Calculating Electric Flux through a Disk

Suppose a disk with area $A$ is placed in a uniform electric field of magnitude $E$. The disk is oriented so that the vector normal to its surface, $\hat{n}$, makes an angle $\theta$ with the electric field, as shown in the figure.

**Part A**

What is the electric flux $\Phi_E$ through the surface of the disk that is facing right (the normal vector to this surface is shown in the figure)? Assume that the presence of the disk does not interfere with the electric field.

**Express your answer in terms of $E$, $A$, and $\theta$**

You did not open hints for this part.

**ANSWER:**

$$\Phi_E = EA \cos \theta$$

**Exercise 22.7**

Human nerve cells have a net negative charge and the material in the interior of the cell is a good conductor.

**Part A**

If a cell has a net charge of -8.65 pC, what are the magnitude of the net flux through the cell boundary?
Express your answer to three significant figures and include the appropriate units.

ANSWER:

\[ \Phi_E = 0.977 \]

\[ \Phi_E = \frac{Q_{\text{enc}}}{\varepsilon_0} = \frac{-8.65 \times 10^{-12}}{8.85 \times 10^{-12}} = -0.977 \]

**Part B**

What is the sign of the net flux through the cell boundary?

ANSWER:

- minus
- plus

---

**Exercise 22.9**

A charged paint is spread in a very thin uniform layer over the surface of a plastic sphere of diameter 10.0 cm, giving it a charge of -16.0 \( \mu \text{C} \).

**Part A**

Find the electric field just inside the paint layer.

Express your answer using three significant figures.

ANSWER:

\[ |E| = 0 \]

\[ \Rightarrow E = 0 \]

**Part B**

Find the electric field just outside the paint layer.

Express your answer using three significant figures.

ANSWER:

\[ |E| = 5.76 \times 10^7 \]

\[ \Rightarrow |E| = \frac{1}{4\pi \varepsilon_0} \frac{Q_{\text{enc}}}{y^2} \]

\[ = \frac{9 \times 10^{-9}}{(5 \times 10^{-6})^2} \frac{-16 \times 10^{-6}}{9 \times 10^{-9}} = 5.76 \times 10^7 \]
Part C

Find the electric field 9.00 cm outside the surface of the paint layer.

Express your answer using three significant figures.

ANSWER:

\[ |E| = 7.3 \times 10^6 \text{ N/C} \]

Exercise 22.2

A flat sheet is in the shape of a rectangle with sides of lengths 0.400 m and 0.600 m. The sheet is immersed in a uniform electric field of magnitude 77.9 N/C that is directed at 20° from the plane of the sheet (the figure).

Part A

Find the magnitude of the electric flux through the sheet.

Express your answer using two significant figures.

ANSWER:

\[ \Phi = 6.3 \text{ N} \cdot \text{m}^2/\text{C} \]

A Charged Sphere with a Cavity

An insulating sphere of radius \( a \), centered at the origin, has a uniform volume charge density \( \rho \).
Part A

Find the electric field \( \vec{E}(\vec{r}) \) inside the sphere (for \( r < a \)) in terms of the position vector \( \vec{r} \).

Express your answer in terms of \( \vec{r} \), \( \rho \) (Greek letter rho), and \( \varepsilon_0 \).

**Hint 1. How to approach the problem**

Apply Gauss’s law, which states that for a closed surface, the integral of the scalar product of the surface area and the electric field is directly proportional to the enclosed charge: \( \oint \vec{E} \cdot d\vec{A} = \frac{q_{\text{enc}}}{\varepsilon_0} \).

Because this problem deals with a uniform spherically symmetric volume charge density, a logical Gaussian surface in this case is a sphere of radius \( r \), with \( r < a \). Find the integral \( \oint \vec{E} \cdot d\vec{A} \) (which will involve \( \vec{E}(\vec{r}) \)) and the enclosed charge. Then solve for \( \vec{E} \).

**Hint 2. Determine the enclosed charge**

What is the charge \( q_{\text{enc}} \) enclosed by a Gaussian sphere centered at the origin with radius \( r \) (for \( r < a \))?

Express your answer in terms of \( r \) (the magnitude of \( \vec{r} \)) and \( \rho \) (Greek letter rho).

**ANSWER:**

\( q_{\text{enc}} = \)

**Hint 3. Calculate the integral over the Gaussian surface**

Because of the symmetry of the problem, the value of \( \vec{E} \cdot d\vec{A} \) is constant over the entire Gaussian surface. In particular, \( \oint \vec{E} \cdot d\vec{A} = E(r)A(r) \), where \( E(r) \) is the magnitude of the electric field at radius \( r \), and \( A(r) \) is the surface area of the Gaussian sphere of radius \( r \). Find an expression for \( A(r) \).

Express your answer in terms of \( r \) and \( \pi \).

**ANSWER:**

\( A(r) = \)

\[ \Phi_E = \oint \vec{E} \cdot d\vec{A} = \oint |\vec{E}| \cdot |d\vec{A}| = |\vec{E}| \cdot |\vec{A}| = |\vec{E}| \cdot 4\pi r^2 \]

\[ = \frac{q_{\text{enc}}}{\varepsilon_0} = \frac{4}{3} \pi r^3 \rho \]

\[ \Rightarrow |\vec{E}| = \frac{1}{4\pi \varepsilon_0} \times \frac{4}{3} \pi r^3 \rho = \frac{r \rho}{3\varepsilon_0} \]
Part B

A spherical cavity is excised from the inside of the sphere. The cavity has radius \( \frac{a}{4} \) and is centered at position \( \vec{h} \), where \( |\vec{h}| < \frac{3}{4} a \), so that the entire cavity is contained within the larger sphere. Find the electric field inside the cavity.

Express your answer as a vector in terms of any or all of \( \rho \) (Greek letter rho), \( \varepsilon_0 \), \( \vec{r} \), and \( \vec{h} \).

---

**Hint 1. How to approach the problem**

Use the principle of superposition. A region of zero charge behaves just like a region with equal amounts of positive and negative charge. Consider the field produced by an imaginary sphere the size of the cavity, with charge density opposite that of the larger sphere. If you add the field from the imaginary sphere to the field produced by the original, intact, sphere, you will obtain the field produced by the sphere with the cavity.

**Hint 2. Find the field due to the imaginary sphere**

Consider an imaginary sphere with charge density \( -\rho \) and radius \( \frac{a}{4} \) centered at \( \vec{h} \). Ignoring the actual sphere, what is the field \( \vec{E}_{\text{imag}}(\vec{r}) \) inside the imaginary sphere?

Express your answer as a vector in terms of \( \vec{r} \), \( \vec{h} \), \( \rho \) (Greek letter rho) and \( \varepsilon_0 \).

**Hint 1. Use the result for a uniformly charged sphere**

The imaginary sphere is just like the uniformly charged sphere studied in Part A of this problem, except that it has a different charge density and different position. Therefore, you can use the result you already obtained from the uniformly charged sphere if you use the new charge density and if you replace \( \vec{r} \) with a new vector \( \vec{s} \) that represents the displacement from the center of the imaginary sphere to \( \vec{r} \).

**What is \( \vec{s} \) in terms of \( \vec{r} \) and \( \vec{h} \)?**

**ANSWER:**
A Conducting Shell around a Conducting Rod

An infinitely long conducting cylindrical rod with a positive charge \( \lambda \) per unit length is surrounded by a conducting cylindrical shell (which is also infinitely long) with a charge per unit length of \(-2\lambda\) and radius \( r_1 \), as shown in the figure.

Part A

What is \( E(r) \), the radial component of the electric field between the rod and cylindrical shell as a function of the distance \( r \) from the axis of the cylindrical rod?

Express your answer in terms of \( \lambda \), \( r \), and \( \epsilon_0 \), the permittivity of free space.
ANSWER:

\[ E(r) = \frac{1}{2\pi\varepsilon_0} \frac{\lambda}{r} \]

\[ \Phi_E = \oint \vec{E} \cdot d\vec{A} = \oint E_r r d\phi = \oint E_r L = \frac{Q_{shell}}{\varepsilon_0} = \frac{L \cdot \lambda}{\varepsilon_0} \]

\[ E \mid = \frac{1}{2\pi\varepsilon_0} \frac{\lambda}{r} \]

Part B

What is \( \sigma_{\text{inner}} \), the surface charge density (charge per unit area) on the inner surface of the conducting shell?

You did not open hints for this part.

ANSWER:

\[ \sigma_{\text{inner}} = -\frac{\lambda}{2\pi r_1} \]

\[ \Phi_E = \oint \vec{E} \cdot d\vec{A} = 0 = \frac{Q_{\text{inner}}}{\varepsilon_0} = \frac{L \cdot \lambda_{\text{rod}} + L \cdot \lambda_{\text{inner}}}{\varepsilon_0} \]

\[ \Rightarrow \lambda_{\text{rod}} + \lambda_{\text{inner}} = \frac{\lambda}{2} \]

\[ \sigma_{\text{inner}} = \frac{Q_{\text{inner}}}{A_{\text{inner}}} = \frac{L \cdot (-\lambda)}{2\pi r_1 \cdot L} \]

Part C

What is \( \sigma_{\text{outer}} \), the surface charge density on the outside of the conducting shell? (Recall from the problem statement that the conducting shell has a total charge per unit length given by \(-2\lambda\).)

You did not open hints for this part.

ANSWER:

\[ \lambda_{\text{shell}} = \lambda_{\text{inner}} + \lambda_{\text{outer}} \]

\[ -2\lambda \quad -\lambda \]

\[ \Rightarrow \lambda_{\text{outer}} = -\lambda \]

\[ \sigma_{\text{outer}} = \sigma_{\text{inner}} = \frac{Q_{\text{outer}}}{A_{\text{outer}}} = \frac{L \cdot (-\lambda)}{2\pi r_1 L} = \frac{\lambda}{2\pi r_1} \]

Part D

What is the radial component of the electric field, \( E(r) \), outside the shell?

You did not open hints for this part.

ANSWER:

\[ \Phi_E = \oint \vec{E} \cdot d\vec{A} = \oint E_r r d\phi = \oint E_r L = \frac{Q_{\text{outer}}}{\varepsilon_0} = \frac{L \cdot \lambda_{\text{rod}} + L \cdot \lambda_{\text{shell}}}{\varepsilon_0} \]

\[ = \frac{L \cdot \lambda + L \cdot (-2\lambda)}{\varepsilon_0} \]

\[ = -\frac{L \lambda}{\varepsilon_0} \]
\[ E(r) = \frac{-1}{2\pi \varepsilon_0} \frac{e}{r} \]

Exercise 22.3

You measure an electric field of \(1.30 \times 10^6 \text{N/C}\) at a distance of 0.147 m from a point charge. There is no other source of electric field in the region other than this point charge.

Part A

What is the electric flux through the surface of a sphere that has this charge at its center and that has radius 0.147 m?

**ANSWER:**

\[ \Phi = 3.5 \times 10^5 \text{ N} \cdot \text{m}^2/\text{C} \]

Part B

What is the magnitude of the charge?

**ANSWER:**

\[ q = 3.1 \times 10^{-6} \text{ C} \]

Exercise 22.12

The nuclei of large atoms, such as uranium, with 92 protons, can be modeled as spherically symmetric spheres of charge. The radius of the uranium nucleus is approximately \(7.4 \times 10^{-15} \text{ m}\).

Part A

What is the electric field this nucleus produces just outside its surface?

**Express your answer using two significant figures.**

**ANSWER:**
\[ E = 2.4 \times 10^4 \quad \text{N/C} \]

\[ E = \frac{k}{r^2} \]
\[ = \frac{9 \times 10^9 \times 9.2 \times 1.6 \times 10^{-9}}{(1.7 \times 10^{-10})^2} \]
\[ = 2.4 \times 10^3 \text{ N/C} \]

**Part B**

What magnitude of electric field does it produce at the distance of the electrons, which is about \(1.7 \times 10^{-10}\) m?

Express your answer using two significant figures.

Answer:

\[ E = 4.6 \times 10^2 \quad \text{N/C} \]

**Part C**

The electrons can be modeled as forming a uniform shell of negative charge. What net electric field do they produce at the location of the nucleus?

Express your answer using two significant figures.

Answer:

\[ E_{\text{net}} = 0 \quad \text{N/C} \]

**PSS 22.1: Gauss’s Law**

**Learning Goal:**

To practice Problem-Solving Strategy 22.1: Gauss’s Law.

An infinite cylindrical rod has a uniform volume charge density \(\rho\) (where \(\rho > 0\)). The cross section of the rod has radius \(r_0\). Find the magnitude of the electric field \(E\) at a distance \(r\) from the axis of the rod. Assume that \(r < r_0\).

**Problem-Solving Strategy 22.1: Gauss’s Law**

**IDENTIFY the relevant concepts:**

Gauss’s law is most useful in situations where the charge distribution has spherical or cylindrical symmetry or is distributed uniformly over a plane. In these situations we determine the direction of \(\vec{E}\) from the symmetry of the charge distribution. If we are given the charge distribution, we can use Gauss’s law to find the magnitude of \(\vec{E}\). Alternatively, if we are given the field, we can use Gauss’s law to determine the details of the charge distribution. In either case, begin your analysis by asking the question: What is the symmetry?

**SET UP the problem using the following steps:**
1. Select the surface that you will use with Gauss's law. We often call it a Gaussian surface. If you are trying to find the field at a particular point, then that point must lie on your Gaussian surface.
2. The Gaussian surface does not have to be a real physical surface. Often the appropriate surface is an imaginary geometric surface.
3. If the charge distribution has cylindrical or spherical symmetry, choose the Gaussian surface to be a coaxial cylinder or a concentric sphere, respectively.

EXECUTE the solution as follows:

1. Carry out the integral
   \[ \Phi_E = \oint E \cos \phi \, dA = \oint E_\perp \, dA = \oint \vec{E} \cdot d\vec{A} = \frac{Q_{\text{enc}}}{\varepsilon_0} \]
   The symmetry of the charge distribution and your careful choice of a Gaussian surface make the integration straightforward.
2. Often you can think of the closed Gaussian surface as being made up of several separate surfaces. The integral \( \oint E_\perp \, dA \) over the entire closed surface is always equal to the sum of the integrals over all the separate surfaces.
3. If \( \vec{E} \) is perpendicular (normal) at every point to a surface with area \( A \), if it points outward from the interior of the surface, and if it also has the same magnitude at every point on the surface, then \( E_\perp = E = \text{constant} \), and \( \int E_\perp \, dA \) over that surface is equal to \( EA \). If instead \( \vec{E} \) is perpendicular and inward, then \( E_\perp = -E \) and \( \int E_\perp \, dA = -EA \).
4. If \( \vec{E} \) is tangent to a surface at every point, then \( E_\perp = 0 \) and the integral over that surface is zero.
5. If \( \vec{E} = 0 \) at every point on a surface, the integral is zero.
6. In \( \oint E_\perp \, dA \), \( E_\perp \) is always the perpendicular component of the total electric field at each point on the closed Gaussian surface. In general, this field may be caused partly by charges within the surface and partly by charges outside it. Even when there is no charge within the surface, the field at points on the Gaussian surface is not necessarily zero. In that case, however, the integral over the Gaussian surface—that is, the total electric flux through the Gaussian surface—is always zero.
7. Once you have evaluated the integral, use the equation \( \Phi_E = \oint \vec{E} \cdot d\vec{A} = Q_{\text{enc}} / \varepsilon_0 \) to solve for your target variable.

EVALUATE your answer:
Often your result will be a function that describes how the magnitude of the electric field varies with position. Examine this function with a critical eye to see whether it makes sense.

IDENTIFY the relevant concepts
The charge distribution described in this problem has cylindrical symmetry. The electric field must point away from the positive charges distributed on the rod. Since you are given information on the charge distribution, use Gauss's law to determine the magnitude of \( \vec{E} \) at the required position.

SET UP the problem using the following steps

Part A
Considering the symmetry of the charge distribution, choose one of the following options as the most appropriate choice of Gaussian surface to use in this problem.

ANSWER:
Part B

In which direction is the electric field on the cylindrical Gaussian surface?

Check all that apply.

ANSWER:

- perpendicular to the curved wall of the cylindrical Gaussian surface
- tangential to the curved wall of the cylindrical Gaussian surface
- perpendicular to the flat end caps of the cylindrical Gaussian surface
- tangential to the flat end caps of the cylindrical Gaussian surface

EXECUTE the solution as follows

Part C

Find the magnitude $E$ of the electric field at a distance $r$ from the axis of the cylinder for $r < r_0$.

Remember that we've chosen the label $l$ to represent the length of the cylindrical Gaussian surface.

Express your answer in terms of some or all of variables $\rho$, $r$, $r_0$, $l$, and $\varepsilon_0$.

**Hint 1.** How to approach the problem

Apply Gauss's law: First calculate the net electric flux (as a function of the given variables) through the cylindrical Gaussian surface described in Part A by evaluating $\oint E_\perp \ dA$; then make the flux equal to the total charge enclosed by the Gaussian surface divided by $\varepsilon_0$, and solve for the electric field. Keep in mind that we've chosen the label $l$ to represent the length of the cylindrical Gaussian surface. Use this notation through the remainder of this problem, however, note that your final answer should not depend on $l$.

**Hint 2.** Find the net electric flux by evaluating $\oint E_\perp \ dA$ over the Gaussian surface

Find the net electric flux $\Phi_e$ through the Gaussian cylinder of length $l$ by evaluating the integral $\oint E_\perp \ dA$. The information found in Part B will make the integration straightforward.
Express your answer in terms of some or all of the variables $E$, $r$, $l$, and appropriate constants.

**Hint 1.** Evaluating surface integrals and fluxes

You can think of the closed Gaussian cylinder as being made up of three separate surfaces: the curved wall and the two end caps. The integral $\oint E_{\perp} \, dA$ over the entire closed cylindrical surface must, then, be equal to the sum of the integrals over the curved wall and each end cap. To evaluate these integrals, make use of the information found in Part B and review points 3 and 4 in the strategy above.

**Hint 2.** Evaluate $\oint E_{\perp} \, dA$ over the curved cylindrical wall

As you found in Part B, the electric field is perpendicular to the curved wall of your Gaussian surface and point outward. If the surface area of the curved wall is $A$, what is $\oint E_{\perp} \, dA$ calculated over $A$? Use $E$ for the magnitude of the electric field.

Express your answer in terms of some or all of the variables $E$ and $A$.

**ANSWER:**

\[
\oint E_{\perp} \, dA =
\]

**Hint 3.** Find the surface area of the curved cylindrical wall

Find the surface area $A$ of the curved wall of your cylindrical Gaussian surface.

Express your answer in terms of some or all of the variables $r$ and $l$ and appropriate constants.

**ANSWER:**

\[
A =
\]

**Hint 4.** Evaluate $\oint E_{\perp} \, dA$ over the end caps of the Gaussian cylinder

As you found in Part B, the electric field is tangent to the flat cap ends of your Gaussian surface. If the surface area of each cap end is $A$, evaluate $\oint E_{\perp} \, dA$ over $A$.

Express your answer in terms of some or all of the variables $E$ and $A$.

**ANSWER:**

\[
\oint E_{\perp} \, dA =
\]
ANSWER:

\[ \Phi_0 = \oint E_{\perp} \, dA = \]

**Hint 3.** Find the charge enclosed

Considering that the rod has a uniform volume charge density \( \rho \), find the charge \( Q_{\text{encl}} \) enclosed by the Gaussian cylinder of length \( l \).

Express your answer in terms of some or all of the variables \( \rho \), \( r \), \( l \), and appropriate constants.

**Hint 1.** Find the volume enclosed

Find the volume \( V \) enclosed by the Gaussian surface.

Express your answer in terms of some or all of the variables \( r \), \( l \), and appropriate constants.

ANSWER:

\[ V = \]

\[ \vec{E}_E = \oint E \cdot dA = |\vec{E}| \cdot 2\pi r \cdot l \]

\[ = \frac{Q_{\text{encl}}}{\varepsilon_0} = \frac{V \cdot \rho}{\varepsilon_0} = \frac{\pi r^2 \cdot l \cdot \rho}{\varepsilon_0} \]

ANSWER:

\[ Q_{\text{encl}} = \]

\[ |\vec{E}| = \frac{\gamma \rho}{2 \varepsilon_0} \]

ANSWER:

\[ E = \frac{\gamma \rho}{2 \varepsilon_0} \]

EVALUATE your answer

**Part D**

If you repeated your calculation from Part C for \( r = r_0 \), you would find that the magnitude of the electric field on the surface of the rod is

\[ E_{\text{surface}} = \rho \frac{r_0}{2\varepsilon_0} \]
Now rewrite the expression for $E_{\text{surface}}$ in terms of $\lambda$, the linear charge density on the rod.

Express your answer in terms of $\lambda$, $r_0$, and $\varepsilon_0$. Your answer should not contain the variable $\rho$.

**Hint 1.** Find $\lambda$ in terms of $\rho$

Find the linear charge density $\lambda$ on the rod in terms of the rod's volume charge density $\rho$.

Express your answer in terms of $\rho$ and $r_0$.

**Hint 1.** Charge and charge density

Consider a segment of the rod of length $l$. The volume of that segment is $\pi r_0^2 l$, and therefore the charge contained within the segment is $Q_i = \rho \pi r_0^2 l$. The charge $Q_1$ is also equal to the linear charge density times the length of the segment: $Q_i = \lambda l$. Combining these two formulas for $Q_1$ will lead to the desired answer.

\[
\rho = \frac{Q}{V} = \frac{\lambda l}{\pi r_0^2 l} = \frac{\lambda}{\pi r_0^2}
\]

**ANSWER:**

\[
\lambda = \frac{\rho \pi r_0^2}{l}
\]

**ANSWER:**

\[
E_{\text{surface}} = \frac{\rho r_0}{2\varepsilon_0} = \frac{\lambda}{\pi r_0^2} \cdot \frac{r_0}{2\varepsilon_0} = \frac{1}{2\pi \varepsilon_0} \frac{\lambda}{r_0}
\]

---

**Exercise 22.5**

A hemispherical surface with radius $r$ in a region of uniform electric field $\vec{E}$ has its axis aligned parallel to the direction of the field.

**Part A**

Calculate the flux through the surface.

Express your answer in terms of the given quantities and appropriate constants.

**ANSWER:**

\[
|\Phi| = |\vec{E}| \cdot \pi r^2
\]
Calculating Flux for Hemispheres of Different Radii

Learning Goal:

To understand the definition of electric flux, and how to calculate it.

Flux is the amount of a vector field that "flows" through a surface. We now discuss the electric flux through a surface (a quantity needed in Gauss's law): \( \Phi_E = \int \vec{E} \cdot d\vec{A} \), where \( \Phi_E \) is the flux through a surface with differential area element \( d\vec{A} \), and \( \vec{E} \) is the electric field in which the surface lies. There are several important points to consider in this expression:

1. It is an integral over a surface, involving the electric field at the surface.
2. \( d\vec{A} \) is a vector with magnitude equal to the area of an infinitesimal surface element and pointing in a direction normal (and usually outward) to the infinitesimal surface element.
3. The scalar (dot) product \( \vec{E} \cdot d\vec{A} \) implies that only the component of \( \vec{E} \) normal to the surface contributes to the integral. That is, \( \vec{E} \cdot d\vec{A} = |\vec{E}| |d\vec{A}| \cos(\theta) \), where \( \theta \) is the angle between \( \vec{E} \) and \( d\vec{A} \).

When you compute flux, try to pick a surface that is either parallel or perpendicular to \( \vec{E} \), so that the dot product is easy to compute.

Two hemispherical surfaces, 1 and 2, of respective radii \( r_1 \) and \( r_2 \), are centered at a point charge and are facing each other so that their edges define an annular ring (surface 3), as shown. The field at position \( \vec{r} \) due to the point charge is:

\[
\vec{E}(\vec{r}) = \frac{C}{r^2} \hat{r}
\]

where \( C \) is a constant proportional to the charge, \( r = |\vec{r}| \), and \( \hat{r} = \vec{r}/r \) is the unit vector in the radial direction.

Part A

What is the electric flux \( \Phi_3 \) through the annular ring, surface 3?

Express your answer in terms of \( C, r_1, r_2, \) and any constants.

You did not open hints for this part.

ANSWER:
\[ \Phi_3 = 0 \quad (\vec{E} \perp d\vec{A} \text{ on surface } 3) \]

**Part B**

What is the electric flux \( \Phi_1 \) through surface 1?

Express \( \Phi_1 \) in terms of \( C, r_1, r_2 \), and any needed constants.

You did not open hints for this part.

**ANSWER:**

\[ \Phi_1 = 2\pi C \]

**Part C**

What is the electric flux \( \Phi_2 \) passing outward through surface 2?

Express \( \Phi_2 \) in terms of \( r_1, r_2, C \), and any constants or other known quantities.

You did not open hints for this part.

**ANSWER:**

\[ \Phi_2 = 2\pi C \]

---

**Flux out of a Cube**

A point charge of magnitude \( q \) is at the center of a cube with sides of length \( L \).

**Part A**

What is the electric flux \( \Phi \) through each of the six faces of the cube?

Use \( \varepsilon_0 \) for the permittivity of free space.
You did not open hints for this part.

**ANSWER:**

\[ \Phi = \frac{\mathbf{E}}{6 \varepsilon_0} \quad \text{N} \cdot \text{m}^2 / \text{C} \]

\[ \Phi = \frac{Q_{\text{enc}}}{\varepsilon_0} \]

\[ = \frac{Q}{\varepsilon_0} \]

*6 faces are identical and equal distance to \( Q \).*

\[ \Phi_{\text{each face}} = \frac{\Phi}{6} = \frac{\mathbf{E}}{6 \varepsilon_0} \]

**Part B**

What would be the flux \( \Phi_1 \) through a face of the cube if its sides were of length \( L_1 \)?

Use \( \varepsilon_0 \) for the permittivity of free space.

You did not open hints for this part.

**ANSWER:**

\[ \Phi_1 = \frac{\mathbf{E}}{6 \varepsilon_0} \quad \text{N} \cdot \text{m}^2 / \text{C} \]

---

**Gauss's Law**

**Learning Goal:**

To understand the meaning of the variables in Gauss’s law, and the conditions under which the law is applicable.

Gauss’s law is usually written

\[ \Phi_E = \oint \mathbf{E} \cdot d\mathbf{A} = \frac{q_{\text{enc}}}{\varepsilon_0} , \]

where \( \varepsilon_0 = 8.85 \times 10^{-12} \text{ C}^2 / (\text{N} \cdot \text{m}^2) \) is the permittivity of vacuum.

---

**Part A**

How should the integral in Gauss’s law be evaluated?

**ANSWER:**

- around the perimeter of a closed loop
- over the surface bounded by a closed loop
- over a closed surface
Part B

In Gauss's law, to what does \( q_{\text{enc}} \) refer?

**ANSWER:**

- [x] the net charge inside the closed surface
- [ ] the charge residing on insulators inside the closed surface
- [ ] all the charge in the physical system
- [ ] any charge inside the closed surface that is arranged symmetrically

---

**The Electric Field inside and outside a Charged Insulator**

A slab of insulating material of uniform thickness \( d \), lying between \(-d/2\) to \(+d/2\) along the x axis, extends infinitely in the y and z directions, as shown in the figure. The slab has a uniform charge density \( \rho \). The electric field is zero in the middle of the slab, at \( x = 0 \).

---

Part A

Which of the following statements is true of the electric field \( \vec{E} \) at the surface of one side of the slab?

**ANSWER:**

- [ ] The direction of \( \vec{E} \) is constant but its magnitude varies across the surface.
- [x] Both the magnitude and the direction of \( \vec{E} \) are constant across the entire surface.
- [ ] The direction of \( \vec{E} \) varies across the surface but its magnitude is constant.
- [ ] Both the magnitude and the direction of \( \vec{E} \) vary across the surface.

---

Part B
What is the angle $\theta$ that the field $\vec{E}$ makes with the surface of the slab, which is perpendicular to the $x$ direction?

**Express your answer in radians.**

**ANSWER:**

$$\theta = \frac{\pi}{2} \text{ rad}$$

**Part C**

What is $E_{\text{out}}$, the magnitude of the electric field outside the slab?

As implied by the fact that $E_{\text{out}}$ is not given as a function of $x$, this magnitude is constant everywhere outside the slab, not just at the surface.

**Express your answer in terms of $d$, $\rho$, and $\varepsilon_0$.**

**Hint 1. How to approach the problem**

Apply Gauss's law. You already know, by symmetry, that the electric field is perpendicular to the slab, and that it has constant magnitude across the surface of the slab. Use this information to choose an appropriate Gaussian surface.

**Hint 2. Gauss's law**

Gauss's law can be written as $\Phi_E = q/\varepsilon_0$, where $\Phi_E$ is the electric flux through a Gaussian surface (which depends on $E_{\text{out}}$), and $q$ is the total charge enclosed by the surface.

**Hint 3. A Gaussian surface for this problem**

Construct a Gaussian surface that is a rectangular box with sides oriented parallel to the electric field. There are two equally good choices for where to place the ends of the box:

- Put one end of the box outside the slab, perpendicular to the electric field, and the other end exactly in the middle of the slab (where the electric field is zero).
- Put one end of the box outside the slab and the other end an equal distance on the other side of the slab, both perpendicular to the electric field. By symmetry, the field at both ends of the...
box must be equal in magnitude and opposite in direction. Both choices will lead to the correct answer, but the remaining hints will assume that you have chosen a Gaussian surface with one end in the middle of the slab.

**Hint 4. Calculate the enclosed charge**

What is \( q \), the total charge enclosed within the Gaussian surface?

**Express your answer in terms of \( d \), \( \rho \), and the cross-sectional area \( A \) of the Gaussian surface.**

**Hint 1. How to find the enclosed charge**

The enclosed charge is equal to the volume of the slab that is contained within the Gaussian surface times the charge density \( \rho \).

**ANSWER:**

\[
q = \ldots
\]

**Hint 5. Compute the electric flux**

Let \( E_{\text{out}} \) denote the magnitude of the electric field at the end of the Gaussian surface outside the slab. What is \( \Phi_E \), the total electric flux through the Gaussian surface?

**Express your answer in terms of \( E_{\text{out}} \) and the cross-sectional area \( A \) of the Gaussian surface.**

**Hint 1. How to compute the flux**

The sides of the box that we have chosen as our Gaussian surface are parallel to the electric field. The ends of the box are perpendicular to the field. Where the electric field is perpendicular to the surface, the flux is given by \( EA \), where \( E \) is the magnitude of the electric field and \( A \) is the area of the surface. Where the electric field is parallel to the surface, the flux is zero.

**ANSWER:**
\[ \Phi_E = \oint \mathbf{E} \cdot d\mathbf{A} \]
\[ = |\mathbf{E}| \cdot A + |\mathbf{E}^\prime| \cdot A \]
\[ = 2|\mathbf{E}| \cdot A \]
\[ = \frac{Q_{\text{enc}}}{\varepsilon_0} \]
\[ = \frac{A \cdot d \cdot \mathbf{p}}{\varepsilon_0} \]
\[ \Rightarrow \mathbf{E} = \frac{d \cdot \mathbf{p}}{2\varepsilon_0} \]

**Part D**

What is \( E_{\text{in}}(x) \), the magnitude of the electric field inside the slab as a function of \( x \)?

**Hint 1. How to approach the problem**

As in the calculation of the field outside the slab, apply Gauss’s law. The procedure is exactly the same as before, but you will need to choose a different Gaussian surface.

**Hint 2. A Gaussian surface for this problem**

Try using a Gaussian surface in the shape of a box with cross-sectional area \( A \). Put one side of the box in the middle of the slab \((x = 0)\), where the electric field is zero. Put the other end of the box at some arbitrary location \( x \) inside the slab to determine the field \( \mathbf{E}_{\text{in}}(x) \) at that position.

**Hint 3. Calculate the enclosed charge**

What is \( q \), the charge enclosed within the Gaussian surface?

Express your answer in terms of the cross-sectional area of the Gaussian surface \( A \), \( \rho \), and \( x \).

**ANSWER:**

\[ q = \]

**Hint 4. Compute the flux**

Let \( E_{\text{in}} \) denote the magnitude of the electric field inside the slab at position \( x \). What is \( \Phi_E \), the total electric flux through the Gaussian surface?

Express your answer in terms of \( E_{\text{in}} \) and \( A \).

**ANSWER:**
The Electric Field of a Ball of Uniform Charge Density

A solid ball of radius $r_b$ has a uniform charge density $\rho$.

**Part A**

What is the magnitude of the electric field $E(r)$ at a distance $r > r_b$ from the center of the ball?

Express your answer in terms of $\rho$, $r_b$, $r$, and $\varepsilon_0$.

You did not open hints for this part.

**ANSWER:**

$$E(r) = \frac{r_b^3 \rho}{3 \varepsilon_0} \cdot \frac{1}{r^2}$$

$$\Rightarrow |E| = \frac{r_b^3 \rho}{3 \varepsilon_0} \cdot \frac{1}{r^2}$$

**Part B**

What is the magnitude of the electric field $E(r)$ at a distance $r < r_b$ from the center of the ball?

Express your answer in terms of $\rho$, $r$, $r_b$, and $\varepsilon_0$.

You did not open hints for this part.

**ANSWER:**

$$E(r) = \frac{\rho}{3 \varepsilon_0}$$

$$\Rightarrow |E| = \frac{\rho}{3 \varepsilon_0}$$
Part C

Let $E(r)$ represent the electric field due to the charged ball throughout all of space. Which of the following statements about the electric field are true?

Check all that apply.

You did not open hints for this part.

ANSWER:

- $E(0) = 0$.
- $E(r_b) = 0$.
- $\lim_{r \to \infty} E(r) = 0$.
- The maximum electric field occurs when $r = 0$.
- The maximum electric field occurs when $r = r_b$.
- The maximum electric field occurs as $r \to \infty$.

Score Summary:

Your score on this assignment is 0.0%.
You received 0 out of a possible total of 15 points.