v = 1.97 \times 10^4 \ \text{m/s} \]

\[ V = V_0 + \frac{E}{m_p} \cdot t \]

\[ = 0 + \frac{\frac{2}{3} \frac{1}{1.67 \times 10^{-37}} \times (1.57 \times 10^{-8})^2}{\frac{2}{3} \times 1.67 \times 10^{-37}} \]

**Exercise 21.53**

A ring-shaped conductor with radius \( a = 2.80 \text{cm} \) has a total positive charge \( Q = 0.123 \text{nC} \) uniformly distributed around it.
\[ E_r = k \frac{2 \pi x}{(a^2 + x^2)^{3/2}} \]
\[ = 9 \times 10^9 \frac{0.123 \times 10^{-7} \cdot 39 \times 10^{-2}}{(2.8 \times 10^2)^2 + (39 \times 10^{-2})^2}^{3/2} \]
\[ = \frac{4.3197 \times 10^{-2}}{(0.1529)^{3/2}} = 7.22 \]

**Part A**

What is the magnitude of the electric field at point \( P \), which is on the positive \( x \)-axis at \( x = 39.0 \) cm?

**ANSWER:**

\[ E = 7.22 \text{ N/C} \]

**Part B**

What is the direction of the electric field at point \( P \)?

**ANSWER:**

\( \bigcirc \) +x-direction

\( \bigcirc \) -x-direction

**Part C**

A particle with a charge of \( -2.90 \mu C \) is placed at the point \( P \) described in part A. What is the magnitude of the force exerted by the particle on the ring?

\[ F = q \vec{E} = -2.9 \mu C \cdot 7.22 = 2.9 \times 10^{-6} \times 7.22 \]

**ANSWER:**

\[ F = 2.09 \times 10^{-5} \text{ N} \]
Part D

What is the direction of the force exerted by the particle on the ring?

[Answer Options]
- +x-direction
- -x-direction

Exercise 21.57

Point charges \( q_1 = -4.20 \, \text{nC} \) and \( q_2 = +4.20 \, \text{nC} \) are separated by a distance of 3.20 mm, forming an electric dipole.

Part A

Find the magnitude of the electric dipole moment.

\[
|\vec{P}| = q \cdot d = 4.2 \times 10^{-9} \times 3.2 \times 10^{-3} = 1.34 \times 10^{-11} \, \text{C} \cdot \text{m}
\]

Part B

Find the direction of the electric dipole moment?

[Answer Options]
- from \( q_1 \) to \( q_2 \)
- from \( q_2 \) to \( q_1 \)

Part C

The charges are in a uniform electric field whose direction makes an angle of 37.0° with the line connecting the charges. What is the magnitude of this field if the torque exerted on the dipole has magnitude 7.70 \times 10^{-9} \, \text{N} \cdot \text{m}?

\[
\vec{E} = \vec{P} \times \vec{E}
\]

\[
|\vec{E}| = |\vec{P}| \cdot |\vec{E}| \sin \theta
\]

7.7 \times 10^{-9} = 1.34 \times 10^{-11} (\vec{E}) \cdot \sin 37°

\[
|\vec{E}| = 9.5 \times 10^2 \, \text{N/C}
\]
Problem 21.82

Two tiny spheres of mass $m = 5.90\text{mg}$ carry charges of equal magnitude, $72.0\text{ nC}$, but opposite sign. They are tied to the same ceiling hook by light strings of length 0.530 m. When a horizontal uniform electric field $E$ that is directed to the left is turned on, the spheres hang at rest with the angle $\theta$ between the strings equal to $50.0^\circ$ in the following figure.

![Diagram of two spheres attached to strings with an electric field $E$ and angle $\theta$.]

**Part A**

Which ball (the one on the right or the one on the left) has positive charge?

**ANSWER:**

- The one on the left
- The one on the right

**Part B**

What is the magnitude $E$ of the field?

Express your answer with the appropriate units.

**ANSWER:**

$$E = 3.6 \times 10^3$$

**Score Summary:**

Your score on this assignment is 0.0%.
You received 0 out of a possible total of 15 points.